



AN EOQ MODEL FOR THREE PARAMETER WEIBULL DETERIORATING ITEM WITH PARTIAL BACKLOGGING

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ABSTRACT. Background: Business organisations are facing a lot of competition during these days. To withstand the competition and to remain in the front row, an enterprise should have optimum profitable plan for his business. Researchers in recent years have developed various inventory models for deteriorating items considering various practical situations. Partial backlogging is considerably a new concept introduced in developing various models for Weibull deteriorating items.

Methodology: In this paper an inventory model has been developed considering three parameter Weibull deterioration of a single item with partial backlogging. Here demand rate is considered to be constant and lead time is zero. During the stock out period the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment.

Results and conclusion: Optimal order quantity and total variable cost during a cycle has been derived for the proposed inventory model considering three parameter Weibull deteriorating item with partial backlogging. The results obtained in this paper are illustrated with the help of a numerical example and sensitivity analysis..

Key words: EOQ, Weibull deterioration, partial backlogging.

INTRODUCTION

It is generally observed in various markets and super markets that the demand rate is usually influenced by the amount of the stock level. When a business runs out of stock we consider it as a negative aspect, but in certain situations this shortage may actually prove beneficial for the business, because backlogging of demand allows him to order a larger lot size and hence allows for a larger cycle inventory and reduces cost for him. Partial backlogging is a function of waiting time where as complete backlogging is independent of waiting time. Looking at various situations researchers have studied in this direction and have developed various inventory models. Optimal pricing and lot-

sizing under conditions of perishability and partial backordering was developed by Abad [1996] and Abad [2001]. Chang and Dye [1999] studied an EOQ model for deteriorating items with time varying demand and partial backlogging. Then Dye, C. [2007-a], developed a model of optimal selling price and lot-size with a varying rate of deterioration and exponential partial backlogging. A deterministic inventory model for deteriorating items with capacity constraint and time proportional backlogging rate was studied by Dye [2007] et. al. Then Hung, K., [2011] developed an inventory model with generalized type demand, deterioration and back order rates. Earlier Park, K.S. [1982] proposed an inventory model with partial back orders and Jalan [1996] et.al studied an EOQ model for items with Weibull distribution deterioration, shortages and trended demand.

Ouyang et.al [2006] have established a model considering optimal ordering policy for deteriorating items with partial backlogging under permissible delay in payments. Shah, N. and Shukla, K., [2009] studied a deteriorating inventory model for waiting time partial backlogging. An EOQ inventory model with Weibull distribution deterioration, ramp type demand and partial backlogging was studied by Singh, S.R. and Singh, T.J. [2007]. Then Singh, S.R. and Singh, C., [2008] established a Perishable inventory model with quadratic demand, partial backlogging and permissible delay in payments. Skouri, et.al [2009] developed an inventory model with ramp type demand rate, partial backlogging and Weibull deterioration rate. Teng et.al [2007], made a comparison between two pricing and lot-sizing models with partial back logging and deteriorated items. Tripathy C.K. and Pradhan L.M., [2010], developed an EOQ model for Weibull deteriorating items with power demand and partial back logging. In that paper they have allowed shortages which are partially backlogged. Tripathy, C. K., and Mishra, U., [2010] have studied an inventory model with time dependent linear deteriorating items with partial backlogging. Wu and Cheng, [2005] established an inventory model for deteriorating items with exponential declining demand and Partial back logging.

In the present paper an economic ordered quantity model has been developed considering three parameter Weibull deterioration where shortages are allowed and are partially backlogged. The holding cost and demand rate are assumed to be constant for this model. In section 2 assumptions and notations required for the development of the model are given. The optimum cycle time, holding cost, optimal ordered quantity and total average cost of the model are derived in the Section 3. An illustrative numerical example, sensitivity analysis and conclusion are given in section 4, 5 and 6 respectively.

BASIC ASSUMPTIONS AND NOTATIONS

The following are the assumptions required for development of the model:

1. The model deals with a single item.
2. Demand rate for the product is known and constant.
3. Planning horizon is infinite.
4. Lead time is zero.
5. Once a unit of the product is produced, it is available to meet the demand.
6. The backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. For the negative inventory the backlogging rate is
$$B(t) = \frac{1}{1 + \delta(T - t)},$$
 $\delta > 0$ denotes the backlogging parameter.
7. Deterioration rate is a three parameter Weibull

The notations that are employed here:

- A : ordering cost per order.
 a : constant demand rate.
 C : Purchase cost per unit.
 h : Inventory holding cost per unit per unit time.
 θ : Weibull three parameter deterioration rate (unit/unit time), $\theta = \alpha \beta (t - \gamma)^{\beta-1}$, where $0 < \alpha \ll 1$, $\beta > 0$, and $0 < \gamma < 1$, where α is called scale parameter β is called shape parameter and γ is called the location parameter.
 π_b : Backordered cost per unit short per time unit.
 π_l : Cost of lost sales per unit.
 t_1 : The time at which the inventory level reaches zero, $t_1 \geq 0$.
 t_2 : The length of period during which shortages are allowed, $t_2 \geq 0$.
 T : The length of cycle time, i.e. $T = t_1 + t_2$.
 $I_1(t)$: The level of positive inventory at time t , $0 \leq t \leq t_1$
 $I_2(t)$: The level of negative inventory at time t , $t_1 \leq t \leq t_1 + t_2$
 IM : Maximum inventory level of the product during $[0, T]$.
 IB : Maximum backordered units during stock out period.

Q : Order quantity during a cycle of length T ,
 i.e. $Q = IM + IB$.

TC : Total average cost per time unit.

MATHEMATICAL MODEL

The initial inventory level or the maximum level of inventory $I_1(0) = IM$, decreases due to the combined effect of demand and deterioration during the time $[0, t_1]$. Thus, the inventory level of the product at time t over the period $[0, t_1]$ can be represented by the following differential equation

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -a, \quad 0 \leq t \leq t_1$$

Using the value of $\theta = \alpha \beta (t - \gamma)^{\beta-1}$, $0 < \alpha \ll 1$, $\beta > 0$ and $0 < \gamma < 1$ called the scale, shape and location parameter respectively.

$$\frac{dI_1(t)}{dt} + \alpha \beta (t - \gamma)^{\beta-1} I_1(t) = -a, \quad 0 \leq t \leq t_1 \quad (1)$$

Inventory level reaches to zero at time t_1 . After that shortages occur. During the interval $[t_1, t_1 + t_2]$, the inventory level depends on demand and a fraction of demand is backlogged. The state of inventory during $[t_1, t_1 + t_2]$ can be represented by the differential equation,

$$\frac{dI_2(t)}{dt} = -\frac{a}{1 + \delta(t_1 + t_2 - t)}, \quad t_1 \leq t \leq t_1 + t_2 \quad (2)$$

Here the boundary conditions are $I_1(t_1) = I_2(t_1) = 0$

Equation (1) is a linear differential equation. Its integrating factor is given by

$$= e^{\int \alpha \beta (t - \gamma)^{\beta-1} dt} = e^{\alpha (t - \gamma)^\beta}$$

Hence the solution of equation (1) can be written as

$$I_1(t) e^{\alpha (t - \gamma)^\beta} = \int -a e^{\alpha (t - \gamma)^\beta} dt + c, \quad \text{where 'c', is the constant of integration.}$$

Since, $0 < \alpha \ll 1$ so taking the first two terms from the series expansion of the

exponential function and then integrating we get

$$I_1(t) e^{\alpha (t - \gamma)^\beta} = -a \left(t + \frac{\alpha (t - \gamma)^{\beta+1}}{\beta+1} \right) + c$$

Using the given boundary condition $I_1(t_1) = 0$ in the above we get the required solution of equation (1) as

$$I_1(t) e^{\alpha (t - \gamma)^\beta} = -a \left(t + \frac{\alpha (t - \gamma)^{\beta+1}}{\beta+1} \right) + a \left(t_1 + \frac{\alpha (t_1 - \gamma)^{\beta+1}}{\beta+1} \right) \\ \Rightarrow I_1 = a e^{-\alpha (t - \gamma)^\beta} \left[t_1 - t + \frac{\alpha}{\beta+1} \left((t_1 - \gamma)^{\beta+1} - (t - \gamma)^{\beta+1} \right) \right]$$

Again taking the first two terms from the series expansion of the exponential function and neglecting the higher power of α that is the power greater than or equal to 2, in the above the solution of equation (1) can be rewritten as

$$I_1(t) = a \left[t_1 - t + \frac{\alpha}{\beta+1} \left((t_1 - \gamma)^{\beta+1} - (t - \gamma)^{\beta+1} \right) \right], \\ -\alpha t_1 (t - \gamma)^\beta + \alpha t (t - \gamma)^\beta \\ 0 \leq t \leq t_1 \quad (3)$$

Similarly the solution of equation (2) can be written as

$$I_2(t) = \frac{a}{\delta} \ln \{ 1 + \delta (t_1 + t_2 - t) \} + c$$

Using the boundary condition $I_2(t_1) = 0$ in the above, required solution of equation (2) is

$$I_2(t) = \frac{a}{\delta} \left[\ln \{ 1 + \delta (t_1 + t_2 - t) \} - \ln (1 + \delta t_2) \right], \\ t_1 \leq t \leq t_1 + t_2 \quad (4)$$

The maximum positive inventory is

$$IM = I_1(0) = a \left[t_1 + \frac{\alpha}{\beta+1} \left((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1} \right) - \alpha t_1 (-\gamma)^\beta \right] \quad (5)$$

The maximum backordered units are given by,

$$IB = -I_2(t_1 + t_2) = -\frac{a}{\delta} \left[\ln \{ 1 + \delta (t_1 + t_2 - t_1 - t_2) \} - \ln (1 + \delta t_2) \right] \\ = \frac{a}{\delta} \ln (1 + \delta t_2) \quad (6)$$

The order size during $[0, T]$ is
 $Q = IM + IB$
 $\Rightarrow Q = a \left[t_1 + \frac{\alpha}{\beta+1} ((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}) \right. \\ \left. - \alpha t_1 (-\gamma)^\beta + \frac{1}{\delta} \ln(1 + \delta t_2) \right]$ (7)

Ordering cost per cycle is

$$OC = A$$
 (8)

Inventory holding cost per cycle is

$$IHC = h \int_0^{t_1} I_1(t) dt \\ = ha \int_0^{t_1} \left[t_1 - t + \frac{\alpha}{\beta+1} ((t_1 - \gamma)^{\beta+1} - (t - \gamma)^{\beta+1}) \right. \\ \left. - \alpha t_1 (t - \gamma)^\beta + \alpha t (t - \gamma)^\beta \right] dt \\ = ha \left[\frac{t_1^2}{2} + \frac{\alpha t_1 (t_1 - \gamma)^{\beta+1}}{\beta+1} + \frac{\alpha t_1 (-\gamma)^{\beta+1}}{\beta+1} \right. \\ \left. + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right]$$
 (9)

Backordered cost per cycle is

$$BC = \pi_b \int_{t_1}^{t_1+t_2} -I_2(t) dt \\ = -\pi_b \int_{t_1}^{t_1+t_2} \frac{a}{\delta} [\ln\{1 + \delta(t_1 + t_2 - t)\} - \ln(1 + \delta t_2)] dt \\ = \frac{-\pi_b a}{\delta} [\delta t_2 - (1 + \delta t_2 + t_2) \ln(1 + \delta t_2)]$$
 (10)

Cost due to lost sales per cycle is given by,

$$LS = \pi_l a \int_{t_1}^{t_1+t_2} \left(1 - \frac{1}{1 + \delta(t_1 + t_2 - t)} \right) dt \\ = \frac{\pi_l a}{\delta} [\delta t_2 - \ln(1 + \delta t_2)]$$
 (11)

Purchase cost per cycle is,

$$PC = C \times Q \\ = Ca \left[t_1 + \frac{\alpha}{\beta+1} ((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}) \right. \\ \left. - \alpha t_1 (-\gamma)^\beta + \frac{1}{\delta} \ln(1 + \delta t_2) \right]$$
 (12)

Therefore the total average cost per time unit is

$$TC = \frac{1}{t_1 + t_2} [OC + IHC + BC + LS + PC] \\ = \frac{1}{t_1 + t_2} \left[A + ha \left\{ \frac{t_1^2}{2} + \frac{\alpha t_1 (t_1 - \gamma)^{\beta+1}}{\beta+1} \right. \right. \\ \left. \left. + \frac{\alpha t_1 (-\gamma)^{\beta+1}}{\beta+1} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right\} \right. \\ \left. - \frac{\pi_b a}{\delta} \{\delta t_2 - (1 + \delta t_2 + t_2) \ln(1 + \delta t_2)\} + \frac{\pi_l a}{\delta} \{\delta t_2 - \ln(1 + \delta t_2)\} \right. \\ \left. + Ca \left\{ t_1 + \frac{\alpha}{\beta+1} ((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}) \right. \right. \\ \left. \left. - \alpha t_1 (-\gamma)^\beta + \frac{1}{\delta} \ln(1 + \delta t_2) \right\} \right]$$
 (13)

The necessary condition for the total average cost to be minimized is

$$\frac{\partial TC}{\partial t_1} = 0 \\ \Rightarrow \frac{-1}{(t_1 + t_2)^2} \left[A + ha \left\{ \frac{t_1^2}{2} + \frac{\alpha t_1 (t_1 - \gamma)^{\beta+1}}{\beta+1} \right. \right. \\ \left. \left. + \frac{\alpha t_1 (-\gamma)^{\beta+1}}{\beta+1} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right\} \right. \\ \left. - \frac{\pi_b a}{\delta} \{\delta t_2 - (1 + \delta t_2 + t_2) \ln(1 + \delta t_2)\} + \frac{\pi_l a}{\delta} \{\delta t_2 - \ln(1 + \delta t_2)\} \right. \\ \left. + Ca \left\{ t_1 + \frac{\alpha}{\beta+1} ((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1}) - \alpha t_1 (-\gamma)^\beta + \frac{1}{\delta} \ln(1 + \delta t_2) \right\} \right. \\ \left. + \frac{1}{(t_1 + t_2)} \left[ha \left\{ t_1 + \alpha t_1 (t_1 - \gamma)^\beta - \frac{\alpha(t_1 - \gamma)^{\beta+1}}{\beta+1} + \frac{\alpha(-\gamma)^{\beta+1}}{\beta+1} \right\} \right. \right. \\ \left. \left. + Ca \{1 + \alpha(t_1 - \gamma)^\beta - \alpha(-\gamma)^\beta\} \right] \right] = 0$$
 (14)

and $\frac{\partial TC}{\partial t_2} = 0$

$$\Rightarrow \frac{-1}{(t_1 + t_2)^2} \left[A + ha \left\{ \frac{t_1^2}{2} + \frac{\alpha t_1 (t_1 - \gamma)^{\beta+1}}{\beta+1} + \frac{\alpha t_1 (-\gamma)^{\beta+1}}{\beta+1} \right. \right. \\ \left. \left. + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{2\alpha(t_1 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right\} \right. \\ \left. - \frac{\pi_b a}{\delta} \{\delta t_2 - (1 + \delta t_2 + t_2) \ln(1 + \delta t_2)\} \right. \\ \left. + \frac{\pi_l a}{\delta} \{\delta t_2 - \ln(1 + \delta t_2)\} \right]$$

$$+Ca \left\{ \begin{array}{l} t_1 + \frac{\alpha}{\beta+1} \left((t_1 - \gamma)^{\beta+1} - (-\gamma)^{\beta+1} \right) \\ -\alpha t_1 (-\gamma)^\beta + \frac{1}{\delta} \ln(1 + \delta t_2) \end{array} \right\} + \frac{1}{(t_1 + t_2)} \left[\begin{array}{l} \pi_b a \left\{ \frac{t_2}{1 + \delta t_2} + \left(1 + \frac{1}{\delta} \right) \ln(1 + \delta t_2) \right\} \\ + \frac{\pi_l a \delta t_2}{1 + \delta t_2} + \frac{Ca}{1 + \delta t_2} \end{array} \right] = 0 \quad (15)$$

Provided,

$$\frac{\partial^2 TC}{\partial t_1^2} \times \frac{\partial^2 TC}{\partial t_2^2} - \frac{\partial^2 TC}{\partial t_1 \partial t_2} > 0,$$

for obtained value of (t_1, t_2) . (16)

The equations (14) and (15) are highly non linear. They can be solved by mathematica-5.1 software for a given set of known parameters. The obtained values of t_1 and t_2 must satisfy equation (16) to minimize the total cost per time unit of the inventory system. To illustrate these we have given a numerical example and a sensitivity analysis in the following sections.

NUMERICAL EXAMPLE

Let us consider an inventory system with the following parametric values in their proper units.

$$[A, C, h, \pi_b, \pi_l, a, \delta, \alpha, \beta, \gamma] = [3000, 100, 0.4, 20, 60, 12, 8, 0.08, 4, 0.4]$$

Using these values in equation (14) and (15) we get, $t_1^* = 2.02662$ and $t_2^* = 7.99722$ respectively. Putting the optimum values of t_1^* and t_2^* in equation (7) and (13) we get, $Q^* = 32.7191$ and $TC^* = 1883.92$ respectively.

SENSITIVITY ANALYSIS

For study of sensitivity analysis let us change one parameter at a time keeping the other parameters unchanged. The original values of all the parameters for sensitivity analysis have been taken from the example given in section 4 above. Sensitivity analysis is

performed by changing the values of all the parameters from -50% to +50%, one by one in the model which are given in the following table-1.

From the table-1 we can conclude the following:

The optimal time t_1^* increases as $A, \pi_b, \pi_l, \delta, \gamma$ increases but it decreases as C, h, a, α, β increases. Similarly the optimal time t_2^* increases as C, h, A, α, β increases but it decreases as $\delta, \pi_b, \pi_l, a, \gamma$ increases. Again the optimal ordered quantity Q^* per cycle increases as $A, \pi_b, \pi_l, a, \gamma$ increases but it decreases as $C, h, \delta, \alpha, \beta$ increases. The total average cost TC^* of the system increases as $A, C, h, \pi_b, \pi_l, a, \delta, \alpha, \beta$ increases but it decreases as γ increases.

CONCLUSION

In the present paper an economic ordered quantity model has been developed for an item with three parameter Weibull deterioration where shortages are allowed and are partially backlogged. The optimal cycle time, optimal ordered quantity per cycle and total optimal cost has been derived for the model. Sensitivity analysis shows how the different parameters affect the optimal cycle time, ordered quantity per cycle and total optimal cost. It can be concluded that to minimise the total cost, it is required to minimize the ordering cost, purchase cost, holding cost, back ordered cost, the cost of lost sales per unit, the demand rate, backlogging parameter, scale parameter and shape parameter, whereas we need to maximise the value of the location parameter.

Table 1. sensitivity analysis
Tabela 1. Analiza wrażliwości

Parameter	%change	t_1^*	t_2^*	Q^*	TC^*
A	-50	1.89117	3.63945	29.1727	1684.3
	-25	1.9731	5.74291	31.2537	1798.8
	25	2.06539	10.3503	33.8383	1951.05
	50	2.09533	12.772	34.738	2006.2
C	-50	2.54094	3.27636	44.0224	1636.46
	-25	2.2634	5.60016	37.1571	1785.84
	25	1.78866	10.3209	29.0519	1953.9
	50	1.48812	12.4773	25.0357	2006.01
h	-50	2.03057	7.98001	32.7899	1883.36
	-25	2.0286	7.98862	32.7545	1883.64
	25	2.02465	8.0058	32.6839	1884.2
	50	2.02268	8.01436	32.6487	1884.47
π_b	-50	1.6455	22.2254	28.0624	1437.75
	-25	1.8906	12.6267	30.9932	1683.57
	25	2.11579	5.38981	33.8749	2045.6
	50	2.17794	3.79444	34.6628	2174.26
π_l	-50	1.82391	10.6775	29.6568	1603.36
	-25	1.93729	9.33534	31.3401	1746.49
	25	2.09972	6.68938	33.8686	2014.54
	50	2.16055	5.43897	34.8166	2136.88
a	-50	2.13955	17.7537	18.0635	1046.59
	-25	2.07617	11.151	25.6189	1477.92
	25	1.98545	6.18381	39.48	2272.11
	50	1.95005	5.02146	45.9739	2647.08
δ	-50	2.00394	8.14827	36.5811	1846.8
	-25	2.01406	8.13586	34.0411	1863.16
	25	2.03869	7.83093	31.9329	1904.32
	50	2.04972	7.66401	31.4266	1923.33
α	-50	2.32032	7.51128	36.4918	1867.7
	-25	2.12485	7.80192	34.2163	1877.52
	25	1.94169	8.14224	31.6209	1888.57
	50	1.87552	8.25643	30.7625	1892.19
β	-50	2.99484	7.03768	47.1619	1850.9
	-25	2.29666	7.78465	37.0219	1876.94
	25	1.87709	8.16831	30.4969	1889.4
	50	1.78572	8.25674	29.0756	1892.2
γ	-50	1.83439	8.43644	30.5891	1897.78
	-25	1.93027	8.21756	31.661	1890.96
	25	2.12382	7.76999	33.7506	1876.46
	50	2.22234	7.52859	34.7362	1868.3

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MODEL EKONOMICZNEJ WIELKOŚCI PARTII DLA TRZECH PARAMETRÓW UTRATY WARTOŚCI WEIBULLA Z CZĘŚCIOWYMI ZALEGŁOŚCIAMI

STRESZCZENIE. Wstęp: W obecnych czasach przedsiębiorstwa muszą sprostać wielu wymaganiom stawianym przez konkurencyjny rynek. Aby pokonować konkurencję pozostać liderem, przedsiębiorstwo powinno opierać się na optymalnym planie zyskowności swojej działalności. W ostatnich latach naukowcy opracowali wiele modeli dla asortymentów podlegających zużyciu praktycznie dla każdej występującej w rzeczywistości sytuacji. Częściowe zaległości są nową koncepcją wprowadzoną w rozwijanych modelach Weibulla.

Metody: Model zapasów asortymentu dla którego występują częściowe zaległości został opracowany dla trzech parametrów utraty wartości Weibulla. Założono, że popyt jest stały oraz czas realizacji równa się zero. W okresie braków współczynnik zaległości jest zmienny i zależy od czasu oczekiwania na uzupełnienie zapasów.

Wyniki i wnioski: Optymalna wielkość zamówienia oraz całkowity koszt zmienny w czasie cyklu zostały opracowane dla proponowanego modelu zapasów uwzględniającego trzy parametry zużycia Weibulla z częściowymi zaległościami. Wyniki zostały zaprezentowane przy pomocy przykładu liczbowego oraz analizy wrażliwości.

Słowa kluczowe: ekonomiczna wielkość partii, zużycie Weibulla, częściowe zaległości

MODELL DER WIRTSCHAFTLICHEN LOSGRÖßE EINER BESTELLUNG FÜR DREI PARAMETER DES WEIBULL-WERTVERLUSTES MIT TEILHAFTEN RÜCKSTÄNDEN

ZUSAMMENFASSUNG. Einleitung: Der Wettbewerbsmarkt stellt heutzutage vor Unternehmen viele wichtige Herausforderungen. Um die Konkurrenz zu überholen und die Führungsposition zu bewahren, sollten sich die Unternehmen auf den optimalen Plan der Rentabilität ihrer Betätigung stützen. In den letzten Jahren haben die Wissenschaftler viele Modelle für die Sortimente, die einem Verschleiß unterliegen, praktisch für jede in der Wirklichkeit auftretende Situation ausgearbeitet. Das Modell mit den teilhaften Rückständen stellt ein neues, in den entwickelten Weibull-Modellen eingeführtes Konzept dar.

Methoden: Das Modell der Sortimentsvorräte, bei denen die teilhaften Rückständen auftreten, wurde für die drei Parameter des Weibull-Wertverlustes konzipiert. Man hat angenommen, dass die Nachfrage konstant bleibt und die Ausführungszeit der Bestellung dem Null gleicht. In der Zeit der auftretenden Defizite ist der Koeffizient der Rückstände variabel und hängt von der Wartezeit beim Nachschub der Vorräte ab.

Ergebnisse und Fazit: Die optimale Losgröße der Bestellung sowie die variablen Gesamtkosten innerhalb eines Zyklus wurden für die Zwecke des vorgeschlagenen Vorratsmodells, welches die drei Parameter des Weibull-Wertverlustes mit teilhaften Rückständen berücksichtigt, ausgearbeitet. Die Ergebnisse wurden anhand eines Zahlenbeispiels und der Empfindlichkeitsanalyse dargestellt.

Codewörter: wirtschaftliche Losgröße, Weibull-Wertverlust, teilhafte Rückstände

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