



## A PRODUCTION INVENTORY MODEL FOR AN ITEM WITH THREE PARAMETER WEIBULL DETERIORATION AND PRICE DISCOUNT

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**ABSTRACT. Background:** Deterioration is a natural process for most of the items as such it cannot be ignored in study of inventory control and management. In recent years great deal of study is devoted in developing inventory models for deteriorating items considering various practical situations. Price discount for partially deteriorated items is considerably a new concept introduced in developing various models.

**Methods:** This paper deals with the development of an inventory model for Weibull deteriorating items. Here production and demand rate are considered to be constant and the holding cost per unit is assumed to be constant with respect to time. Completely deteriorated units are discarded and partially deteriorated items are allowed to carry a discount. Shortages are not allowed.

**Results and conclusions:** A Production Inventory model for an item with three parameter Weibull deterioration with price discount for partially deteriorated item have been proposed in this paper. Here the optimal cycle time for the model has been derived and the result is illustrated with the help of numerical example. Sensitivity analysis has been carried out to analyze the changes in the optimal solution with respect to the change in other parameters.

**Key words:** Production quantity model, Weibull deterioration, Price discount.

### INTRODUCTION

One of the important concerns of the inventory management is to decide as to when and how much is to be ordered or manufactured so that the total cost associated with the inventory system can be kept at minimum. When the inventory is subject to significant deterioration, it becomes more important as loss due to deterioration cannot be ignored in this case. Study in this direction have resulted in continuous modification of inventory modelling for deteriorating items by including more and more practical features. The impact of product deterioration should not be neglected in the decision process of production lot size. Researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, medicines, electronic components, fashion goods, fruits, vegetables, etc. An order level inventory model with constant deterioration was developed by Aggarwal [1978]. Earlier some researchers like Ghare [1963] considered exponentially decaying inventory for a constant demand. Optimum production planning for a deteriorating item was developed by Hwang [1986]. A production inventory model for decaying raw materials and a decaying single finished product system was developed by Raffat [1985], [1991]. Optimal pricing and lot sizing under conditions of perishability and partial backordering was studied by Misra [1975] and Abad [1996]. Goyal and Giri [2001] have also given many deteriorating inventory models. Then Economic lot scheduling problem was studied by Gary et.al [2005] and Deng et.al [2007]. Yang and Wee [2003] developed an integrated multilot-size production inventory model. Then Sugapriya and Jeyaraman [2008] studied a common production cycle time for an EPQ model of non instantaneous deteriorating items allowing price discount using permissible delay in payments. Inventory management of time dependent deteriorating with salvage value was developed by Mishra [2008]. Optimal policy for a deteriorating

item with finite replenishment and with price dependent demand rate was studied by Sabahno [2008]. Shah and Acharya [2008] have established a model of time dependent deterioration with exponential demand. An EPQ model under stock dependent demand, Weibull distribution deterioration and shortage was developed by Roy and Choudhuri [2009]. Tripathy C.K. and Pradhan L.M., [2010] developed a Production Inventory model for Weibull deteriorating Items allowing price discount & permissible delay in payments. Tripathy C.K. and Pradhan L.M., [2011] then studied optimal Pricing & Ordering Policy for three parameter Weibull deterioration under trade credit. Tripathy C. K., and Mishra U., [2011] developed an EOQ model with time dependent Weibull deterioration and ramp type demand. Tripathy C.K., Pradhan L.M, and Mishra U, [2010] studied an EPQ Model for Linear deteriorating Item with Variable Holding cost. Meher M.K et.al [2012] have proposed an inventory model with Weibull deterioration rate considering delay in payment for declining market.

In the present paper a production inventory model has been developed considering three parameter Weibull deterioration with price discount for partially deteriorated items. The holding cost is assumed to be constant and shortages are not allowed for this model. In section 2 assumptions and notations required for the development of the model are given. The optimum cycle time, holding cost and total variable cost of the model is derived in the Section 3. An illustrative numerical example, a sensitivity table and conclusion are given in section 4, 5 and 6 respectively.

## BASIC ASSUMPTIONS AND NOTATIONS

The following are the assumptions required for development of the model:

1. The demand rate for the product is known and finite.
2. Shortage is not allowed.
3. Planning horizon is infinite.
4. Once a unit of the product is produced, it is available to meet the demand.
5. Price discount is allowed for partially deteriorated items.
6. There is no replacement or repair for a deteriorated item.

The notations that are employed here:

- $p$  : Production rate per unit time.  
 $d$  : Actual demand of the product per unit time  
 $A$  : Set up cost  
 $\theta$  : Weibull three parameter deterioration rate (unit/unit time),  $\theta = \alpha \beta (t - \gamma)^{\beta-1}$ , where  $0 < \alpha < 1$ ,  $\beta > 1$ ,  $0 < \gamma < 1$ , where  $\alpha$  is called scale parameter and  $\beta$  is called shape parameter and  $\gamma$  is called the location parameter.  
 $h$  : Inventory carrying cost per unit per unit time which is constant.  
 $k$  : Production cost per unit.  
 $l$  : Price discount per unit cost.  
 $T$  : Optimal cycle time.  
 $T_1$  : Production period.  
 $T_2$  : Time during which there is no production. i.e.,  $T_1 = T - T_2$ .  
 $I_1(t)$  : Inventory level for product during the production period, i.e.  $0 \leq t \leq T_1$ .  
 $I_2(t)$  : Inventory level of the product during the period when there is no production i.e.  $T_1 \leq t \leq T_2$ .  
 $I(M)$  : Maximum inventory level of the product.  
 $TVC(T)$  : Total cost/unit time.

## MATHEMATICAL MODEL

At  $t = 0$ , the inventory level is zero. The production and supply start simultaneously and the production stops when the maximum inventory  $I(M)$  is reached at time  $t = T_1$ . During this period of time inventory built up at a rate  $p - d$  and there is no deterioration. After time  $T_1$ , the produced units start deterioration and supply is continued at the discount rate. As the demand remains constant for the product the inventory level reduces to zero and then the production run begins. Thus the inventory level of the product at time  $t$  over the period  $[0, T]$  can be represented by the following differential equations

$$\frac{dI_1(t)}{dt} = p - d \quad 0 \leq t \leq T_1 \quad (1)$$

and

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \quad 0 \leq t \leq T_2 \quad (2)$$

Where  $\theta = \alpha \beta (t - \gamma)^{\beta-1}$ , where  $0 < \alpha < 1$ ,  $\beta > 1$ ,  $0 < \gamma \ll 1$   
 $\alpha$  is called scale parameter and  $\beta$  is called shape parameter and  $\gamma$  is called the location parameter.

Here the boundary conditions are  $I_1(0) = I_2(T_2) = 0$

Solving equation (1) and (2), we get

$$I_1(t) = (p - d)t \quad , 0 \leq t \leq T_1 \quad (3)$$

$$I_2(t) = d \left[ T_2 - t + \frac{\alpha}{\beta+1} (T_2 - \gamma)^{\beta+1} - \frac{\alpha}{\beta+1} (t - \gamma)^{\beta+1} - \alpha T_2 (t - \gamma)^\beta + \alpha t (t - \gamma)^\beta + \alpha (t - \gamma)^{2\beta+1} \right] \quad , 0 \leq t \leq T_2 \quad (4)$$

The **production cost** per unit time is

$$PC = pk \frac{T_1}{T} \quad (5)$$

The **set up cost** per unit time is

$$SC = \frac{A}{T} \quad (6)$$

The **Holding Cost** is

$$HC = \frac{1}{T} \left[ \int_0^{T_1} h(t) I_1(t) dt + \int_0^{T_2} h(t) I_2(t) dt \right] = \frac{1}{T} \left[ h \int_0^{T_1} (p - d) t dt \right] + h d \int_0^{T_2} \left[ T_2 - t + \frac{\alpha}{\beta+1} (T_2 - \gamma)^{\beta+1} - \frac{\alpha}{\beta+1} (t - \gamma)^{\beta+1} - \alpha T_2 (t - \gamma)^\beta + \alpha t (t - \gamma)^\beta + \alpha (t - \gamma)^{2\beta+1} \right] dt$$

Integrating the above we get

$$\begin{aligned}
 HC &= \frac{h(p-d)T_1^2}{2T} + \\
 \frac{hd}{T} &\left[ \frac{T_2^2}{2} - \frac{2\alpha(T_2 - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{\alpha T_2 (T_2 - \gamma)^{\beta+1}}{(\beta+1)} + \frac{\alpha(T_2 - \gamma)^{2\beta+2}}{2(\beta+1)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right] \\
 &+ \frac{hd}{T} \left[ \frac{\alpha T_2 (-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(-\gamma)^{2\beta+2}}{2(\beta+1)} \right] \quad (7)
 \end{aligned}$$

Let us express  $T_1$  and  $T_2$  in terms of  $T$

We know  $I_1(T_1) = I_2(0)$

$$(p-d)T_1 = d \left[ T_2 + \frac{\alpha}{\beta+1} (T_2 - \gamma)^{\beta+1} - \frac{\alpha}{\beta+1} (-\gamma)^{\beta+1} - \alpha T_2 (-\gamma)^\beta + \alpha (-\gamma)^{2\beta+1} \right]$$

Neglecting the terms involving second and higher power of  $\gamma$  as  $0 < \gamma \ll 1$  and  $T_2$  from the right hand side to get a suitable solution, we have

$$(p-d)T_1 = d T_2$$

$$(p-d)(T - T_2) = d T_2$$

$$T_2 = \frac{(p-d)}{p} T = xT \quad , \text{ where, let } x = \frac{p-d}{p} \quad (8)$$

$$T_1 = \frac{dT}{p} \quad (9)$$

Using these values of  $T_1$  and  $T_2$  in equation (7) we get

$$\begin{aligned}
 HC &= \frac{hd xT}{2} - \frac{2\alpha hd (xT - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd \alpha x (xT - \gamma)^{\beta+1}}{(\beta+1)} + \frac{hd \alpha (xT - \gamma)^{2\beta+2}}{2(\beta+1)T} \\
 &+ \frac{2\alpha hd (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd \alpha x (-\gamma)^{\beta+1}}{(\beta+1)} - \frac{hd \alpha (-\gamma)^{2\beta+2}}{2(\beta+1)T} \quad (10)
 \end{aligned}$$

### Deterioration cost

The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given

$$\begin{aligned}
 \text{as } DC &= \frac{k}{T} \left[ I_2(0) - \int_0^{T_2} d dt \right] \\
 &= \frac{k d}{T} \left[ \frac{\alpha (T_2 - \gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha (-\gamma)^{\beta+1}}{(\beta+1)} - \alpha T_2 (-\gamma)^\beta + \alpha (-\gamma)^{2\beta+1} \right] \quad (11)
 \end{aligned}$$

### Price discount

Price discount is offered as a fraction of production cost for the units in the Period  $[0, T_2]$

$$\begin{aligned}
 PD &= \frac{k l T_2}{T} \int_0^{T_2} d dt \\
 &= \frac{k l d T_2}{T}
 \end{aligned} \tag{12}$$

Therefore the **average total cost** per unit time is given by

$$\begin{aligned}
 TVC(T) &= PC + SC + HC + PD + DC \\
 &= \frac{pkT_1}{T} + \frac{A}{T} + \frac{hd xT}{2} - \frac{2\alpha hd(xT - \gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd \alpha x(xT - \gamma)^{\beta+1}}{(\beta+1)} + \frac{hd \alpha (xT - \gamma)^{2\beta+2}}{2(\beta+1)T} \\
 &\quad + \frac{2\alpha hd(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{hd \alpha x(-\gamma)^{\beta+1}}{(\beta+1)} - \frac{hd \alpha (-\gamma)^{2\beta+2}}{2(\beta+1)T} + \frac{k l d T_2}{T} \\
 &\quad + \frac{k d}{T} \left[ \frac{\alpha (T_2 - \gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha (-\gamma)^{\beta+1}}{(\beta+1)} - \alpha T_2 (-\gamma)^\beta + \alpha (-\gamma)^{2\beta+1} + \right]
 \end{aligned} \tag{13}$$

Putting the values of  $T_2$  and  $T_1$  in terms of  $T$  from equation (8) and (9) respectively, equation (13) becomes

$$\begin{aligned}
 TVC(T) &= kd + \frac{A}{T} + \frac{hd xT}{2} - \frac{2\alpha hd}{(\beta+1)(\beta+2)T} \left( (xT - \gamma)^{\beta+2} - (-\gamma)^{\beta+2} \right) \\
 &\quad + \frac{hd \alpha}{2(\beta+1)T} \left( (xT - \gamma)^{2\beta+2} - (-\gamma)^{2\beta+2} \right) + \frac{hd \alpha x}{(\beta+1)} \left( (xT - \gamma)^{\beta+1} + (-\gamma)^{\beta+1} \right) \\
 &\quad + \frac{kd \alpha}{(\beta+1)T} \left( (xT - \gamma)^{\beta+1} - (-\gamma)^{\beta+1} \right) + \frac{\alpha kd (-\gamma)^{2\beta+1}}{T} + k l d x - \alpha k dx (-\gamma)^\beta
 \end{aligned} \tag{14}$$

To find the minimum total cost, we calculate the value of  $T$  from

$$\begin{aligned}
 \frac{d}{dT}(TVC(T)) &= 0 \\
 \Rightarrow \frac{-A}{T^2} + \frac{hd x}{2} - \frac{2\alpha hd}{(\beta+1)(\beta+2)T^2} &\left[ Tx(\beta+2)(xT - \gamma)^{\beta+1} - (xT - \gamma)^{\beta+2} + (-\gamma)^{\beta+2} \right] \\
 &+ \frac{\alpha hd}{2(\beta+1)T^2} \left[ Tx(2\beta+2)(xT - \gamma)^{2\beta+1} - (xT - \gamma)^{2\beta+2} + (-\gamma)^{2\beta+2} \right] \\
 &+ \frac{\alpha kd}{(\beta+1)T^2} \left[ Tx(\beta+1)(xT - \gamma)^\beta - (xT - \gamma)^{\beta+1} + (-\gamma)^{\beta+1} \right] \\
 &+ \alpha hd x^2 (xT - \gamma)^\beta - \frac{\alpha kd (-\gamma)^{2\beta+1}}{T^2} = 0
 \end{aligned} \tag{15}$$

The value of  $T$  calculated from (16) will minimize the  $TVC$  if

$$\begin{aligned} & \frac{d^2}{dT^2}(TVC(T)) > 0 \\ \Rightarrow & \frac{2A}{T^3} - \frac{2\alpha h d x}{(\beta + 1)T^2} [Tx(\beta + 1)(xT - \gamma)^\beta - (xT - \gamma)^{\beta+1}] \\ & + \frac{2\alpha h d}{(\beta + 1)(\beta + 2)T^4} [T^2 x(\beta + 2)(xT - \gamma)^{\beta+1} - 2(xT - \gamma)^{\beta+2}T] + \frac{4\alpha h d (-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T^3} \\ & + \frac{\alpha h d x}{T^2} [Tx(2\beta + 1)(xT - \gamma)^{2\beta} - (xT - \gamma)^{2\beta+1}] \\ - & \frac{\alpha h d}{2(\beta + 1)T^4} [T^2 x(2\beta + 2)(xT - \gamma)^{2\beta+1} - 2(xT - \gamma)^{2\beta+2}T] - \frac{\alpha h d (-\gamma)^{2\beta+2}}{(\beta + 1)T^3} \\ & + \frac{\alpha k d x}{T^2} [Tx\beta(xT - \gamma)^{\beta-1} - (xT - \gamma)^\beta] \\ - & \frac{\alpha k d}{(\beta + 1)T^4} [T^2 x(\beta + 1)(xT - \gamma)^\beta - 2(xT - \gamma)^{\beta+1}T] \\ - & \frac{2\alpha k d (-\gamma)^{\beta+1}}{(\beta + 1)T^3} + \alpha h d \beta x^3 (xT - \gamma)^{\beta-1} + \frac{2\alpha k d (-\gamma)^{2\beta+1}}{T^3} > 0 \end{aligned} \quad (16)$$

## NUMERICAL EXAMPLE

Let  $A = Rs\ 2000$  /set up,  $p = 200$  units/unit time,  $d = 50$  unit/unit time,  $\alpha = 0.6$ ,  $\beta = 10$ ,  $\gamma = 0.4$ ,  $k = Rs\ 60$  /unit,  $l = 0.05$ ,  $h = 2$ . Using equation (15), (10), (14) and (16) we get the optimum values of  $T^* = 1.83038$ ,  $HC^* = 72.1144$ ,  $TVC^* = 4343.27$ ,  $\frac{d^2}{dT^2}(TVC(T)) = 4776.65 > 0$  respectively.

## SENSITIVITY ANALYSIS

We now perform the sensitivity analysis of the optimal solution of the model for changes in  $A$ ,  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $k$ ,  $p$  and  $d$  parameter values associated with the system. We change one parameter at a time keeping the other parameters unchanged for study of sensitivity analysis. The original values of all the parameters for sensitivity analysis are taken from the example given above. Sensitivity analysis is performed by changing the values of all the parameters from -50% to +50%, one by one in the model which are given in the following table 1.

Table 1. Sensitivity Analysis  
Tabela 1. Analiza wrażliwości

Changing parameter	% change	Change In T*	change in HC*	change in TVC*
<i>A</i>	-50	1.74415	66.8461	3785.18
	-40	1.76679	68.0679	3899.09
	-30	1.78593	69.1759	4011.66
	-20	1.80254	70.2063	4123.12
	-10	1.81722	71.1808	4233.62
	10	1.8423	73.0169	4452.18
	20	1.8532	73.8963	4560.42
	30	1.86325	74.759	4668.04
	40	1.87258	75.6099	4775.11
<i>p</i>	50	1.88127	76.4507	4881.66
	-50	2.73961	70.7141	3918.37
	-40	2.35044	71.1952	4060.01
	-30	2.13391	71.5286	4161.18
	-20	1.99599	71.7741	4237.05
	-10	1.90046	71.9636	4296.06
	10	1.77676	72.2362	4381.9
	20	1.73443	72.3382	4414.09
	30	1.70015	72.4237	4441.32
<i>d</i>	40	1.67183	72.497	4464.67
	50	1.64804	72.5606	4484.9
	-50	1.64423	41.0672	2898.62
	-40	1.6722	47.2891	3193.9
	-30	1.70513	53.52	3485.39
	-20	1.74256	59.7403	3773.82
	-10	1.78431	65.9401	4059.67
	10	1.88088	78.2594	4624.88
	20	1.93604	84.3737	4904.68
<i>α</i>	30	1.99619	90.4568	5182.82
	40	2.06173	96.5071	5459.4
	50	2.13317	102.524	5734.52
	-50	1.91322	75.8871	4296.11
	-40	1.89113	74.8473	4308.32
	-30	1.87261	73.9958	4318.75
	-20	1.85669	73.2771	4327.87
	-10	1.84276	72.658	4335.97
	10	1.81925	71.6307	4349.91
<i>β</i>	20	1.80915	71.1957	4356.00
	30	1.79991	70.8008	4361.63
	40	1.7914	70.4396	4366.87
	50	1.78352	70.1071	4371.76
	-50	1.81262	74.3514	4419.78
	-40	1.81706	73.5764	4395.05
	-30	1.82098	73.0561	4377.45
	-20	1.82453	72.6585	4363.39
	-10	1.82764	72.3561	4352.36
<i>γ</i>	10	1.83278	71.9177	4335.75
	20	1.8349	71.7543	4329.38
	30	1.83678	71.6164	4323.94
	40	1.83847	71.4995	4319.23
	50	1.83997	71.3973	4315.11
	-50	1.58625	63.9971	4529.37
	-40	1.63485	65.5722	4487.27
	-30	1.68357	67.1745	4447.83
	-20	1.7324	68.8004	4410.81
-10	1.78134	70.4479	4376.02	
<i>k</i>	10	1.87951	73.7974	4312.4
	20	1.92873	75.4954	4283.25
	30	1.97804	77.2068	4255.68
	40	2.02743	78.9287	4229.54
	50	2.0769	80.6587	4204.68
	-50	1.90347	78.8454	2473.8
	-40	1.88504	76.8311	3065.86
	-30	1.86892	75.2701	3386.59
	-20	1.85465	74.0176	3706.28
-10	1.84189	72.9849	4025.12	
<i>k</i>	10	1.8199	71.366	4660.85
	20	1.81031	70.7139	4977.94
	30	1.80148	70.1383	5294.61
	40	1.79329	69.6238	5610.92
	50	1.78567	69.1604	5926.91

From the table 1 we can conclude the following:

- i.  $T^*$  is directly proportional to  $A, d, \beta, \gamma$  but inversely proportional to  $p, \alpha, k$ .
- ii.  $HC^*$  is directly proportional to  $A, p, d, \gamma$  but inversely proportional to  $\alpha, \beta, k$ .
- iii.  $TVC^*$  is directly proportional to  $A, d, p, \alpha, k$  but inversely proportional to  $\beta, \gamma$ .

## CONCLUSION

Here, a Production inventory model has been developed for an item with three parameter Weibull deterioration where the holding cost is constant per unit, per unit time. We have assumed here that the production and demand rate are constant and shortages are not allowed. Completely deteriorated items are discarded and partially deteriorated items are offered for sale with a discount meeting the demand. The optimum production cycle time, holding cost and total variable cost has been derived for the developed model. Sensitivity analysis shows how the different parameters affect the production cycle time, holding cost and total variable cost. It is clearly seen from the table that to minimise the total cost, the set up cost, production rate, demand rate, scale parameter and the production cost per unit should be minimised whereas the value of the shape parameter and location parameter should be maximised.

## REFERENCES

- Abad P.L., 1996, Optimal pricing and lot-sizing under conditions of perishability and partial backordering, *Management Science*, 42, 1093 -1104.
- Aggarwal S.P., 1978. A note on an order-level model for a system with constant rate of deterioration, *Opsearch*, 15, 184-187.
- Deng P. S., Lin R. H.-J., Chu P., 2007, A note on the inventory models for deteriorating items with ramp type demand rate, *European Journal of Operational Research*, 178 112-120.
- Ghare P.N., and Schrader G.F., 1963. A model for exponentially decaying inventories, *J.Ind. Eng.*, 15, 238-243.
- Goyal S.K., Giri B.C., 2001, Recent trends in modeling of deteriorating inventory, *European Journal of Operational Research*, 134, 1 -16.
- Hwang C.H, 1986. Optimization of production planning problem with continuously distributed time-lags, *Int. J. Syst. Sci.*, 17, 1499-1508.
- Lin G.C., Kroll D.E., Lin C.J., 2005. Determining a common production cycle for an economic lot scheduling problem with deteriorating items, *European journal of Operational Research*. 101, 369-384.
- Maxwell W.L., 1964, The Scheduling of economic lot sizes, *naval Research Logistics Quarterly*, 11 (2-3), 89-124.
- Meher M.K., Panda G.Ch., Sahu S.K., 2012, An inventory model with Wiebull deterioration rate under the dealy in payment in demand decling market. *Applied Mathematical Sciences*, 6, 23, 1121-1133.
- Mishra P., Shah N.H., 2008. Inventory management of time dependent deteriorating with salvage value. *Applied Math. Sci.*, 2, 793-798.



- Misra R.B, 1975, Optimum production lot size model for a system with deteriorating inventory. *International J. of Production Research*. 13(5), 495- 505.
- Raafat F. 1985. A production-inventory model for decaying raw materials and a decaying single finished product system, *Int. J. Syst. Sci.*, 16, 1039-1044.
- Raafat F., 1991, Survey of literature on continuously deteriorating inventory models, *Journal of the Operational Research Society*, 40, 27- 37.
- Roy T., Chaudhuri K.S., 2009, A production-inventory model under stock-dependent demand, Weibull distribution deterioration and shortage, *Intl. Trans. in Op. Res.* 16, 325-346.
- Sabahno H, 2008. Optimal policy for a deteriorating inventory model with finite replenishment rate and with price dependent demand rate and cycle length dependent price. *Proc. World Acad. Sci. Eng. Technol.*, 34, 219-223.
- Shah N.H., Acharya A.S, 2008. A time dependent deteriorating order level inventory model for exponentially declining demand. *Applied Math. Sci.*, 2, 2795-2802.
- Sugapriya C., Jeyaraman K, 2008. Determining a common production cycle time for an EPQ model for non-instantaneous deteriorating items allowing price discount using permissible delay in payments. *APRN. J. Eng. Applied Sci.*, 3, 26-30.
- Tripathy C.K. Pradhan L.M., 2010. A Production Inventory model for Weibull deteriorating Items allowing price discount & permissible delay in payments. *Global Journal of Mathematical Sciences: Theory & practical*, vol2, Number-1, 1-12.
- Tripathy C.K., Pradhan L.M., 2011. Optimal Pricing & Ordering Policy for three parameter Weibull deterioration under trade credit. *International Journal of Mathematical Analysis*, Vol.5, No6, 275-284.
- Tripathy C.K., Pradhan L.M, Mishra U, 2010. An EPQ Model for Linear deteriorating Item with Variable Holding cost. *International Journal of computational & Applied Mathematics*, Vol.5, No2, 199-205.
- Tripathy C. K., Mishra U., 2011. An EOQ model with time dependent Weibull deterioration and ramp type demand, *International Journal of Industrial Engineering Computations*, 2, 307-318.
- Yang P. Wee H., 2003. An integrated multilot- size production inventory model for deteriorating item, *Comput. Operat. Res.*, 30, 671-682.

## **MODEL ZARZĄDZANIA PRODUKCJĄ Z TRÓJPARAMETROWYM WPŁYWEM PSUCIA SIĘ PRODUKTÓW I UPUSTEM CENOWYM**

**STRESZCZENIE.** Wstęp: Psucie się jest naturalnym procesem większości produktów i nie może być ignorowane przez zarządzających produkcją. W ostatnich latach opublikowano wiele prac poświęconych różnym modelom zarządzania zapasem i produkcją, uwzględniających psucie się produktów w różnych warunkach praktycznych. Upust cenowy stosowany dla częściowo zepsutych produktów jest stosunkowo nową koncepcją, wprowadzoną w wielu rozwijanych obecnie modelach. **Metody:** Praca ta porusza zagadnienia związane z opracowaniem modelu dla artykułów podlegających psuciu się według modelu Weibulla. Wielkość produkcji i popytu są wielkościami stałymi, zakłada się, że koszt utrzymania jednostki towaru jest stały w czasie. Całkowicie zepsute artykuły są usuwane z zapasu, natomiast częściowo zepsute mogą być sprzedane z upustem cenowym. Nie dopuszcza się braków towarowych. **Wyniki i wnioski:** Został zaproponowany model zarządzania produkcją uwzględniający trójparametrowe psucie się towaru oraz możliwość upustów cenowych dla towarów częściowo zepsutych. Ustalono optymalną długość cyklu uzupełniania. Wyniki przedstawiono za pomocą przykładu. Analiza wrażliwości została przeprowadzona w celu stworzenia optymalnego rozwiązania uwzględniającego zmiany w innych parametrach..

**Słowa kluczowe:** model zarządzania produkcją, proces psucia się Weibulla, upust cenowy.

## MANAGEMENT-MODELL FÜR DIE DURCH DEN DREI-PARAMETER-VERDERB DER PRODUKTE UND DEN DADURCH BEDINGTEN PREISNACHLASS CHARAKTERISIERTE PRODUKTION

**ZUSAMMENFASSUNG. Einleitung:** Verderben ist bei den meisten Produkten ein natürlicher Prozess und darf daher von Produktionsmanagern nicht ignoriert werden. In den letzten Jahren veröffentlichte man viele Arbeiten, welche unterschiedlichen Modellen für Bestands- und Produktionsmanagement unter Berücksichtigung des Produktverderbs in verschiedenen praktischen Bedingungen gewidmet waren. Der bei den teilweise verdorbenen Produkten angewendete Preisnachlass stellt ein neues, in vielen, heutzutage entwickelten Modellen eingeführtes Konzept dar.

**Methoden:** Die vorliegende Arbeit berührt die mit Entwicklung eines brauchbaren Modells für die einem Verderb gemäß dem Weibull-Modells unterliegenden Artikel verbundenen Fragen. Produktion und Nachfrage sind konstante Größen, ferner nimmt man an, dass Kosten des Unterhalts einer Wareneinheit in der Zeit als ebenfalls konstante Größe anzusehen ist. Die total verdorbenen Produkte werden vom Bestand entfernt, dagegen die teilweise verdorbenen dürfen mit dem Preisnachlass verkauft werden. Die Warenmängel werden ausgeschlossen.

**Ergebnisse und Fazit:** Es wurde ein Management-Modell für die Produktion unter der Berücksichtigung des Drei-Parameter-Verderbs der Produkte sowie der Möglichkeit der Anwendung von Preisnachlässen bei den teilweise verdorbenen Waren vorgestellt. Es wurde eine optimale Länge des Nachschubzyklus festgelegt. Die Ergebnisse stellte man anhand eines Beispiels dar. Zwecks Ausarbeitung einer optimalen, die Veränderungen innerhalb anderer Parameter berücksichtigenden Lösung wurde auch die Empfindlichkeitsanalyse durchgeführt.

**Codewörter:** Modell für Produktionsmanagement, Verderbnisprozess gemäß dem Weibull-Modell, Preisnachlass.

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