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AN EOQ MODEL FOR TIME DEPENDENT WEIBULL DETERIORATION WITH LINEAR DEMAND AND SHORTAGES

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ABSTRACT. Background. The study of control and maintenance of production inventories of deteriorating items with and without shortages has grown in its importance recently. The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose.

Methods: In this article order level inventory models have been developed for deteriorating items with linear demand and Weibull deterioration. In developing the model we have assumed that the production rate and the demand rate are time dependent. The unit production cost is inversely proportional to demand. Inventory-production system has two parameters Weibull deterioration.

Results and conclusions: Two models have been developed considering without shortage cases and with shortage case where the shortages are completely backlogged. The objective of the model is to develop an optimal policy that minimizes the total average cost. Sensitivity analysis has been carried out to show the effect of changes in the parameter on the optimum total average cost.

Key words: demand, Weibull deterioration, unit production cost, shortage.

INTRODUCTION

In recent years, many researchers are interested in the study of control and maintenance of production inventories of deteriorating items with and without shortages because most physical goods deteriorate over time. The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item cannot be used for its original purpose. Food items, drugs, pharmaceuticals, radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Research in this direction began with the work of Whitin [1957] who considered fashion goods deteriorating at the end of a prescribed storage period. Ghare and Schrader [1963] developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal [1977]. Aggarwal [1978] reconsidered this model by rectifying the error in the work of Shah and Jaiswal [1977] in calculating the average inventory holding cost. In all models discussed above, the demand rate and the deterioration rate were assumed to be constant, the replenishment rate was infinite and shortage in inventory ware not allowed.

Researchers started to develop inventory systems allowing time variability in one or more than one parameters. Dave and Patel [1981], He, Wang and Lai [2010] and Chuang [2012] discussed an inventory model for replenishment. This was followed by another model by Dave [1986] with variable instantaneous demand, discrete opportunities with shortages. Bahari-Kashani [1989] discussed a heuristic model with time-proportional demand. An Economic Order Quantity (EOQ) model for

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deteriorating items with shortages and linear trend in demand was studied by Goswami and Chaudhuri [1991] and Meher, Panda and Sahu [2012]. On all these inventory systems, the deterioration rate was considered to be constant.

Berrotoni [1962] observed, while discussing the difficulties of fitting empirical data to mathematical distributions, that both leakage failure of dry batteries and life expectancy of ethical drugs could be expressed in term of Weibull distribution. In both cases, the rate of deterioration increased with age or the longer the items remained unused, the higher the rate at which they failed. At some point in time, all units that had not been used would have failed. Perhaps the work of Berrotoni [1962] prompted Covert and Philip [1973] to develop an inventory model for deteriorating items with variable rate of deterioration. They used the two-parameter Weibull distribution to represent the distribution of deterioration. Two-parameter Weibull distribution deterioration is a generalized form of exponentially decaying functions. The main reason for choosing the Weibull distribution deterioration lies in its convenient generalized properties. The novelty we will be taking into consideration in this research is that the time of deterioration is a random variable following the two-parameter Weibull distribution. This distribution can be used to model either increasing or decreasing rate of deterioration, according to the choice of the parameters. The instantaneous rate function for a two-parameter Weibull distribution is given by:

$$Z(t) = \alpha\beta\, t^{\beta-1}$$

where α is the scale parameter, $\alpha > 0$; β is the shape parameter , $\beta > 0$; t is time of deterioration $t > 0$.

It is seen that the two-parameter Weibull distribution is appropriate for an item with decreasing rate of deterioration only if the initial rate of deterioration is extremely high. Similarly, this distribution can also be used for an item with increasing rate of deterioration only if the initial rate is approximately zero.

Another class of inventory models has been developed by different researchers with time-dependent deterioration rate. Mishra [1975] analyzed an inventory model with a variable rate of deterioration, finite rate of replenishment for no shortage case, but only a special case of the model was solved under very restrictive assumptions. Deb and Chaudhuri [1986] studied a model with a finite rate of production and a time-proportional deterioration rate, allowing backlogging. Goswami and Chaudhuri [1992] assumed that the demand rate, production rate and deterioration rate were all time dependent. Detailed information regarding inventory modelling for deteriorating items was given in the review articles of Nahmias [1982] and Rafaat [1991]. An order-level inventory model for deteriorating items without shortages has been developed by Jalan and Chaudhuri [1999].

Further related works in this line are due to Papachristos and Skouri [2000], Goyal and Giri [2001], Giri [et al. 2003], Ghosh [et al. 2006], Roy and Chaudhuri [2009] and Tripathy and Mishra [2011], Tripathy and Pradhan [2011a], Tripathy and Mishra [2010], Tripathi [2011], Tripathy and Pradhan [2011b], as well as Tripathi and Kumar [2011].

In this paper, we have tried to developed EOQ models for time-dependent deteriorating items with linear demand rate. The production rate is finite and proportional demand rate. The unit production cost is assumed to be inversely proportional to the demand rate. The first model discussed here is for without shortage case and then the model is extended to cover the case of inventory with shortages which are completely backlogged. The procedure of solving the model is illustrated with the help of two numerical examples. Sensitivity analysis has been carried out to show the effect of changes in the parameter on the optimum total average cost.

MODEL I: MODEL FOR WITHOUT SHORTAGE CASE

We need the following assumptions and notation for developing our inventory model.

(i) Lead time is zero.

(ii) $R = f(t) = a + bt$: Demand rate at any time t where, $a \geq 0,\ b > 0,\ t \geq 0$

(iii) $K = \delta\, f(t)$: Production rates where $(\delta > 1)$, is a constant.

(iv) $\theta(t) = \alpha\beta\, t^{\beta-1}$: Deterioration rate which follows a two parameter Weibull distribution, where $\alpha(0 < \alpha << 1)$ is the scale parameter, $\beta > 1$ is the shape parameter. It is assumed that the deterioration of units increases with time $t > 0$.

(v) c_1 : Constant holding cost per item per unit of time.

(vi) c_2 : Infinite shortage cost i.e. shortages are not permitted.

(vii) c_3 : Constant deterioration cost per unit per unit of time.

(viii) C : Total average cost for a production cycle.

(ix) $v = \alpha_1 R^{-\gamma}$: Unit production cost which is inversely related to the demand rate where $\alpha_1 > 0,\ \ \gamma > 0\ \ and\ \ \gamma \neq 2$.

Here we have,

$$\frac{dv}{dR} = -\alpha_1 \gamma R^{-(\gamma+1)} < 0,$$

$$\frac{d^2 v}{dR^2} = \alpha_1 \gamma(\gamma+1) R^{-(\gamma+2)} > 0.$$

Hence, we observe that the marginal unit cost of production is an increasing function of R. As increase in demand rate results in decreases of the unit cost of production at an increasing rate. This encourages the manufacturer to produce more as the demand for the item increases.

At initial time $t = 0$, the production starts with zero inventory and the production stops as the stock reches level S at time $t = t_1$. Due to demand and deterioration inventory level gradually diminishes during the time period $t_1 \le t \le t_2$ which ultimately falls to zero at time $t = t_2$, after which the next cycle begains. During the early stage of inventory, the intensity of deterioration is very low because t is small. However, the intensity increases with time, but $\theta(t)$ remains bounded for $t >> 1$ since $0 < \alpha << 1$.

Let $Q(t)$ be the inventory level of the system at any time $t\,(0 \le t \le t_2)$. The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, t_2]$ are given by

$$\frac{dQ(t)}{dt} + \theta(t)\, Q(t) = K - f(t), \qquad 0 \le t \le t_1. \tag{1}$$

$$\frac{dQ(t)}{dt} + \theta(t)\, Q(t) = -f(t), \qquad t_1 \le t \le t_2. \tag{2}$$

Using $\theta(t) = \alpha\beta t^{\beta-1}$ and $f(t) = a + bt$, (1) and (2) become respectively

$$\frac{dQ(t)}{dt} + \alpha\beta\, t^{\beta-1} Q(t) = (\delta - 1)(a + bt), \qquad 0 \le t \le t_1. \tag{3}$$

with the conditions $Q(0) = 0$ and $Q(t_1) = S$, and

$$\frac{dQ(t)}{dt} + \alpha\beta\, t^{\beta-1}\, Q(t) = -(a + bt), \qquad t_1 \le t \le t_2. \tag{4}$$

with the conditions $Q(t_1) = S$ and $Q(t_2) = 0$.

The solution of equation (3) using $Q(0) = 0$, is

$$Q(t) = (\delta - 1)\left\{ a\left(t - \alpha t^{\beta+1} + \frac{\alpha t^{\beta+1}}{\beta+1} \right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{\alpha t^{\beta+2}}{2} \right) \right\}, \qquad 0 \le t \le t_1. \tag{5}$$

As $0 < \alpha << 1$, neglecting powers of α higher than the first, this approximation is followed throughout the subsequent calculations. The solution of (4) using the condition

$Q(t_1) = S$, is

$$Q(t) = S\left\{1+\alpha\left(t_1^{\beta}-t^{\beta}\right)\right\}+a\left\{t_1-t+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)+\alpha\left(t^{\beta+1}-t_1t^{\beta}\right)\right\}$$
$$+b\left\{\frac{t_1^{2}}{2}-\frac{t^2}{2}+\frac{\alpha}{\beta+2}\left(t_1^{\beta+2}-t^{\beta+2}\right)+\frac{\alpha}{2}\left(t^{\beta+2}-t_1^{2}t^{\beta}\right)\right\}, \qquad t_1 \le t \le t_2. \qquad (6)$$

Since $Q(t_2) = 0$, we get from equation (6),

$$S\left\{1+\alpha\left(t_1^{\beta}-t_2^{\beta}\right)\right\}+a\left\{t_1-t_2+\frac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t_2^{\beta+1}\right)+\alpha\left(t_2^{\beta+1}-t_1t_2^{\beta}\right)\right\}$$
$$+b\left\{\frac{t_1^{2}}{2}-\frac{t_2^{2}}{2}+\frac{\alpha}{\beta+2}\left(t_1^{\beta+2}-t_2^{\beta+2}\right)+\frac{\alpha}{2}\left(t_2^{\beta+2}-t_1^{2}t_2^{\beta}\right)\right\}=0$$

Neglecting powers of α higher than the first, after simplification this result reduces to

$$S = a\left\{t_2-t_1+\frac{\alpha}{\beta+1}\left(t_2^{\beta+1}-t_1^{\beta+1}\right)+\alpha\left(t_1^{\beta+1}-t_1^{\beta}t_2\right)\right\}+b\left\{\frac{t_2^{2}}{2}-\frac{t_1^{2}}{2}+\frac{\alpha}{\beta+2}\left(t_2^{\beta+2}-t_1^{\beta+2}\right)+\frac{\alpha}{2}\left(t_1^{\beta+2}-t_1^{\beta}t_2^{2}\right)\right\}$$

Therefore

$$Q(t)=\begin{cases}(\delta-1)\left\{a\left(t-\alpha t^{\beta+1}+\dfrac{\alpha t^{\beta+1}}{\beta+1}\right)+b\left(\dfrac{t^2}{2}+\dfrac{\alpha t^{\beta+2}}{\beta+2}-\dfrac{\alpha t^{\beta+2}}{2}\right)\right\}, & \text{if} \quad 0\le t\le t_1\\ S\left\{1+\alpha\left(t_1^{\beta}-t^{\beta}\right)\right\}+a\left\{t_1-t+\dfrac{\alpha}{\beta+1}\left(t_1^{\beta+1}-t^{\beta+1}\right)+\alpha\left(t^{\beta+1}-t_1t^{\beta}\right)\right\} & \\ +b\left\{\dfrac{t_1^{2}}{2}-\dfrac{t^2}{2}+\dfrac{\alpha}{\beta+2}\left(t_1^{\beta+2}-t^{\beta+2}\right)+\dfrac{\alpha}{2}\left(t^{\beta+2}-t_1^{2}t^{\beta}\right)\right\}, & \text{if} \quad t_1\le t\le t_2\end{cases} \qquad (7)$$

Thus the total inventory in the cycle is $\int_0^{t_1} Q(t)dt + \int_{t_1}^{t_2} Q(t)dt$

$$=(\delta-1)\left\{a\left(\frac{t_1^{2}}{2}-\frac{\alpha t_1^{\beta+2}}{\beta+2}+\frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right)+b\left(\frac{t_1^{3}}{6}+\frac{\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha t_1^{\beta+3}}{2(\beta+3)}\right)\right\}$$
$$+a\left\{\frac{t_2^{2}}{2}+\frac{t_1^{2}}{2}+t_1t_2+\frac{\alpha}{\beta+1}\left(t_2^{\beta+3}-t_1t_2^{\beta+2}+t_1^{\beta+3}-t_1^{\beta+2}-\frac{t_2^{\beta+2}}{\beta+2}+\frac{t_1^{\beta+2}}{\beta+2}+t_2t_1^{\beta+1}-t_2t_1^{\beta+2}\right)+\alpha\left(\quad 2t_2t_1^{\beta+1}\right.\right.$$
$$\left.\left.-\frac{t_2^{\beta+2}}{\beta+1}+\frac{2t_2t_1^{\beta+1}}{\beta+1}+\frac{t_2^{\beta+2}}{\beta+2}-\frac{t_1^{\beta+2}}{\beta+2}-\frac{t_1t_2^{\beta+1}}{\beta+1}\right)\right\}+b\left\{\frac{t_2^{3}}{3}+\frac{t_1^{3}}{6}-\frac{t_2^{2}t_1}{2}+\frac{\alpha}{\beta+2}\left(t_2^{\beta+3}-t_1t_2^{\beta+2}-\frac{t_2^{\beta+2}}{\beta+2}+\frac{t_1^{\beta+2}}{\beta+2}\right)\right.$$
$$\left.+\frac{\alpha}{2}\left(\frac{t_2^{\beta+3}}{\beta+3}-\frac{t_1^{\beta+3}}{\beta+3}-\frac{t_2^{\beta+3}}{\beta+1}+\frac{t_2^{2}t_1^{\beta+1}}{\beta+1}\right)\right\}$$

Total number of deteriorated items in $[0,t_2]$ is given by production in $[0,t_1]-$ Demand in $[0,t_2]$, i.e.

$$\delta\int_0^{t_1}(a+bt)dt-\int_0^{t_2}(a+bt)dt=a\left(\delta t_1-t_2\right)+\frac{1}{2}b\left(\delta t_1^{2}-t_2^{2}\right) \qquad (8)$$

Since the production in $[u,\ u+du]$ is *Kdu*, the cost of production in $[u,u+du]$ is:

$$K\nu\, du = \frac{\alpha_1 \delta R}{(a+bu)^{\gamma}}du = \frac{\alpha_1\delta}{(a+bu)^{\gamma-1}}du.$$

Hence the production cost in $[0, t_1]$ is given by

$$\int_0^{t_1}\frac{\alpha_1\delta}{(a+bu)^{\gamma-1}}du = \frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{2-\gamma}-(a+bt_1)^{2-\gamma}\right], \qquad \gamma \neq 2. \tag{9}$$

Now the total average cost of the system is

$$C = \frac{1}{t_2}\left[c_1(\delta-1)\left\{a\left(\frac{t_1^{\,2}}{2}-\frac{\alpha t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}\right)+b\left(\frac{t_1^{\,3}}{6}+\frac{\alpha t_1^{\,\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha t_1^{\,\beta+3}}{2(\beta+3)}\right)\right\}\right.$$
$$+c_1a\left\{\frac{t_2^{\,2}}{2}+\frac{t_1^{\,2}}{2}+t_1t_2+\frac{\alpha}{\beta+1}\left(t_2^{\,\beta+3}-t_1t_2^{\,\beta+2}+t_1^{\,\beta+3}-t_1^{\,\beta+2}-\frac{t_2^{\,\beta+2}}{\beta+2}+\frac{t_1^{\,\beta+2}}{\beta+2}+t_2t_1^{\,\beta+1}-t_2t_1^{\,\beta+2}\right)+\alpha\left(\ 2t_2t_1^{\,\beta+1}\right.\right.$$
$$\left.\left.-\frac{t_2^{\,\beta+2}}{\beta+1}+\frac{2t_2t_1^{\,\beta+1}}{\beta+1}+\frac{t_2^{\,\beta+2}}{\beta+2}-\frac{t_1^{\,\beta+2}}{\beta+2}-\frac{t_1t_2^{\,\beta+1}}{\beta+1}\right)\right\}+c_1b\left\{\frac{t_2^{\,3}}{3}+\frac{t_1^{\,3}}{6}-\frac{t_2^{\,2}t_1}{2}+\frac{\alpha}{\beta+2}\left(t_2^{\,\beta+3}-t_1t_2^{\,\beta+2}-\frac{t_2^{\,\beta+2}}{\beta+2}+\frac{t_1^{\,\beta+2}}{\beta+2}\right)\right.$$
$$\left.+\frac{\alpha}{2}\left(\frac{t_2^{\,\beta+3}}{\beta+3}-\frac{t_1^{\,\beta+3}}{\beta+3}-\frac{t_2^{\,\beta+3}}{\beta+1}+\frac{t_2^{\,2}t_1^{\,\beta+1}}{\beta+1}\right)\right\}+c_3a(\delta t_1-t_2)+\frac{1}{2}c_3b\left(\delta t_1^{\,2}-t_2^2\right)$$
$$\left.+\frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{2-\gamma}-(a+bt_1)^{2-\gamma}\right]\right] \tag{10}$$

Optimum values of t_1 and t_2 for minimum average cost 'C' are the solutions of the equations

$$\frac{\partial C}{\partial t_1}=0 \text{ and } \frac{\partial C}{\partial t_2}=0. \tag{11}$$

Provided

$$\frac{\partial^2 C}{\partial t_1^{\,2}}>0, \quad \frac{\partial^2 C}{\partial t_2^{\,2}}>0 \quad \text{and} \quad \frac{\partial^2 C}{\partial t_1^{\,2}}\frac{\partial^2 C}{\partial t_2^{\,2}}-\frac{\partial^2 C}{\partial t_1\partial t_2}>0.$$

Equation (11) is

$$\frac{\partial C}{\partial t_1}=c_1(\delta-1)\left\{a\left(t_1-\alpha t_1^{\,\beta+1}+\frac{\alpha t_1^{\,\beta+1}}{\beta+1}\right)+b\left(\frac{t_1^{\,2}}{2}+\frac{\alpha t_1^{\,\beta+2}}{\beta+2}-\frac{\alpha t_1^{\,\beta+2}}{2}\right)\right\}$$
$$+c_1a\left[t_1+t_2+\frac{\alpha}{\beta+1}\left\{(\beta+3)t_1^{\,\beta+2}-t_2^{\,\beta+2}-(\beta+2)t_1^{\,\beta+1}+t_1^{\,\beta+1}-(\beta+2)t_1^{\,\beta+1}t_2+(\beta+1)t_1^{\,\beta}t_2\right\}\right.$$
$$\left.+\alpha\left\{2(\beta+1)t_2t_1^{\,\beta}+2t_1^{\,\beta}t_2t_1^{\,\beta+1}-\frac{t_2^{\,\beta+1}}{\beta+1}\right\}\right]+c_1b\left\{\frac{t_1^{\,2}}{2}-\frac{t_2^{\,2}}{2}+\frac{\alpha}{\beta+2}\left(t_1^{\,\beta+1}-t_2^{\,\beta+2}\right)+\frac{\alpha}{2}\left(t_2^{\,2}t_1^{\,\beta}-t_1^{\,\beta+2}\right)\right\}$$
$$+c_3a\delta+c_3b\delta t_1+\alpha_1\delta(a+bt_1)^{1-\gamma}=0 \tag{12}$$

$$\frac{\partial C}{\partial t_2}=-\frac{1}{t_2^{\,2}}\left[c_1(\delta-1)\left\{a\left(\frac{t_1^{\,2}}{2}-\frac{\alpha t_1^{\,\beta+2}}{\beta+2}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}\right)+b\left(\frac{t_1^{\,3}}{6}+\frac{\alpha t_1^{\,\beta+3}}{(\beta+2)(\beta+3)}-\frac{\alpha t_1^{\,\beta+3}}{2(\beta+3)}\right)\right\}\right.$$
$$+c_1a\left\{\frac{t_1^{\,2}}{2}+\frac{\alpha}{\beta+1}\left(t_1^{\,\beta+3}-t_1^{\,\beta+2}+\frac{t_1^{\,\beta+2}}{\beta+2}\right)-\frac{\alpha t_1^{\,\beta+2}}{\beta+2}\right\}+c_1b\left\{\frac{t_1^{\,3}}{6}+\frac{\alpha t_1^{\,\beta+2}}{(\beta+2)(\beta+2)}-\frac{\alpha t_1^{\,\beta+3}}{2(\beta+3)}\right\}$$
$$\left.+c_3a\delta t_1+\frac{1}{2}c_3b\delta t_1^{\,2}+\frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{2-\gamma}-(a+bt_1)^{2-\gamma}\right]\right]+c_1a\left[\frac{1}{2}+\frac{\alpha}{\beta+2}\left\{\ (\beta+2)t_2^{\,\beta+1}-(\beta+1)t_1t_2^{\,\beta}\right.\right.$$

$$-\frac{(\beta+1)t_2^{\beta}}{\beta+2}\Bigg\}+\alpha\left\{\frac{(\beta+1)t_2^{\beta}}{\beta+2}-\frac{\beta t_1 t_2^{\beta-1}}{\beta+1}-t_2^{\beta}\right\}\Bigg]+c_1 b\left[\frac{2t_2}{3}-\frac{t_1}{2}+\frac{\alpha}{\beta+2}\Bigg\{\;(\beta+2)t_2^{\beta+1}-(\beta+1)t_1t_2^{\beta}\right.$$

$$\left.-\frac{(\beta+1)t_2^{\beta}}{\beta+2}\Bigg\}+\frac{\alpha}{2}\left\{\frac{(\beta+2)t_2^{\beta+1}}{\beta+3}-\frac{(\beta+2)t_2^{\beta+1}}{\beta+1}+\frac{t_1^{\beta+1}}{\beta+1}\right\}\right]-\frac{1}{2}bc_3=0 \qquad (13)$$

MODEL II: MODEL FOR WITH SHORTAGE

Here we develop an order-level model for deteriorating items with a finite rate of replenishment allowing shortages, which are completely backlogged. We will use the same notation and assumption of model-I replacing (vii) by: c_2 be the constant shortage cost per unit per unit of time. Initially at time $t=0$ we start with zero inventories. Then the production starts and continues up to time $t=t_1$ when the stock reaches level S after meeting the demand during this period. Inventory accumulated in $[0,t_1]$ after meeting the demands is used in $[t_1,t_2]$. The stock reaches the zero level at time $t=t_2$. Now shortages start to develop and accumulate to the level P at $t=t_3$. Production starts at time t_3. The running demands as well as the backlog for $[t_2,t_3]$ are satisfied in $[t_3,t_4]$. The inventory again reaches the zero level at time $t=t_4$, Then the next cycle starts. Our objective is to determine the optimum values of C, t_1, t_2, t_3 and t_4 subject to the assumptions stated above.

$Q(t)$ be the instantaneous inventory level at any time $t\,(0\le t\le t_4)$. Thus the instantaneous states of $Q(t)$ is governed by the following differential equations:

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}Q(t)=(\delta-1)(a+bt), \qquad 0\le t\le t_1. \qquad (14)$$

Subject to the conditions $Q(0)=0$ and $Q(t_1)=S.$

$$\frac{dQ(t)}{dt}+\alpha\beta\, t^{\beta-1}Q(t)=-(a+bt), \qquad t_1\le t\le t_2. \qquad (15)$$

Subject to the conditions $Q(t_1)=S$ and $Q(t_2)=0.$

$$\frac{dQ(t)}{dt}=-(a+bt), \qquad t_2\le t\le t_3. \qquad (16)$$

Subject to the conditions $Q(t_2)=S$ and $Q(t_3)=-S.$

$$\frac{dQ(t)}{dt}=(\delta-1)(a+bt), \qquad t_3\le t\le t_4. \qquad (17)$$

Subject to the conditions $Q(t_3)=-P$ and $Q(t_4)=0.$

Following the derivation as in section 2 we get the solutions of (14) and (15) similar to (7). The solutions of (16) and (17) using conditions $Q(t_2)=0$ and $Q(t_4)=0$ will be

$$Q(t)=a(t_2-t)+\frac{b}{2}\left(t_2^{2}-t^{2}\right), \qquad t_2\le t\le t_3. \qquad (18)$$

and

$$Q(t)=a(\delta-1)(t-t_4)+\frac{b}{2}(\delta-1)(t^{2}-t_4^{2}), \qquad t_3\le t\le t_4. \qquad (19)$$

As there is no inventory during the period $[t_2, t_4]$, there is no deterioration. Hence total number of deteriorated items in $[0, t_4]$ is the same as given in (8).

Total shortage during $[t_2, t_4]$ is

$$\int_{t_2}^{t_4} -[Q(t)]dt = \int_{t_2}^{t_3} [-Q(t)]dt + \int_{t_3}^{t_4} -[Q(t)]dt$$

$$= \frac{a}{2}\left[t_2^{\,2} - 2t_2t_3 + \delta t_3^{\,2} + (\delta-1)t_4^{\,2} - 2(\delta-1)t_4t_3\right] + \frac{b}{6}\left[2t_2^{\,3} - 3t_2^{\,2}t_3 + \delta t_3^{\,3} + 2(\delta-1)t_4^{\,3} - 3(\delta-1)t_4^{\,2}t_3\right]$$

And the production cost during $[t_3, t_4]$ is,

$$\int_{t_3}^{t_4} Kvdu = \alpha_1\delta\int_{t_3}^{t_4} (a+bu)^{1-\gamma}du = \frac{\alpha_1\delta}{b(\gamma-2)}\left[(a+bt_3)^{2-\gamma} - (a+bt_4)^{2-\gamma}\right], \qquad \gamma \neq 2.$$

Therefore the production cost during $[0, t_4]$ is

$$\frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{2-\gamma} - (a+bt_1)^{2-\gamma} + (a+bt_3)^{2-\gamma} - (a+bt_4)^{2-\gamma}\right], \qquad \gamma \neq 2.$$

Thus the total average cost of the system during $[0, t_4]$ is,

$$C = \frac{1}{t_4}\Bigg[c_1(\delta-1)\left\{a\left(\frac{t_1^{\,2}}{2} - \frac{\alpha t_1^{\,\beta+2}}{\beta+2} + \frac{\alpha t_1^{\,\beta+2}}{(\beta+1)(\beta+2)}\right) + b\left(\frac{t_1^{\,3}}{6} + \frac{\alpha t_1^{\,\beta+3}}{(\beta+2)(\beta+3)} - \frac{\alpha t_1^{\,\beta+3}}{2(\beta+3)}\right)\right\}$$

$$+ c_1a\left\{\frac{t_2^{\,2}}{2} + \frac{t_1^{\,2}}{2} + t_1t_2 + \frac{\alpha}{\beta+1}\left(t_2^{\,\beta+3} - t_1t_2^{\,\beta+2} + t_1^{\,\beta+3} - t_1^{\,\beta+2} - \frac{t_2^{\,\beta+2}}{\beta+2} + \frac{t_1^{\,\beta+2}}{\beta+2} + t_2t_1^{\,\beta+1} - t_2t_1^{\,\beta+2}\right) + \alpha\left(2t_2t_1^{\,\beta+1}\right.\right.$$

$$\left.\left. - \frac{t_2^{\,\beta+2}}{\beta+1} + \frac{2t_2t_1^{\,\beta+1}}{\beta+1} + \frac{t_2^{\,\beta+2}}{\beta+2} - \frac{t_1^{\,\beta+2}}{\beta+2} - \frac{t_1t_2^{\,\beta+1}}{\beta+1}\right)\right\} + c_1b\left\{\frac{t_2^{\,3}}{3} + \frac{t_1^{\,3}}{6} - \frac{t_2^{\,2}t_1}{2} + \frac{\alpha}{\beta+2}\left(t_2^{\,\beta+3} - t_1t_2^{\,\beta+2} - \frac{t_2^{\,\beta+2}}{\beta+2} + \frac{t_1^{\,\beta+2}}{\beta+2}\right)\right.$$

$$\left. + \frac{\alpha}{2}\left(\frac{t_2^{\,\beta+3}}{\beta+3} - \frac{t_1^{\,\beta+3}}{\beta+3} - \frac{t_2^{\,\beta+3}}{\beta+1} + \frac{t_2^{\,2}t_1^{\,\beta+1}}{\beta+1}\right)\right\} + \frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{\,2-\gamma} - (a+bt_1)^{2-\gamma} + (a+bt_3)^{2-\gamma} - (a+bt_4)^{2-\gamma}\right]$$

$$+ \frac{c_2a}{2}\left[t_2^{\,2} - 2t_2t_3 + \delta t_3^{\,2} + (\delta-1)t_4^{\,2} - 2(\delta-1)t_4t_3\right] + \frac{c_2b}{6}\left[2t_2^{\,3} - 3t_2^{\,2}t_3 + \delta t_3^{\,3} + 2(\delta-1)t_4^{\,3} - 3(\delta-1)t_4^{\,2}t_3\right]$$

$$+ c_3a\left(\delta t_1 - t_2\right) + \frac{1}{2}c_3b\left(\delta t_1^{\,2} - t_2^2\right)\Bigg], \gamma \neq 2 \tag{20}$$

The required optimum values of t_1, t_2, t_3 and t_4 which minimize the cost function C can be obtained from the solution of the following equations,

$$\frac{\partial C}{\partial t_1} = 0, \ \frac{\partial C}{\partial t_2} = 0, \frac{\partial C}{\partial t_3} = 0 \text{ and } \frac{\partial C}{\partial t_4} = 0. \tag{21}$$

Provided these values of $t_i (i = 1,2,3,4)$ obtain above equation satisfy the conditions $D_i > 0 (i = 1,2,3,4)$, where D_i is the Hessian determinant of order i given by:

$$D_i = \begin{vmatrix} c_{11} & c_{12} \ldots\ldots & c_{1i} \\ c_{21} & c_{22} \ldots\ldots & c_{2i} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ c_{i1} & c_{i2} \ldots & c_{ii} \end{vmatrix}$$

$$c_{ij} = \frac{\partial^2 C}{\partial t_i \partial t_j} \qquad (i, j = 1, 2, 3, 4)$$

We can expand the equations of (21) as follows,

$$\frac{\partial C}{\partial t_1} = c_1(\delta - 1)\left\{ a\left(t_1 - \alpha t_1^{\beta+1} + \frac{\alpha t_1^{\beta+1}}{\beta+1} \right) + b\left(\frac{t_1^2}{2} + \frac{\alpha t_1^{\beta+2}}{\beta+2} - \frac{\alpha t_1^{\beta+2}}{2} \right) \right\}$$
$$+ c_1 a\left[t_1 + t_2 + \frac{\alpha}{\beta+1}\left\{ (\beta+3)t_1^{\beta+2} - t_2^{\beta+2} - (\beta+2)t_1^{\beta+1} + t_1^{\beta+1} - (\beta+2)t_1^{\beta+1}t_2 + (\beta+1)t_1^{\beta}t_2 \right\} \right.$$
$$\left. + \alpha\left\{ 2(\beta+1)t_2 t_1^{\beta} + 2t_1^{\beta}t_2 - t_1^{\beta+1} - \frac{t_2^{\beta+1}}{\beta+1} \right\} \right] + c_1 b\left\{ \frac{t_1^2}{2} - \frac{t_2^2}{2} + \frac{\alpha}{\beta+2}\left(t_1^{\beta+1} - t_2^{\beta+2} \right) + \frac{\alpha}{2}\left(t_2^2 t_1^{\beta} - t_1^{\beta+2} \right) \right\}$$
$$+ c_3 a\delta + c_3 b\delta t + \alpha_1\delta\left(a + bt_1 \right)^{1-\gamma} = 0 \tag{22}$$

$$\frac{\partial C}{\partial t_2} = c_1 a\left[t_1 + t_2 + \frac{\alpha}{\beta+1}\left\{ (\beta+3)t_2^{\beta+2} - (\beta+2)t_1 t_2^{\beta+1} - t_2^{\beta+1} + t_1^{\beta+1} - t_1^{\beta+2} \right\} + \alpha\left\{ 2t_1^{\beta+1} - \frac{(\beta+2)t_2^{\beta+1}}{\beta+1} \right.\right.$$
$$\left.\left. + t_2^{\beta+1} - t_1 t_2^{\beta} + \frac{2t_1^{\beta+1}}{\beta+1} \right\} \right] + c_1 b\left\{ t_2^2 - t_1 t_2 + \frac{\alpha}{\beta+1}\left((\beta+3)t_2^{\beta+2} - (\beta+2)t_1 t_2^{\beta+1} - t_2^{\beta+1} \right) \right.$$
$$\left. + \frac{\alpha}{2}\left(t_2^{\beta+2} - \frac{(\beta+3)t_2^{\beta+2}}{\beta+1} + \frac{2t_2 t_1^{\beta+2}}{\beta+1} \right) \right\} + ac_2(t_2 - t_3) + bc_2(t_2^2 - t_2 t_3) - c_3 a - c_3 b t_2 = 0$$

(23)

$$\frac{\partial C}{\partial t_3} = c_2 a\left[\delta t_3 - t_2 - (\delta - 1)t_4 \right] + \frac{c_2 b}{2}\left[\delta t_3^2 - t_2^2 - (\delta - 1)t_4^2 \right] - \alpha_1\delta\left(a + bt_3 \right)^{1-\gamma} = 0 \tag{24}$$

and

$$\frac{\partial C}{\partial t_4} = -\frac{1}{t_4^2}\left[c_1(\delta - 1)\left\{ a\left(\frac{t_1^2}{2} - \frac{\alpha t_1^{\beta+2}}{\beta+2} + \frac{\alpha t_1^{\beta+2}}{(\beta+1)(\beta+2)} \right) + b\left(\frac{t_1^3}{6} + \frac{\alpha t_1^{\beta+3}}{(\beta+2)(\beta+3)} - \frac{\alpha t_1^{\beta+3}}{2(\beta+3)} \right) \right\} \right.$$
$$+ c_1 a\left\{ \frac{t_2^2}{2} + \frac{t_1^2}{2} + t_1 t_2 + \frac{\alpha}{\beta+1}\left(t_2^{\beta+3} - t_1 t_2^{\beta+2} + t_1^{\beta+3} - t_1^{\beta+2} - \frac{t_2^{\beta+2}}{\beta+2} + \frac{t_1^{\beta+2}}{\beta+2} + t_2 t_1^{\beta+1} - t_2 t_1^{\beta+2} \right) + \alpha\left(2t_2 t_1^{\beta+1} \right.\right.$$
$$\left.\left. - \frac{t_2^{\beta+2}}{\beta+1} + \frac{2t_2 t_1^{\beta+1}}{\beta+1} + \frac{t_2^{\beta+2}}{\beta+2} - \frac{t_1^{\beta+2}}{\beta+2} - \frac{t_1 t_2^{\beta+1}}{\beta+1} \right) \right\} + c_1 b\left\{ \frac{t_2^3}{3} + \frac{t_1^3}{6} - \frac{t_2^2 t_1}{2} + \frac{\alpha}{\beta+2}\left(t_2^{\beta+3} - t_1 t_2^{\beta+2} - \frac{t_2^{\beta+2}}{\beta+2} + \frac{t_1^{\beta+2}}{\beta+2} \right) \right.$$
$$\left. + \frac{\alpha}{2}\left(\frac{t_2^{\beta+3}}{\beta+3} - \frac{t_1^{\beta+3}}{\beta+3} - \frac{t_2^{\beta+3}}{\beta+1} + \frac{t_2^2 t_1^{\beta+1}}{\beta+1} \right) \right\} + \frac{\alpha_1\delta}{b(\gamma-2)}\left[a^{2-\gamma} - \left(a + bt_1 \right)^{2-\gamma} + \left(a + bt_3 \right)^{2-\gamma} \right]$$
$$\left. + \frac{c_2 a}{2}\left[t_2^2 - 2t_2 t_3 + \delta t_3^2 \right] + \frac{c_2 b}{6}\left[2t_2^3 - 3t_2^2 t_3 + \delta t_3^3 \right] + c_3 a\left(\delta t_1 - t_2 \right) + \frac{1}{2}c_3 b\left(\delta t_1^2 - t_2^2 \right) \right] + \frac{c_2 a}{2}[\delta - 1] \tag{25}$$

NUMERICAL EXAMPLES

Example-1: Consider $a = 8000$, $b = 80$, $c_1 = 4, c_3 = 60$, $\alpha = 0.01$, $\beta = 32$, $\alpha_1 = 16$, $\gamma = 0.8$ and $\delta = 5$ in appropriate units. By the help of Mathematica 5.1, we obtain the optimum solution for t_1 and t_2 of Equation (12) and (13) of Model-I, as $t_1^* = 1.39585$ and $t_2^* = 0.0881263$. Putting t_1^* and t_2^* in (10), we get the optimum average cost as $C^* = 39186600$.

Example-2: Consider $a = 8000$, $b = 80$, $c_1 = 4, c_2 = 400, c_3 = 60, \alpha = 0.01$, $\beta = 32, \alpha_1 = 16$, $\gamma = 0.8$ and $\delta = 5$ in appropriate units. By the help of Mathematica 5.1, we obtain the optimum solution for t_1, t_2, t_3 and t_4 of Equation (22)-(25) of Model-II as $t_1^* = 1.3057$, $t_2^* = 0.0630253$, $t_3^* = 1.29626$ and $t_4^* = 1.60219$. Putting t_1^*, t_2^*, t_3^* and t_4^* in (20), we get the optimum average cost as $C^* = 3882610$.

SENSITIVITY ANALYSIS

Table 1. The summary of the sensitivity analysis when shortage is not permitted
Tabela 1. Podsumowanie analizy wrażliwości, w przypadku gdy braki nie są dopuszczne

Parameter	% Change	t_1^*	t_2^*	C^*
a	+50	1.39582	0.0879105	58783300
	+25	1.39584	0.0879968	78985000
	-25	1.39588	0.0883921	29388200
	-50	1.39594	0.0887738	19589600
b	+50	1.39589	0.0884499	39180100
	+25	1.39587	0.0882881	39183400
	-25	1.39583	0.0879645	39189900
	-50	1.39581	0.0878028	39193200
c_1	+50	1.37953	0.0873744	39905300
	+25	1.38681	0.087713	39539700
	-25	1.40773	0.0886577	38854700
	-50	1.42482	0.0894039	38563800
c_3	+50	1.41265	0.0888746	58123400
	+25	1.40504	0.0885385	48646500
	-25	1.38422	0.0875933	29747200
	-50	1.36831	0.086339	20333100
α	+50	1.37862	0.0873913	39008000
	+25	1.38634	0.0877217	39087900
	-25	1.40821	0.0886481	39315600
	-50	1.42582	0.0893836	39500600
β	+50	1.26053	0.0572185	54651400
	+25	1.31376	0.0694663	46867000
	-25	1.53962	0.1196	31750300
	-50	1.85897	0.180803	25142600
α_1	+50	1.39586	0.0881265	39190500
	+25	1.39586	0.0881264	39188500
	-25	1.39585	0.0881262	39184700
	-50	1.39585	0.0881261	39182800
γ	+50	1.39585	0.088126	39179200
	+25	1.39585	0.088126	39180200
	-25	1.39589	0.0881282	39225200
	-50	1.39614	0.0881395	39458600
δ	+50	1.39454	0.136574	38010300
	+25	1.39506	0.112353	38471300
	-25	1.39721	0.0638857	40446500
	-50	1.40011	0.0396085	43256500

Table 2. The summary of the results when shortage is permitted
Tabela 2. Podsumowanie analizy wrażliwości, w przypadku gdy braki są dopuszczne

Parameter	% Change	t_1^*	t_2^*	t_3^*	t_4^*	C^*
a	+50	1.30545	0.0627687	1.28755	1.59218	5810750
	+25	1.30555	0.0628722	1.29108	1.59624	4846650
	-25	1.30595	0.0632788	1.30481	1.61201	2918670
	-50	1.30645	0.0637925	1.32236	1.63214	1955050
b	+50	1.30620	0.0634873	1.31364	1.62257	3896440
	+25	1.30596	0.0632647	1.30541	1.61295	3889470
	-25	1.30538	0.0627482	1.28508	1.58889	3875860
	-50	1.3049	0.0623699	1.26845	1.56884	3869390
c_1	+50	1.28972	0.0616404	1.30032	1.6076	3901160
	+25	1.29688	0.0622994	1.29788	1.60439	3890840
	-25	1.3172	0.0638555	1.29599	1.60165	3879820
	-50	1.33362	0.064873	1.29827	1.60424	3879560
c_2	+50	1.3124	0.0669311	1.05714	1.30316	4708070
	+25	1.30934	0.0652184	1.15706	1.42816	4318250
	-25	1.30133	0.0601288	1.51315	1.87311	3383250
	-50	1.29659	0.0566277	1.95771	2.4274	2792670
c_3	+50	1.31611	0.0603377	1.62877	2.01708	4792530
	+25	1.31127	0.0615268	1.46377	1.81127	4357670
	-25	1.29887	0.0648439	1.11777	1.37925	3349630
	-50	1.28964	0.0670109	0.915741	1.12677	2727140
α	+50	1.28951	0.062438	1.28678	1.59052	3857560
	+25	1.29676	0.062702	1.29103	1.59575	3868800
	-25	1.31732	0.0634426	1.30306	1.61056	3900500
	-50	1.33386	0.0640314	1.31274	1.62249	3925890
β	+50	1.1962	0.0411447	1.22803	1.52255	3777820
	+25	1.23904	0.0498266	1.25489	1.55389	3819060
	-25	1.42352	0.0852584	1.36861	1.68687	3993770
	-50	1.68695	0.128721	1.53166	1.87934	4242310
α_1	+50	1.30546	0.0628674	1.28759	1.59141	3882290
	+25	1.30558	0.0629467	1.29193	1.59681	3882430
	-25	1.30582	0.0631032	1.30058	1.60756	3882830
	-50	1.30595	0.0631805	1.30488	1.61291	3883100
γ	+50	1.30617	0.0633205	1.31274	1.6227	3883690
	+25	1.30609	0.0632727	1.31004	1.61934	3883960
	-25	1.30355	0.0616149	1.22238	1.51027	3886490
	-50	1.28508	0.0460192	0.719419	0.8857	4571920
δ	+50	1.31468	0.105706	1.68616	1.92709	4890050
	+25	1.31061	0.0841612	1.49773	1.76472	4419500
	-25	1.29924	0.042317	1.0688	1.43945	3243350
	-50	1.28926	0.0219905	0.786866	1.29352	2416700

To study the effect of changes in the system parameters a, b, c_1, c_2, c_3, α, β, , α_1, γ and δ on optimal cost derived by the above proposed method a sensitivity analysis has been performed considering the two numerical examples given at section-4 above. Sensitivity analysis has been done by changing(increasing or decreasing) one parameter at a time by 25% and 50% and keeping the remaining parameters at their original values. Table 1 and Table 2 summarize these results.

Based on the results of Table 1, the following observations can be made.

(i) An increase on the values of any one of the parameters a , c_1, c_3 , α_1 and β will result in an increase on C^*.

(ii) An increase in the values of any one of the parameters b,α , γ,and δ will result in an decrease on C^*.

On the results of Table 2, the following observations can be made.

(i) An increase on the values of any one of the parameters a, b, c_1, c_2, c_3 and δ will result in an increase on C^*.

(ii) An increase in the values of any one of the parameters α, β, α_1 and γ will result in an decrease on C^*.

CONCLUSIONS

Inventory models for deteriorating items with Weibull deterioration rate, time dependent demand rate and unit cost of production have been developed in this paper. The two-parameter Weibull distribution is appropriate for an item with decreasing rate of deterioration only if the initial rate of deterioration is extremely high. Similarly, this distribution can also be used for an item with increasing rate of deterioration only if the initial rate is approximately zero. So these models are not very much relevant for the items which don't confirm these conditions of deterioration. Further it is observed from the sensitivity analysis that the model with shortages and backlogging is considered to be better economically. Potential future research work in this line can be done by further extending the models for items having quadratic demand or power demand with three parameter Weibull distribution..

REFERENCES

Aggarwal S.P., 1978. A note on an order-level inventory model for a system with constant rate of deterioration, Opsearch, 15, 184-187.

Bahari-Kashani H., 1989. Replenishment schedule for deteriorating items with time-proportional demand, Journal of the Operational Research Society, 40, 75-81.

Berrotoni J.N., 1962. Practical applications of Weibull distribution. ASQC Technical Conference Transactions, 303-323.

Chuang J.P.C., 2012, Inventory model with stock dependent selling rate. African Journal of Business Management, 6(3), 780-786.

Covert R.P., Philip G.C., 1973. An EOQ model for items with Weibull distribution deterioration, AIIE Transaction, 5, 323-326.

Dave U., 1986. An order-level inventory model for deteriorating items with variable instantaneous demand and discrete opportunities for replenishment, Opsearch, 23, 244-249.

Dave U., Patel L.K., 1981. (T,Si) policy inventory model for deteriorating items with time proportional demand", Journal of the Operational Research Society, 32, 137-142.

Deb M., Chaudhuri K.S., 1986. An EOQ Model for items with finite rate of production and variable rate of deterioration, Opsearch, 23, 175-181.

Ghare P.M., Schrader G.P., 1963. A model for exponentially decaying inventories, Journal of Industrial Engineering, 14, 238-243.

Ghosh S. K., Goyal S. K., Chaudhuri K. S. 2006. An inventory model with Weibull demand rate, finite rate of production and shortages, International Journal of Systems Science, Vol. 37, No. 14, 15 November, 1003-1009

Giri B.C., Jalan A.K., Chaudhuri K.S. 2003. Economic order quantity model with Weibull deteriorating distribution, shortage and ramp-type demand, International Journal of Systems Science, 34, 237-243.

Goswami A., Chaudhuri K.S., 1991. An EOQ model for deteriorating items with shortages and a linear trend in demand, Journal of the Operational Research Society, 42, 1105-1110.

Goswami A., Chaudhuri K.S., 1992. Variations of order-level inventory models for deteriorating items, International Journal of Production Economics, 27, 111-117.

Goyal S.K., Giri B.C., 2001. Recent trends in modelling of deteriorating inventory, European Journal of Operational Research, 134, 1-26.

He Y., Wang S., Lai K.K., 2010, An optimal production-inventory model for deteriorating items with multiple-market demand. European Journal of Operational Research, 203, 3, 593-600.

Jalan A.K., and Chaudhuri K.S., 1999. Structural properties of an inventory system with deterioration and trended demand, International J. of Systems Science, 30, 627-633.

Meher M.K., Panda G.Ch., Sahu S.K., 2012, An inventory model with Wiebull deterioration rate under the dealy in payment in demand decling market. Applied Mathematical Sciences, 6, 23, 1121-1133.

Mishra R.B., 1975. Optimum production lot-size model for a system with deteriorating inventory, International Journal of Production Research, 13, 495-505.

Nahmias S., 1982. Perishable inventory theory: A review, Operations Research, 30, 680- 708.

Papachristos S., and Skouri K., 2000. An optimal replenishment policy for deteriorating items with time varying demand and partial exponential type-backlogging, Operations Research Letter 27, 4, 175-184.

Philip G.C., 1974. A generalized EOQ model for items with Weibull distribution deterioration, AIIE Transaction, 6, 159-162.

Rafaat F., 1991. Survey of literature on continuously deteriorating inventory model, Journal of the Operational Research Society, 42, 27-37.

Roy T., Chaudhuri K. S., 2009. A production-inventory model under stock-dependent demand, Weibull distribution deterioration and shortage, Intl. Trans. in Op. Res, 16 325-346

Shah Y. K. Jaiswal M. C. 1977. An order-level inventory model for a system with constant rate of deterioration, Opsearch, 14, 174-184.

Tripathy C. K., Mishra U., 2010, An EOQ model for linear deteriorating rates with shortage and permissible delay in payment. The IUP Journal of Operations Management, IX, 4, 7-20.

Tripathy C. K., Mishra U., 2011. An EOQ model with time dependent Weibull deterioration and ramp type demand, International Journal of Industrial Engineering Computations, 2, 307-318.

Tripathy C. K., Pradhan L.M., 2010a, An EOQ model for three parameter Weibull deterioration with permissible delay in payments and associated salvage value. International Journal of Industrial Engineering Computations 3, 2012, 115-122.

Tripathy C. K., Pradhan L.M., 2010b, An EOQ model for Weibull deteriorating items with power demand and partial backlogging. Int.J.Contemp.Math.Sciences, 5, 38, 1895-1904.

Tripathy R.P., Kumar M., 2011, An EOQ model with time-dependent demand rate under trade credits, Int.J.of Mathematical Sciences and Applications, 1, 3.

Tripathy R.P., 2011, EOQ model with time dependent demand rate and time dependent holding cost function. International Journal of Operations Research and Information Systems, 2(3), 79-92.

Whitin T.M., 1957, Theory of Inventory Management, Princeton University Press, Princeton, NJ.

MODEL EOQ USZKODZEŃ WEIBULLA ZE ZMIENNĄ ZALEŻNĄ CZASU ORAZ LINIOWYM POPYTEM I BRAKAMI

STRESZCZENIE. **Wstęp**: W ostatnim czasie coraz większego znaczenia nabierają prace badawcze z zakresu kontroli i utrzymania zapasów towarów łatwo psujących się. Problem psucia się towarów jest bardzo istotnym zagadnieniem w wielu systemach magazynowania. Psucie się definiowane jest jako obniżenie jakości lub uszkodzenia, które powodują, że dany towar nie może być użyty zgodnie z jego pierwotnym przeznaczeniem.
Metody: W pracy opracowano model oparty na systemie poziomu zamówienia dla towarów łatwo psujących się, charakteryzujących się popytem liniowym oraz uszkodzeń Weibulla. Przy opracowaniu modelu założono, że wielkość produkcji i popytu jest zależną czasu. Jednostkowy koszt produkcji jest odwrotnie proporcjonalny do popytu. System produkcyjno-magazynowy obejmuje dwa parametry uszkodzeń Weibulla.
Wyniki i wnioski: Zostały opracowane dwa modele, jeden przeznaczony dla sytuacji bez braków oraz drugi uwzględniający braki, które przyczyniają się do powstawania zaległości. Celem modelu było opracowanie optymalnego sposobu postępowania minimalizującego średni koszt całkowity. Przedstawiono analizę wrażliwości celem wykazania wpływu zmian parametrów na optymalny średni koszt całkowity.

Słowa kluczowe: popyt, uszkodzenie Weibulla, jednostkowy koszt produkcji, braki.

EIN EOQ-MODELL FÜR DIE WEIBULL-BESCHÄDIGUNGEN MIT DER ABHÄNGIGEN VARIABLEN DER ZEIT BEI LINEARER NACHFRAGE UND MANGELWAREN

ZUSAMMENFASSUNG. Einleitung: Forschungsarbeiten im Bereich der Kontrolle und Aufrechterhaltung der Bestände von leicht verderbenden Waren gewinnen in der letzten Zeit deutlich an Bedeutung. Das Problem des Warenverderbs stellt in vielen Lagerungssystemen einen wesentlichen Schwerpunkt dar. Das Verderben selbst wird als Verminderung der Qualität oder Beschädigung definiert. Die Veränderung der Parameter ist die Ursache dafür, dass die verdorbene Ware ihrem primären Verwendungszweck gemäß nicht mehr angewendet werden darf.
Methoden: Im Rahmen der Arbeit hat man ein Modell ausgearbeitet, welches gestützt ist auf das Niveau der Bestellung der leicht verderbenden Waren, die sich durch die lineare Nachfrage und Weibull-Beschädigungen charakterisieren. Beim Konzipieren des Modells hat man angenommen, dass die Produktions- und Nachfragegröße eine Variable der Zeit darstellen. Die Produktionskosten pro Einheit sind umgekehrt proportional zur Nachfrage. Das Produktions -und Lagerungssystem umfasst zwei Parameter der Weibull-Beschädigungen.
Ergebnisse und Fazit: Es wurden zwei Modelle bearbeitet; das eine vorgesehen für die Situation ohne Mangelwaren, das andere für die Situation mit solchen Mangelwaren, die die Entstehung von Rückständen verursachen. Dem Ziel des Modell-Konzeptes lag Bearbeitung einer optimalen Verfahrensweise für die Minimalisierung der durchschnittlichen Gesamtkosten zugrunde. Dabei stelle man die Analyse der Empfindlichkeit zwecks Aufzeigen der Beeinflussung der optimalen, durchschnittlichen Gesamtkosten durch die Veränderung der relevanten Parameter dar.

Codewörter: Nachfrage, Weibull-Beschädigung, Produktionskosten pro Einheit, Mangelwaren.

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