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ORIGINAL PAPER

OPTIMIZATION OF STOCHASTIC PRODUCTION-INVENTORY MODEL FOR DETERIORATING ITEMS IN A DEFINITE CYCLE USING HAMILTON-JACOBI-BELLMAN EQUATION

Bui Minh Hau[1], Hwan-Seong Kim[1], Le Ngoc Bao Long[1], Sam-Sang You[2]

1) Department of Logistics, Korea Maritime and Ocean University, Busan, **Republic of Korea**
2) Division of Mechanical Engineering, Korea Maritime and Ocean University, Busan, **Republic of Korea**

ABSTRACT. Background: Inventory control is essential for a manufacturer to achieve the desired profit in successful supply chain management. This paper deals with the production-inventory system under the decrease in production rate. The model includes three stages: before the decrease in production, after the decrease in production, and after a period of inventory shortage. Throughout the stages, the stochastic inventory model is always affected by random factors and the deterioration of inventory quality.

Method: The article uses the economic order quantity (EOQ) framework to evaluate costs in the production-inventory model. To optimize the manufacturer's profit with the stochastic factor, Hamilton–Jacobi–Bellman (HJB) equation is presented to find the production rate to make the inventory model to guarantee its intended goals in a determined cycle.

Result: Analytical solutions are provided for optimization of the stochastic production-inventory model. Numerical experiments show that inventory level, production rate, and profit over time are based on the optimal initial value of the production rate.

Conclusion: The manufacturer's profit comes from the stages of importing raw materials, processing and producing, storing and supplying items. Finding the initial value of the production rate can make the inventory level and production rate to ensure their desired value and get the target profit within a specified time.

Keywords: Production inventory model, deteriorating items, stochastic optimal control, HJB equation

INTRODUCTION

In commerce, inventory is *a valuable collection of assets* of a manufacturer. By keeping the *optimal stock* levels, the company operates continuously and independently. As illustrated in Figure 1, production-inventory management is *crucial* processes for any business because it can help manufacturers adjust production strategies to ensure business success and *profitability*. Manufacturers want to produce the right amount of goods based on customer demands that generally change over time. In addition, the production rate depends on the quantity of raw materials imported from outside sources. In general, each manufacturer will have the desired production rate and inventory levels targets to guarantee adequate supply to customers and keep businesses running smoothly.

In reality, many deterministic and random factors might affect the production-inventory system. Many items with short life cycles will *often deteriorate during storage* and the original quality will decline *or be lost*, leading to *a small amount of decay*. In a real-life inventory system, the deteriorating inventory is concerned with the marginal value of a commodity caused by damage, obsolescence, rust, humidity, spoilage, etc. The deterioration rate can be equivalent to the maximum useful life of the product. The deteriorating inventory model is widely employed to analyze food

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production, meat, and fast foods with short shelf life. The deteriorating inventory model has appeared for a long time and is becoming a serious *concern* for manufacturers. [Chung & Tsai, 2001] presented the deteriorating model with linear customer demand. [Chen & Lin, 2002] demonstrated the model with a normal distribution on soft life, inventory shortage, and continuous-time demand. Return of goods from the customer also has a significant effect on managing inventory levels. In addition, there will be delays in the supply of raw material *for the production process due to poor supply chain management, typically characterized by an* inability to predict demand. Transportation delays can also increase the related costs, resulting in negative impacts on overall profitability. According to [Hatipoğlu et al., 2022], the cost of flight delays is approximately $ 8.3 billion, and this also causes economic loss in the United States about $ 600 million per year. The impact of supply chain disruption is clearly seen in port congestion at the world's major container *ports* during the Covid-19 pandemic. Many countries have adopted lockdown measures to prevent the disease from spreading. Port congestion occurs due to shortages of maritime participants to handle containers imported into the seaports. The impact of port congestion will lead to difficulty in delivery and significantly to shipping delays and supply chain disruptions. Typically, there are several types of costs associated with inventory management, such as replenishment costs and maintenance costs [Krzyżaniak, 2022]. The economic order quantity (EOQ) is a good approach and highly appreciated method to evaluate total costs for efficient inventory management in the literature. This model mainly is intended to minimize the inventory costs such as ordering cost, holding cost, and shortage cost in business operation. [Jamal et al., 1997] proposed the EOQ inventory model for inventory management. [Wee et al., 2003] developed the EOQ model with a temporary sale price. [Li et al., 2015] used the Hamilton–Jacobi–Bellman (HJB) equation for optimal dynamic pricing in the stochastic inventory system. [Alshamrani, 2013] considered the HJB equation for optimizing the stochastic production–inventory model with deteriorating items. With effective inventory management, businesses would experience higher levels of profit with lower costs, guaranteeing an enhanced competitive advantage over others. Recently, many papers have been presented on the stochastic inventory model summarized in Table 1.

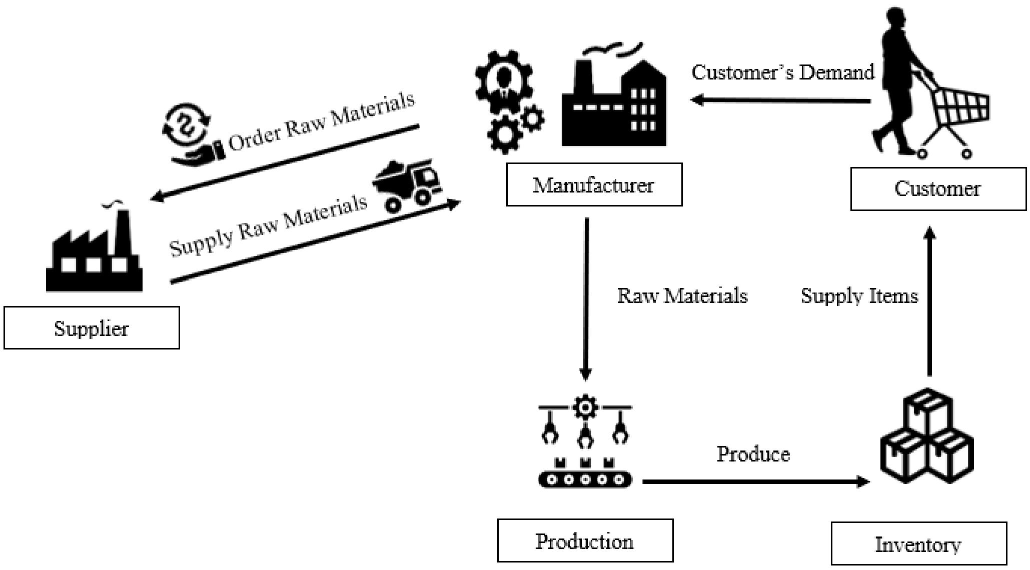

Fig. 1. Generic production–inventory model

Table 1. Literature review of the stochastic inventory model

Article	Deteriorating item	Number of stages in inventory model	Optimization method	Key findings
Li et al. (2015)	No	1	Hamilton-Jacobi-Bellman equation	Dynamic pricing and inventory control for a stochastic inventory system
Alshamrani (2013)	Yes	1	Hamilton-Jacobi-Bellman equation	Optimal expected production rate and expected inventory level
Soni & Suthar (2019)	Yes	3	Two-stage optimization procedure	Optimal price and replenishment cycle of the stochastic demand inventory model
This study	Yes	3	Hamilton-Jacobi-Bellman equation	Optimal initial production rate for the deteriorating inventory model

The main contents of this study are described as follows:

- By considering the deteriorating factor in stochastic production-inventory model, this study extends the model with decreasing coefficient in the production rate.

- The EOQ framework has been employed in the production-inventory model to evaluate costs.

- The optimization problem is analytically solved by the HJB equation, demonstrated by numerical simulations.

This paper is organized as follows. Section 2 provides a problem description in notation. Section 3 presents the production-inventory model and the optimal solution. The article employs the HJB equation to determine the initial production rate to optimize the total profit. Section 4 deals with numerical experiments to demonstrate the feasibility and efficacy of the proposed method. Finally, the conclusions are presented in Section 5.

NOTATIONS AND PROBLEM DEFINITION

This paper will deal with a deteriorating inventory model with a *decrease in supply*. Due to supply chain disruptions, any *delay* in the *receipt* of *raw materials* could affect the costs to order raw materials. When resources cannot be replenished in time, this will result in insufficient inventory level for customer demands, significantly impacting on manufacturer's profits. When there is a shortage of inventory, customers are more likely to cancel orders due to supply issues. Sometimes *customer* retention is less expensive than acquisition. *Rewarding customers* will become a staple of business scheme, by *providing* them with special services and attention, such as discounts, rebates, gifts, etc. In this study, it is assumed that inventory is deteriorating at the beginning of the cycle while customer demand constant. The objective is to find an optimal production-inventory policy to maximize company profit. In addition, this article employs the EOQ framework to evaluate the costs affected by the inventory model. For model development, the *notation* or *symbols are* described in Table 2.

Table 2. Notation of the production-inventory model

Variable	Definition	Unit
$D(t)$	Customer's demand rate at time t	Items/time
$u_1(t)$	Production rate in the interval $(0, t_1)$	Items/time
$u_2(t)$	Production rate in the interval (t_1, t_2)	Items/time
$u_3(t)$	Production rate in the interval (t_2, t_3)	Items/time
$u(t)$	Production rate at time t	Items/time
$I_1(t)$	Inventory level in the interval $(0, t_1)$	Items
$I_2(t)$	Inventory level in the interval (t_1, t_2)	Items
$I_3(t)$	Inventory level in the interval (t_2, t_3)	Items
γ	Decreased coefficient of production rate $0<\gamma<1$	Non-dimensional
u_d	The desired production rate	Items/time
I_d	The desired inventory level	Items
θ	Deteriorating rate of inventory level $0<\theta<1$	Non-dimensional
$dz(t)$	Stochastic variable (Weiner process)	Non-dimensional
σ	Diffusion coefficient	Non-dimensional
o	Ordering cost unit for 1 item	USD
h	Holding cost unit for 1 item	USD
s	Shortage cost unit for 1 item	USD
p	Price unit	USD
α	Increase the coefficient of ordering cost	USD
O	Ordering cost	USD
H	Holding cost	USD
S	Shortage cost	USD
TP	Total profit	USD

MODEL FORMULATION AND SOLUTION APPROACH

With problem descriptions, the behaviors of the inventory model can be categorized in three stages, as shown in Figure 2. Stage 1 is given in the time interval $(0,t_1)$, which is the period prior to inventory problems that cause a delay in the supply of raw materials by the suppliers. Stage 2 is described in the time interval (t_1,t_2). This is the specific period when suppliers are short of raw materials to deliver to manufacturers, and the cost of raw materials could also increase due to competition. The purchasing power of manufacturers will remain unchanged, but the source of raw materials might be decreased. The production rate decreases due to the shortage of production materials, the decreasing coefficient (γ) is used to represent the decrease in the production rate, and the unit price of raw materials will also increase, leading to an increase in ordering cost (O) with the increasing coefficient (α). The last stage is described in the interval (t_2,t_3) when production cannot produce the required number of items; there is a shortage of inventory. At this stage, some customers will continue to wait to receive the goods or try to cancel the orders. The customer cancels orders, leaving the manufacturer lose the opportunity to sell the product.

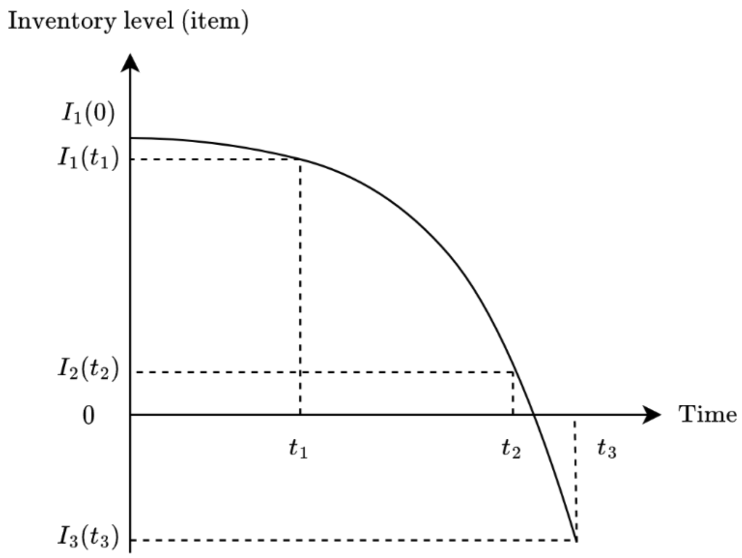

Fig. 2. Pattern of inventory level over time

Several unspecified factors that can influence inventory behavior, such as sales return, inventory spoil, machine breaking down, and human error [Li et al., 2015]. They can be lumped as a Weiner process, $dz(t)$, which is a real-valued *stochastic variable.* The inventory level can be described in terms of three stages:

$$dI = \begin{cases} dI_1 = \left[u(t) - D(t) - \theta I_1(t)\right]dt + \sigma dz(t), & 0 \le t \le t_1 \\ dI_2 = \left[\gamma u(t) - D(t) - \theta I_2(t)\right]dt + \sigma dz(t), & t_1 \le t \le t_2 \\ dI_3 = \left[\gamma u(t) - D(t) - \theta I_3(t)\right]dt + \sigma dz(t), & t_2 \le t \le t_3 \end{cases} \quad (1)$$

where $u(t)$ is the production rate at the time t and it depends on the inventory level at that time. For the interval $(0, t_1)$, there is no decrease in production so that $u_1(t)$, the production rate at stage 1, is equal to $u(t)$. In case of stage 2, the production rate is $u_2(t) = \gamma u(t)$ with $t \in (t_1, t_2)$ due to the decrease in production. Similar to stage 2, the stage 3 has the production rate $u_3(t) = \gamma u(t)$ with $t \in (t_2, t_3)$. The EOQ framework is used to evaluate the costs in the model. For the production-inventory model, there are two common types of costs: ordering cost (O) and holding cost (H). However, at stage 3, the inventory level is possibly negative. If the inventory goes negative, the manufacturer will lose a portion of the profit, which is called the shortage cost (S). The cost (O) includes costs for ordering raw materials and making the product. Manufacturers need to purchase raw materials from suppliers and the pay transportation cost and operation cost. The ordering cost will be based on the quantities of products, u is production rate and u_d is desired production rate. At the interval (t_1, t_3), there is an event that causes increased transportation costs, leading to an increase in the cost of ordering materials,

$$O=\int_0^{t_1} o\left(u-u_d\right)^2 dt+\int_{t_1}^{t_2} o\alpha\left(\gamma u-u_d\right)^2 dt+\int_{t_2}^{t_3} o\alpha\left(\gamma u-u_d\right)^2 dt \tag{2}$$

Items kept in warehouses *must be carefully stored* and managed to protect them from damage or loss while ensuring original quality. There are also warehouse maintenance costs and equipment maintenance fees. These costs are called holding cost (H) in the EOQ model,

$$H=h\left[\int_0^{t_1}\left(I_1-I_d\right)^2 dt+\int_{t_1}^{t_2}\left(I_2-I_d\right)^2 dt+\int_{t_2}^{t_3}\left(I_3-I_d\right)^2 dt\right] \tag{3}$$

The inventory levels in time intervals $(0,t_1)$, (t_1,t_2), (t_2,t_3) are described as I_1, I_2 and I_3 respectively. I_d is the desired inventory level. The shortage cost (S) occurs when the inventory is not enough for the customer demand *as a consequence of a stockout,*

$$S=\int_{t_2}^{t_3} k\left[s\left(-I_3\right)\right] dt \tag{4}$$

where k is a coefficient that depends on the sign of I_3. If $I_3 \le 0$, k will be equal to 1. Otherwise, k will be 0. The gross profit (TP) for the entire cycle T is given by deducting the costs from the total amount earned. To achieve maximum profitability, the research aims to find the initial value of the production rate so that the total cost incurred is zero to maximize profit. In this case, the revenue achieved will be the desired profit. The cycle T is the total time that will be given by, $T=t_3$. Then, the gross profit is described below,

$$\begin{aligned}TP &= Desired\ profit+\int_0^T(-O-H-S)dt\\ &= B_1p_1I_1(t_1)+\int_0^{t_1}\left[-o\left(u-u_d\right)^2-h\left(I_1-I_d\right)^2\right]dt\\ &\quad B_2p_2I_2(t_2)+\int_{t_1}^{t_2}\left[pD-o\alpha\left(\gamma u-u_d\right)^2-h\left(I_2-I_d\right)^2\right]dt\\ &\quad B_3p_3I_3(t_3)+\int_{t_2}^{t_3}\left[pD-o\alpha\left(\gamma u-u_d\right)^2-h\left(I_3-I_d\right)^2+ksI_3\right]dt\end{aligned} \tag{5}$$

where B_1, B_2, and B_3 are the profit rates and p_1, p_2, and p_3 are the price units of items. The objective functions for inventory levels are described in three separate stages:

$$J_1=\max\left\{\int_0^{t_1}\left[-o\left(u-u_d\right)^2-h\left(I_1-I_d\right)^2\right]dt+B_1p_1I_1(t_1)\right\} \tag{6}$$

$$J_2 = \max\left\{\int_{t_1}^{t_2}\left[-o\alpha\left(\gamma u-u_d\right)^2-h\left(I_2-I_d\right)^2\right]dt+B_2p_2I_2\left(t_2\right)\right\} \tag{7}$$

$$J_3 = \max\left\{\int_{t_2}^{t_3}\left[-o\alpha\left(\gamma u-u_d\right)^2-h\left(I_3-I_d\right)^2+ksI_3\right]dt+B_3p_3I_3\left(t_3\right)\right\} \tag{8}$$

Let $V_1\left(I_1,t\right)$ denote the expected value of the objective function J_1 in the interval $\left(0,t_1\right)$ such that it satisfies Hamilton–Jacobi–Bellman (HJB) equation [Sethi & Thompson, 2000] described by,

$$0=\max_{u(t)}\left\{\left[-o\left(u-u_d\right)^2-h\left(I_1-I_d\right)^2\right]+V_t+V_{I_1}\left[u-D-\theta I_1\right]+\frac{1}{2}\sigma^2V_{I_1I_1}\right\} \tag{9}$$

where,

$$V_t=\frac{\partial V_1}{\partial t},\ V_{I_1}=\frac{\partial V_1}{\partial I_1},\ V_{I_1I_1}=\frac{\partial^2V_1}{\partial {I_1}^2}$$

Taking a partial derivative of equation (9) with respect to u gives,

$$u=\frac{V_{I_1}+2ou_d}{2o} \tag{10}$$

Substituting equations (10) into (9) yields

$$0=-o\left(\left(\frac{V_{I_1}+2ou_d}{2o}\right)^2-2u_d\frac{V_{I_1}+2ou_d}{2o}+{u_d}^2\right)-h\left({I_1}^2-2I_dI_1+{I_d}^2\right)$$
$$+V_t+V_{I_1}\left[\frac{V_{I_1}+2ou_d}{2o}-D-\theta I_1\right]+\frac{1}{2}\sigma^2V_{I_1I_1} \tag{11}$$

Let us consider the following set of conditions:

$$V\left(I_1,t\right)=Q_1I_1^2+R_1I_1+M_1 \tag{12}$$

$$V_{I_1}=2Q_1I_1+R_1 \tag{13}$$

$$V_{I_1I_1}=2Q_1 \tag{14}$$

$$V_t=\dot{Q}_1I_1^2+\dot{R}_1I_1+\dot{M}_1 \tag{15}$$

Equation (12) provides the generic form of the expected value $V(I_1,t)$ in stage 1, $t \in (0,t_1)$. Q_1, R_1, and M_1 are variables which will be changed over time in stage 1. is the inventory level at time t of the period $(0,t_1)$. Substituting equations (12), (13), (14), and (15) into (11) gives

$$0 = \left(\frac{Q_1^2}{o} - 2Q_1\theta - h + \dot{Q}_1 \right) I_1^2 + \left(\frac{R_1Q_1}{o} + 2hI_d + \dot{R}_1 - 2Q_1D - \theta R_1 + 2u_dQ_1 \right) I_1 + \frac{R_1^2}{4o} + u_dR_1 - hI_d^2 + \dot{M}_1 - DR_1 + \sigma^2 Q_1 \quad (16)$$

Equation (16) provides all inventory levels, so that

$$\dot{Q}_1 = -\frac{1}{o}Q_1^2 + 2\theta Q_1 + h \quad (17)$$

$$\dot{R}_1 = \left(\theta - \frac{Q_1}{o} \right) R_1 + \left(2Q_1D - 2Q_1u_d - 2hI_d \right) \quad (18)$$

$$\dot{M}_1 = -\frac{R_1^2}{4o} + R_1D - R_1u_d + hI_d^2 - \sigma^2 Q_1 \quad (19)$$

At the time $t = t_1$, the moment at the beginning of the decline in production due to the lack of raw materials, the expected value of $V_1(I_1,t)$ at this time can be described by $B_1p_1I_1(t_1)$,

$$\begin{cases} Q(t_1) = 0 \\ R(t_1) = B_1p_1 \\ M(t_1) = 0 \end{cases} \quad (20)$$

Equation (17) can be solved as follows:

$$\begin{cases} Q_1 = \dfrac{a_1\left(y_1 - 1\right)}{y_1 - \dfrac{a_1}{b_1}} \\ y_1 = e^{\frac{b_1 - a_1}{o}\left(t_1 - t\right)} \\ a_1 = o\theta - o\sqrt{\theta^2 + \dfrac{h}{o}} \\ b_1 = o\theta + o\sqrt{\theta^2 + \dfrac{h}{o}} \end{cases} \tag{21}$$

In addition, equation (18) can be solved by

$$\begin{aligned} R_1 &= e^{-\int\left(\frac{Q_1}{o} - \theta\right)dt}\left[\int\left(2DQ_1 - 2Q_1u_d - 2hI_d\right)e^{\int\left(\frac{Q_1}{o} - \theta\right)dt}dt + C_1\right] \\ &= \frac{\sqrt{y_1}}{y_1 + r_1}\left[\frac{\left(4D - 4u_d\right)a_1o\left(y_1 + 1\right) - 4hI_d o\left(y_1 - r_1\right)}{\left(a_1 - b_1\right)\sqrt{y_1}} + C_1\right] \end{aligned} \tag{22}$$

where

$$\begin{cases} r_1 = -\dfrac{a_1}{b_1} \\ C_1 = \left(1 + r_1\right)B_1p_1 - \dfrac{\left(8D - 8u_d\right)a_1o - 4hI_d o\left(1 - r_1\right)}{a_1 - b_1} \end{cases} \tag{23}$$

According to Equation (10), the production rate in stage 1 is described by

$$u_1 = u = \frac{2Q_1I_1 + R_1 + 2ou_d}{2o} \tag{24}$$

Similarly, the optimal production rates in the intervals $\left(t_1, t_2\right)$ and $\left(t_2, t_3\right)$ will be discussed in the following. The boundary conditions at stages 2 and 3 are, respectively, described below

$$\begin{cases} Q_2\left(t_2\right) = 0; \\ R_2\left(t_2\right) = B_2p_2; \qquad t \in \left(t_1, t_2\right) \\ M_2\left(t_2\right) = 0; \end{cases} \tag{25}$$

$$\begin{cases} Q_3\left(t_3\right) = 0; \\ R_3\left(t_3\right) = B_3p_3; \qquad t \in \left(t_2, t_3\right) \\ M_3\left(t_3\right) = 0; \end{cases} \tag{26}$$

The optimal production rate in the interval $\left(t_1, t_2\right)$ is described by

$$\begin{cases} u = \dfrac{2Q_2 I_2 + R_2 + 2o\alpha u_d}{2o\alpha\gamma} \\ u_2 = \gamma u = \dfrac{2Q_2 I_2 + R_2 + 2o\alpha u_d}{2o\alpha} \\ Q_2 = \dfrac{a_2\left(y_2 - 1\right)}{y_2 - \dfrac{a_2}{b_2}} \\ R_2 = \dfrac{\sqrt{y_2}}{y_2 + r_2}\left[\dfrac{\left(4D - 4u_d\right)a_2 o\alpha\left(y_2 + 1\right) - 4hI_d o\alpha\left(y_2 - r_2\right)}{\left(a_2 - b_2\right)\sqrt{y_2}} + C_2\right] \end{cases} \tag{27}$$

where

$$\begin{cases} y_2 = e^{\frac{b_2 - a_2}{o\alpha}\left(t_2 - t\right)} \\ a_2 = o\alpha\theta - o\alpha\sqrt{\theta^2 + \dfrac{h}{o\alpha}} \\ b_2 = o\alpha\theta + o\alpha\sqrt{\theta^2 + \dfrac{h}{o\alpha}} \\ r_2 = -\dfrac{a_2}{b_2} \\ C_2 = \left(1 + r_2\right)B_2 p_2 - \dfrac{\left(8D - 8u_d\right)a_2 o\alpha - 4hI_d o\alpha\left(1 - r_2\right)}{a_2 - b_2} \end{cases} \tag{28}$$

Then, the optimal production rate in the interval $\left(t_2, t_3\right)$ is given by

$$\begin{cases} u = \dfrac{2Q_3 I_3 + R_3 + 2o\alpha u_d}{2o\alpha\gamma} \\ u_3 = \gamma u = \dfrac{2Q_3 I_3 + R_3 + 2o\alpha u_d}{2o\alpha} \\ Q_3 = \dfrac{a_3\left(y_3 + 1\right)}{y_3 + \dfrac{a_3}{b_3}} \\ R_3 = \dfrac{\sqrt{y_3}}{y_3 + r_3}\left[\dfrac{\left(4D - 4u_d\right)a_3 o\alpha\left(y_3 + 1\right) - \left(4hI_d o\alpha + 2ks\right)\left(y_3 - r_3\right)}{\left(a_3 - b_3\right)\sqrt{y_3}} + C_3\right] \end{cases} \tag{29}$$

where

$$\begin{cases} y_3 = e^{\frac{b_3-a_3}{o\alpha}(t_3-t)} \\ a_3 = o\alpha\theta - o\alpha\sqrt{\theta^2 + \frac{h}{o\alpha}} \\ b_3 = o\alpha\theta + o\alpha\sqrt{\theta^2 + \frac{h}{o\alpha}} \\ r_3 = -\frac{a_3}{b_3} \\ C_3 = (1+r_3)B_3p_3 - \frac{(8D-8u_d)a_3o\alpha - (4hI_do\alpha + 2ks)(1-r_3)}{a_3-b_3} \end{cases} \quad (30)$$

In this study, it is observed that analytical solutions with the closed forms are provided for the optimization of production-inventory model.

NUMERICAL EXPERIMENT

Numerical simulations are performed to verify the effectiveness of the proposed optimal solutions by controlling production rate and the level of inventory. Numerical analysis is performed using MATLAB environment on Microsoft Windows 10 Pro 64-bit computer, 16GB of RAM, AMD Ryzen 5 5600G processor with Radeon Graphics. Inventory management is an essential component to keep supply chains running smoothly. The experiment test scenarios are based on the stochastic inventory model with the annual cycle with a period of 1 year or 365 days. Manufacturers want to find an optimal production policy to meet the needs of the customer. Assume that the average demand rate is 200 items/day and the current inventory level is 200 items. The inventory level at each month is only 65% of the original balances due to a gradual decline in product quality over time. Production rate describes a manufacturer's ability to produce items over time. A part of the product will serve the demand of customers. The rest will be used to maintain inventory levels. In the numerical experiment, the desired production rate $u_d = 275$ items/day would be greater than the desired inventory level $I_d = 220$ items. For the first day of a period up to 150 days $(0, t_1)$, the dynamic model of the supply chain works with the deteriorating inventory system. For the next 90 days (t_1, t_2), there is an increase in the ordering costs for raw material and a decline in the production rate due to risk factors such as transportation, oil or gas prices, and scarcity of raw materials from suppliers. For the last 125 days (t_2, t_3), if the inventory is not enough for the demand (negative inventory), there will be a shortage cost. The experimental model parameters in this case are listed in Table 3. The parameters selected for numerical analysis are typical values for supply chain inventory management.

Table 3. Experiment parameters

Parameter	Value	Parameter	Value
$D(t)$	200 items/day	θ	0.35
$I(0)$	200 items	o	10 USD/item
u_d	275 items/day	s	30 USD/item
I_d	220 items	h	10 USD/item
γ	0.85	$p_1 = p_2 = p_3$	30 USD/item
α	1.15	$B_1 = B_2 = B_3$	1.5
σ	5		

Based on the above background, the stock levels of inventory in three stages are described in Figure 3. The test results show that the inventory valuation is managed to achieve the desired approximation rate $I_d = 220$ (items). Figure 4 illustrates the production rate, in which the model in three stages can be controlled to achieve the approximation of the target rate $u_d = 275$ (items/day). The manufacturer wants to begin the production rate at $u_1 = 291$ (items/day) to optimize the total profit of stage 1. Similarly, the initial production rates for stages 2 and 3 are $u_2 = 277$ and $u_3 = 277$ (items/day), respectively.

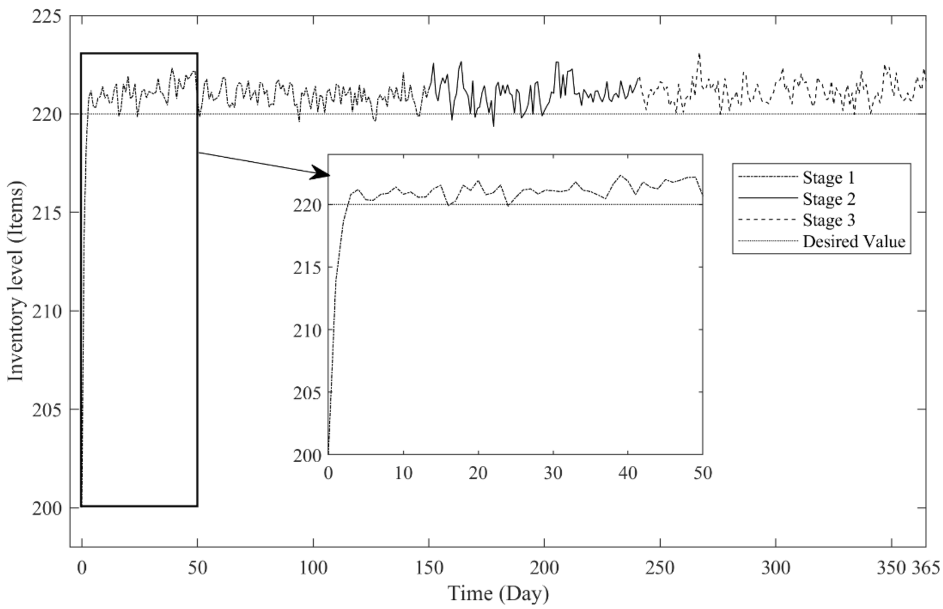

Fig. 3. Stock levels in stochastic inventory management

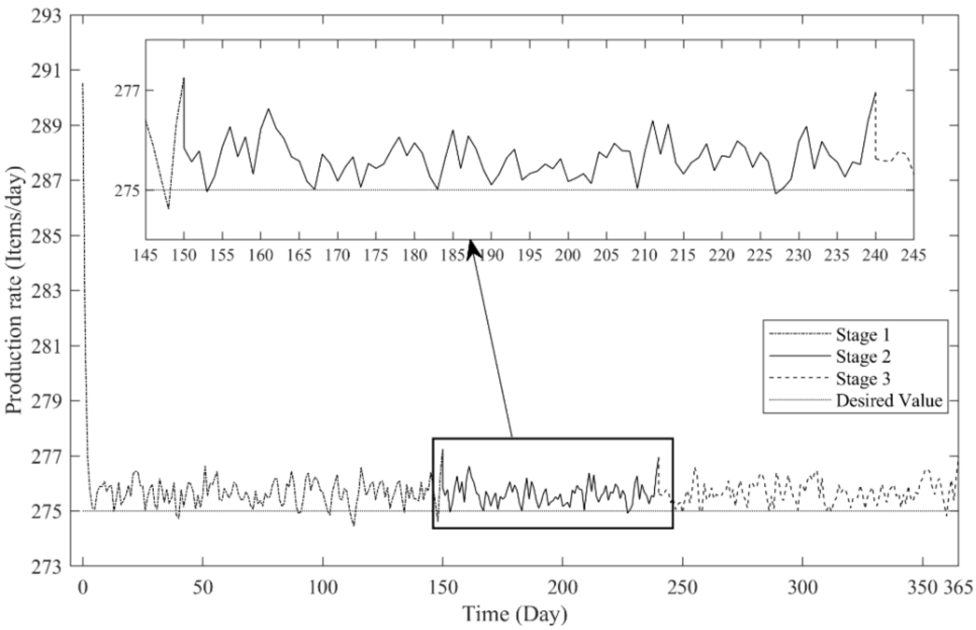

Fig. 4. Production rates of the inventory system

Figure 5 describes the profit of the production-inventory system. Profit depends on the value of the inventory level and the production rate. In this test, the profit rate in three stages is given by $B_1 = B_2 = B_3 = 1.5$. The price units of the items in three stages are $p_1 = p_2 = p_3 = 30$ USD. When the desired inventory level is given by 220 (Items), the desired profit can be given by

$$\text{Desired profit} = B_1 p_1 I_d = 9900 \text{ USD} \tag{31}$$

The values in Figures 3, 4 and 5 cannot be exactly the target rates due to the stochastic factors. However, after optimizing the inventory models, the values do not deviate too much from the target values.

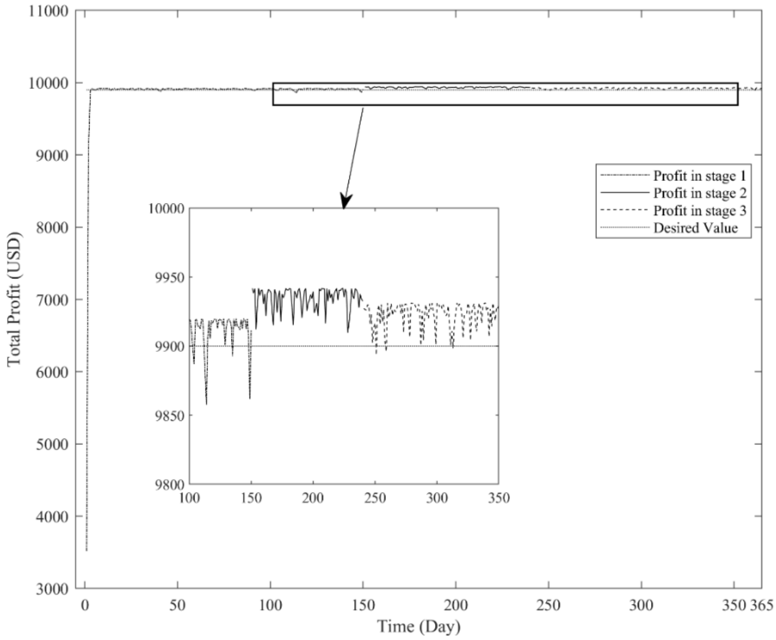

Fig. 5. Total profit in the production-inventory model

CONCLUSION

Efficient inventory management helps manufacturers determine the right production policy to achieve target profits. The manufacturer produces finished goods from raw materials and sells them to retailers or customers to gain *competitive* advantage over others. However, the production-inventory process requires certain costs. The economic order quantity (EOQ) framework with deteriorating item is employed to evaluate the costs in the production inventory model. For solving stochastic *optimal* control *problems*, the study presents the HJB equation to find the initial production rate for the stochastic deterioration model so that the manufacturer can get the desired inventory level, production rate, and profit. Inventory problems and initial production rates at three stages are demonstrated through experiment scenarios. The inventory level, production rate, and the total profit cannot exactly achieve the desired values due to the stochastic factors. However, the numerical experiment shows *that* their *differences* can be negligible by employing a novel inventory management scheme presented in this study. In addition, analytical solutions are provided for the optimization of the production-inventory model. Finally, the efficient inventory management policy will provide the necessary information for many important business decisions.

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Bui Minh Hau
Department of Logistics,
Korea Maritime and Ocean University, Busan, **Republic of Korea**
e-mail: bmh99116@g.kmou.ac.kr

Hwan-Seong Kim ORCID ID: https://orcid.org/0000-0001-7035-7994
Department of Logistics,
Korea Maritime and Ocean University, Busan, **Republic of Korea**
e-mail: kimhs@kmou.ac.kr (corresponding author)

Le Ngoc Bao Long ORCID ID: https://orcid.org/0000-0002-4588-2786
Department of Logistics,
Korea Maritime and Ocean University, Busan, **Republic of Korea**
e-mail: baolong@g.kmou.ac.kr

Professor Sam-Sang You ORCID ID: https://orcid.org/0000-0003-2660-4630
Division of Mechanical Engineering,
Korea Maritime and Ocean University, Busan, **Republic of Korea**
e-mail: ssyou@kmou.ac.kr