LogForum
> Scientific Journal of Logistics <
2022, 18 (4), 379-395
http://doi.org/10.17270/J.LOG.2022.792
http://www.logforum.net p-ISSN 1895-2038 e-ISSN 1734-459X
ORIGINAL PAPER

FACILITY LOCATION PROBLEM MATHEMATICAL MODELS – SUPPLY CHAIN PERSPECTIVE

Adam Redmer

Faculty of Civil and Transport Engineering, Poznan University of Technology, Poznan, **Poland**

ABSTRACT. Background: Supply chains are the networks linking sources of supply with demand points and composed of so-called actors, i.e., producers, distributors/wholesalers, retailers, and customers/consumers. As in every network, supply chains contain vertices and arcs, the former represented by factories and warehouses (including distribution centers). Such facilities cause long-term and expensive investments. As a result, decisions on location and number of them belong to the strategic level of management and require quantitative analysis. To do this, mathematical models of the Facility Location Problem (FLP) are constructed to allow an application of optimization methods.
Methods: Mathematical optimization or programming is the selection of the best solution, with regard to some criterion, from a set of feasible alternatives. The fundamental of mathematical optimization is the formulation of mathematical models of analyzed problems. Mathematical models are composed of objective function, decision variables, constraints, and parameters. These components are presented and compared in the paper concerning FLP from a supply chain perspective.
Results: The ten mathematical models of the FLP are presented, including the two original ones. The models are classified according to such features as facility type they concern, including the desirable, neutral, and undesirable ones. The models and their components are characterized. In addition, their applicability and elasticity are analyzed. Finally, the models are compared and discussed from the supply chain point of view.
Conclusions: However, the FLP mathematical models are relatively similar; the most important element of them for supply chain appropriate representation is an objective function. It strongly influences the possible applicability of FLP models and their solutions, as well. The objective functions having broader applicability turned out to be the maximized number of supply/demand points covered by facilities and the minimized number of facilities necessary to cover supply/demand points. However, not to locate all allowed facilities (use all the location sites) or as many as supply/demand points, but an appropriate number of them, it is necessary to take into account facility fixed costs. Thus, when locating logistics facilities, the minimized total cost of serving supply/demand points is the most appropriate objective function.

Keywords: logistics, distribution network design, facility location, mathematical modelling

INTRODUCTION

The distribution network design belongs to a group of problems so-called Facility Location Problem (FLP) [Eiselt and Marianov, 2011], also known as location analysis or theory, and is a branch of Operational Research (OR). Facility location is a critical component of strategic planning for a wide spectrum of public and private companies [Owen and Daskin, 1998]. The FLP concerns the optimal placement of facilities to minimize transportation costs to, between, and from analyzed facilities along with their variable and fixed costs of operation and (or not) individual minimum/maximum capacities. It leads to Capacitated and an Uncapacitated Facility Location Problems (CFLP/UFLP) [Sliva and Serra, 2007]. One can also distinguish continuous and discrete versions of the FLP. In the continuous FLP (ConFLP), the selection for a new facility can be any location within the space, whereas for the discrete FLP (DisFLP), there is a given set of choices for the facility's location [Eiselt and Marianov, 2011]. On the other hand, demand can also be continuously or discretely distributed in a network, giving a node- or an arc-based FLPs [Lin and Lin, 2018].

Citation: *Redmer A., 2022. Facility Location Problem mathematical models - supply chain perspective. LogForum 18 (4), 379-395*, http://doi.org/10.17270/J.LOG.2022.792
Received: 10.11.2022, Accepted: 05.12.2022, on-line: 30.12.2022

One of the very first problems that can be considered as the FLP was already proposed in the 17th century by P. de Fermat and is known as "a geometric median of three points" [Brimberg, 1995]. The problem was formulated as follows "given three points in a plane, find a fourth point such that the sum of its distances to the three given points is as small as possible" [Dorrie, 1965] and first solved geometrically by E. Torricelli in the year 1645.

In the year 1909 A. Weber used a classical three-point version of the Fermat problem to model possible industrial locations in order to minimize transportation costs from two sources of materials to a single customer or market – so called the Fermat-Weber problem [Drezner and Hamacher, 2004]. This is one of the simplest and first formulations of the ConFLP. A direct numerical, iterative solution method was proposed by E. Weiszfeld only in 1937 [Weiszfeld, 1937]. The method was further developed and popularized by H. Kuhn and R. Kuenne, among others [Kuhn and Kuenne, 1962].

Throughout the years, the FLP got many different mathematical formulations and was solved using numerous different methods [Conceição et al., 2012; Gupta and Könemann, 2011; Ambrosino and Scutellà, 2005; Klose and Drexl, 2005; Vygen, 2005; Drezner and Hamacher, 2004; Klamroth, 2002; Korupolu et al., 2000; Magnanti and Wong, 1984; Or and Pierskalla, 1979].

The purpose of this paper is to present mathematical models of the FLP in the generic and internally consistent manner, allowing for their analysis and comparison from the supply chain perspective. The addition of the models lacking in the already existing literature, but being a logical extension of the models presented there is the second aim of this paper.

The paper is divided into four parts. First, there is the introduction presenting the general background of the raised topic, i.e., the FLP and the most fundamental works related to it. In the next two Sections the particular mathematical formulations of the FLP and their classification along with the comparison are presented, respectively. The paper ends with conclusions and further work planned.

FLP MATHEMATICAL MODELS

Presented in this Section, mathematical models of the FLP are both relatively generic (concern single- and multi-source versions of the problem, by defining decision variable as a binary one or not, also capacitated and incapacitated versions of the problem, by adding or not the constraint (3) and internally consistent too (considering the proposed notation).

Indices:

- predefined supply/demand points i; set I = {1, 2, 3, … i, …},
- predefined facilities j to be located; set J = {1, 2, 3, … j, …}.

Decision variables:

- location variables $x_i \in \{0,1\}$; set X = $\{x_1, x_2, x_3, \dots x_j, \dots\}$,

$$x_j = \begin{cases} 1 \text{ if facility is located at candidate site } j \text{ or if facility } j \text{ is open} \\ 0 \text{ othewise} \end{cases} \quad (1)$$

- assignment variables $y_{ij} \in \{0,1\}$; set Y = $\{y_{11}, y_{12}, y_{13}, \dots y_{ij}, \dots\}$,

$$y_{ij} = \begin{cases} 1 \text{ if supply/demand point } i \text{ is assigned to/served by facility } j \\ 0 \text{ othewise} \end{cases} \quad (2)$$

assuming that $y_{ij} \leq x_j$ for each j.

Parameters:

- D coverage distance (travel time, cost) within which facilities can serve supply/demand points,
- $d_{ij}(x_j)$ distance (travel time, cost or other measure of transport intensity) function between supply/demand point i and facility j located at point x_j,
- $FC_{j\,max}$ maximum capacity of facility j,
- $l_j(x_j)$ total location (opening, building, setup, fixed) cost of facility j located at point x_j,
- P predefined maximum or exact number of facilities j to be located (opened, built, fixed),
- $t_{ij}(x_j)$ unit (per distance d_{ij} and weight w_i), variable transportation cost function of moving a unit weight per unit distance between facility j located at point x_j, and supply/demand point i, or vice versa (usually, in a majority of models, the value of parameter t is predefined, thus, constant for particular pairs of i and j giving the DisFLP),
- W the total value (weight) of the whole industry/market,
- w_i supply/demand point i weight.

Symbols:

- $|\ \ |$ cardinality symbol,
- $\lfloor\ \ \rfloor$ rounding down symbol (towards minus infinity, to the largest integer that does not exceed value in the bracket),
- $\lceil\ \ \rceil$ rounding up symbol (toward plus infinity, to the smallest integer that is not less than the value in the bracket).

Capacity constraints and boundary condition (for the CFLP models):

$$\sum_{i\in I} w_i \cdot y_{ij} \le FC_{j\,max} \ll \sum_{i\in I} w_i \quad \forall j \in J \tag{3}$$

$$\sum_{j\in J} FC_{j\,max} \ge \sum_{i\in I} w_i \tag{4}$$

Minimized distance (travel time or cost) **between a limited number of facilities *j*** (|J| ≪ |I|), that is crucial here, **and supply/demand points *i***. The distance calculated as optionally weighted summed up (5a), average (5b) or maximum (5c) one [Ahmadi-Javid et al., 2017; Boonmee et al., 2017; Klose and Drexl, 2005; Owen and Daskin, 1998; Hakimi, 1964]. The two former correspond to the minsum or *p*-Median Problem, whereas the latter corresponds to the minmax or *p*-Center Problem. Moreover, for the UFLP the supply/demand points *i* are just assigned to the closest facilities *j*.

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{i\in I} \sum_{j\in J} w_i \cdot d_{ij}(x_j) \cdot y_{ij} \tag{5a}$$

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{i\in I} \sum_{j\in J} \left(w_i \cdot d_{ij}(x_j) \cdot y_{ij}\right) \Big/ \sum_{i\in I} (w_i)$$
$$\text{if not weighted divided by } |I| \text{ or even } |J| \text{ instead of } \sum_{i\in I}(w_i) \tag{5b}$$

$$\min_{x_j \in X, y_{ij} \in Y} max\left(w_i \cdot d_{ij}(x_j) \cdot y_{ij}\right) \quad \forall i \in I, \forall j \in J \tag{5c}$$

subject to:

$$\sum_{j\in J} y_{ij} \geq 1 \quad \forall i \in I \tag{5d}$$

and optionally to:

$$\sum_{j\in J}\left[\sum_{i\in I} y_{ij} \Big/ \left(1+\sum_{i\in I} y_{ij}\right)\right] \leq or = P \tag{5e}$$

Minimized pairwise distance (travel time or cost) **between** (minimum 2) **facilities *j***, all located within a limited space. The distance calculated as optionally weighted summed up (6a), average (6b) or maximum (6c) one. To the best of the author's knowledge, it is a new one FLP model, not reported in the literature so far.

$$\min_{x_{j,j'} \in X, y_{ij,j'} \in Y} 0{,}5 \cdot \sum_{j\in J}\sum_{j'\in J}\left\{d_{jj'}(x_j, x_{j'}) \cdot \sum_{i\in I}[w_i \cdot (y_{ij} + y_{ij'})] \cdot \left[\sum_{i\in I} y_{ij} \Big/ \left(1+\sum_{i\in I} y_{ij}\right)\right] \cdot \cdot \left[\sum_{i\in I} y_{ij'} \Big/ \left(1+\sum_{i\in I} y_{ij'}\right)\right]\right\} \quad j \neq j' \tag{6a}$$

$$\min_{x_{j,j'} \in X, y_{ij,j'} \in Y} 0{,}5 \cdot \sum_{j\in J}\sum_{j'\in J}\left\{d_{jj'}(x_j, x_{j'}) \cdot \sum_{i\in I}[w_i \cdot (y_{ij} + y_{ij'})] \cdot \left[\sum_{i\in I} y_{ij} \Big/ \left(1+\sum_{i\in I} y_{ij}\right)\right] \cdot \cdot \left[\sum_{i\in I} y_{ij'} \Big/ \left(1+\sum_{i\in I} y_{ij'}\right)\right]\right\} \Big/ \sum_{i\in I}[w_i \cdot (y_{ij} + y_{ij'})] \quad j \neq j' \tag{6b}$$

$$if\ not\ weighted\ divided\ by\ |I|\ or\ even\ |J|\ instead\ of \sum_{i\in I}[w_i \cdot (y_{ij} + y_{ij'})]$$

$$\min_{x_{j,j'} \in X, y_{ij,j'} \in Y} max\left(d_{jj'}(x_j, x_{j'}) \cdot \sum_{i\in I}[w_i \cdot (y_{ij} + y_{ij'})] \cdot \left[\sum_{i\in I} y_{ij} \Big/ \left(1+\sum_{i\in I} y_{ij}\right)\right] \cdot \cdot \left[\sum_{i\in I} y_{ij'} \Big/ \left(1+\sum_{i\in I} y_{ij'}\right)\right]\right) \quad \forall j \in J, \forall j' \in J, j \neq j' \tag{6c}$$

subject to:

$$\sum_{j\in J} y_{ij} \geq 1 \quad \forall i \in I \tag{6d}$$

and optionally, assuming that all (6e) or specified (6f) facilities *j* or a given (minimum or exact) number of them (6g) are already located (opened, built, fixed) or that specified, the maximum coverage distance (travel time or cost) between facilities *j* and supply/demand points *i* has to be met (6h), subject to:

$$\sum_{i \in I} y_{ij} > 0 \quad \forall j \in J \tag{6e}$$

$$\sum_{i \in I} y_{ij} > 0 \quad \exists j \in J \tag{6f}$$

$$\sum_{j \in J} \left[\sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij} \right) \right] \geq or = P \tag{6g}$$

and/or

$$y_{ij} = 0 \leftrightarrow d_{ij}(x_j) > D \quad \forall i \in I, \forall j \in J \tag{6h}$$

Minimized number of facilities *j* needed (7a) **or the total cost of their location** (opening, building) (7b) **to cover** all demand (or specified level of demand) subject to specified, maximum coverage distance (travel time or cost) between facilities j and supply/demand points i, corresponding to the Set Covering Problem [Boonmee et al., 2017; Klose and Drexl, 2005; Owen and Daskin, 1998].

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{j \in J} \left[\sum_{i \in I} y_{ij} \Big/ \left\lfloor 1 + \sum_{i \in I} y_{ij} \right\rfloor \right] \tag{7a}$$

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{j \in J} l_j(x_j) \cdot \left[\sum_{i \in I} y_{ij} \Big/ \left\lfloor 1 + \sum_{i \in I} y_{ij} \right\rfloor \right] \tag{7b}$$

subject to:

$$\sum_{j \in J} y_{ij} \geq 1 \quad \forall i \in I \tag{7c}$$

$$y_{ij} = 0 \leftrightarrow d_{ij}(x_j) > D \quad \forall i \in I, \forall j \in J \tag{7d}$$

The value of the parameter l_j in fact, depends on the sum of weights w_i of supply/demand points i assigned to facility j, and thus l_j is a function of y_{ij}. As a result, the value of l_j is not strictly constant (however, it can be perceived as a fixed cost but precisely as a step or threshold-like one) and should be scaled. It corresponds to the so-called modular capacity version of the FLP.

Maximized number of (all or new) **supply/demand points *i*** (8a) **or** (all or added) **amount of demand** (8c) **covered** (partially covered if 8b-d is applied) **or** (all or added) **market share gained** (8d) subject to the specified maximum coverage distance (travel time or cost) between a limited number of facilities j ($|J| \ll \infty$), that is crucial here, and supply/demand points i, corresponding to the Maximal Covering Problem or the *p*-Cover Problem [Boonmee et al., 2017; Owen and Daskin, 1998].

$$\max_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} \left\lfloor \sum_{j \in J} y_{ij} \leftrightarrow d_{ij}(x_j) \leq D \right\rfloor \tag{8a}$$

$$\max_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} \left\lceil \sum_{j \in J} y_{ij} \leftrightarrow d_{ij}(x_j) \le D \right\rceil \tag{8b}$$

$$\max_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} \sum_{j \in J} w_i \cdot y_{ij} \leftrightarrow d_{ij}(x_j) \le D \tag{8c}$$

$$\max_{x_j \in X, y_{ij} \in Y} \frac{1}{W} \cdot \sum_{i \in I} \sum_{j \in J} w_i \cdot y_{ij} \leftrightarrow d_{ij}(x_j) \le D \tag{8d}$$

subject to:

$$\sum_{j \in J} y_{ij} \le 1 \quad \forall i \in I \tag{8e}$$

$$\sum_{i \in I} w_i \le W \tag{8f}$$

and optionally to:

$$\sum_{j \in J} \left\lceil \sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij}\right) \right\rceil \le or = P \tag{8g}$$

Minimized number of supply/demand points *i* (9a) **or amount of demand** (9c) **uncovered** (partially uncovered if 9b-d applied) **or lost market share** (9d) subject to specified, minimum (un)coverage distance (travel time or cost) between a limited number of facilities *j* (|J| ≪ |I|), that is crucial here, and supply/demand points *i*, corresponding to the *p*-Cover Problem [Ahmadi-Javid et al., 2017; Boonmee et al., 2017].

$$\min_{x_j \in X, y_{ij} \in Y} |I| - \sum_{i \in I} \left\lfloor \sum_{j \in J} y_{ij} \leftrightarrow d_{ij}(x_j) \le D \right\rfloor \tag{9a}$$

$$\min_{x_j \in X, y_{ij} \in Y} |I| - \sum_{i \in I} \left\lceil \sum_{j \in J} y_{ij} \leftrightarrow d_{ij}(x_j) \le D \right\rceil \tag{9b}$$

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} w_i - \sum_{i \in I} \sum_{j \in J} w_i \cdot y_{ij} \leftrightarrow d_{ij}(x_j) \le D \tag{9c}$$

$$\min_{x_j \in X, y_{ij} \in Y} 1 - \frac{1}{W} \cdot \sum_{i \in I} \sum_{j \in J} w_i \cdot y_{ij} \leftrightarrow d_{ij}(x_j) \le D \tag{9d}$$

subject to:

$$\sum_{j \in J} y_{ij} \le 1 \quad \forall i \in I \tag{9e}$$

$$\sum_{i \in I} w_i \leq W \tag{9f}$$

and optionally to:

$$\sum_{j \in J} \left[\sum_{i \in I} y_{ij} \middle/ \left(1 + \sum_{i \in I} y_{ij} \right) \right] \leq or = P \tag{9g}$$

Minimized total delivery/distribution/ operating cost being a trade-off between fixed and variable or, in accordance with the decision variables, location and assignment, or just warehousing and transportation cost components (10a), corresponding to both the Set Covering Problem and the minsum or *p*-Median Problem, being, to some degree, a combination of them [Ahmadi-Javid et al., 2017; Guastaroba and Speranza, 2014; Rahmani and MirHassani, 2014; Klose and Drexl, 2005; Sridharan, 1995].

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{j \in J} l_j(x_j) \cdot \left[\sum_{i \in I} y_{ij} \middle/ \left[1 + \sum_{i \in I} y_{ij} \right] \right] + \sum_{i \in I} \sum_{j \in J} t_{ij}(x_j) \cdot w_i \cdot d_{ij}(x_j) \cdot y_{ij} \tag{10a}$$

subject to:

$$\sum_{j \in J} y_{ij} \geq 1 \quad \forall i \in I \tag{10b}$$

and optionally to:

$$\sum_{j \in J} \left[\sum_{i \in I} y_{ij} \middle/ \left(1 + \sum_{i \in I} y_{ij} \right) \right] \leq or = P \tag{10c}$$

If it is the DisFLP and weights $w_i = 1$ for all supply/demand points *i*, the problem is called the Simple Plant Location Problem – SPLP [Galli et al., 2018; Cornuejols et al., 1990].

There is also a group of reverse or opposite ways of formulating objective function, and thus defining the so-called obnoxious/undesirable FLP.

Maximized distance (travel time or cost) **between facilities *j* and supply/demand points *i*** all located within a limited space. The distance calculated as optionally weighted summed up (11a), average (11b) or minimum (11c) one [Boonmee et al., 2017; Owen and Daskin, 1998; Kuby, 1987; Drezner and Wesolowsky, 1980; Church and Garfinkel, 1978]. The model corresponding to the maxsum/anti-Median Problem or the maxmin/anti-Center Problem.

$$\max_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} \sum_{j \in J} w_i \cdot d_{ij}(x_j) \cdot y_{ij} \tag{11a}$$

$$\max_{x_j \in X, y_{ij} \in Y} \sum_{i \in I} \sum_{j \in J} \left(w_i \cdot d_{ij}(x_j) \cdot y_{ij} \right) \Big/ \sum_{i \in I} (w_i)$$

$$if\ not\ weighted\ divided\ by\ |I|\ or\ even\ |J|\ instead\ of \sum_{i \in I} (w_i) \tag{11b}$$

$$\max_{x_j \in X, y_{ij} \in Y} min\left(w_i \cdot d_{ij}(x_j) \cdot y_{ij}\right) \quad \forall i \in I, \forall j \in J \tag{11c}$$

optionally, assuming that all (11d) or specified (11e) facilities j or a given (minimum or exact) number of them (11f) is already located (opened, built, fixed) or that specified minimum (un)coverage distance (travel time or cost) between facilities j and supply/demand points i has to be met (11g), subject to:

$$\sum_{i \in I} y_{ij} > 0 \quad \forall j \in J \tag{11d}$$

$$\sum_{i \in I} y_{ij} > 0 \quad \exists j \in J \tag{11e}$$

$$\sum_{j \in J} \left[\sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij}\right) \right] \geq or = P \tag{11f}$$

and/or

$$y_{ij} = 0 \leftrightarrow d_{ij}(x_j) < D \quad \forall i \in I, \forall j \in J \tag{11g}$$

Maximized pairwise distance (travel time or cost) **between** (minimum 2) **facilities *j***, all located within a limited space. The distance calculated as optionally weighted summed up (12a), average (12b), or minimum (12c) one [Owen and Daskin, 1998]. The model corresponding to the *p*-Dispersion Problem.

$$\max_{x_{j,j'} \in X, y_{ij,j'} \in Y} 0{,}5 \cdot \sum_{j \in J} \sum_{j' \in J} \left\{ d_{jj'}(x_j, x_{j'}) \cdot \sum_{i \in I} [w_i \cdot (y_{ij} + y_{ij'})] \cdot \left| \sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij}\right) \right| \cdot \right.$$
$$\left. \cdot \left| \sum_{i \in I} y_{ij'} \Big/ \left(1 + \sum_{i \in I} y_{ij'}\right) \right| \right\} \quad j \neq j' \tag{12a}$$

or

$$\max_{x_{j,j'} \in X, y_{ij,j'} \in Y} 0{,}5 \cdot \sum_{j \in J} \sum_{j' \in J} \left\{ d_{jj'}(x_j, x_{j'}) \cdot \sum_{i \in I} [w_i \cdot (y_{ij} + y_{ij'})] \cdot \left| \sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij}\right) \right| \cdot \right.$$
$$\left. \cdot \left| \sum_{i \in I} y_{ij'} \Big/ \left(1 + \sum_{i \in I} y_{ij'}\right) \right| \right\} \Big/ \sum_{i \in I} [w_i \cdot (y_{ij} + y_{ij'})] \quad j \neq j' \tag{12b}$$

$$if\ not\ weighted\ divided\ by\ |I|\ or\ even\ |J|\ instead\ of \sum_{i \in I} [w_i \cdot (y_{ij} + y_{ij'})]$$

$$\max_{x_{j,j'} \in X, y_{ij,j'} \in Y} max\left(d_{jj'}(x_j, x_{j'}) \cdot \sum_{i \in I} [w_i \cdot (y_{ij} + y_{ij'})] \cdot \left| \sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij}\right) \right| \cdot \right. \tag{12c}$$

$$\cdot\left[\sum_{i\in I} y_{ij'}\Big/\left(1+\sum_{i\in I} y_{ij'}\right)\right]\Bigg) \quad \forall j \in J, \forall j' \in J, j \neq j'$$

subject to:

$$\sum_{j\in J} y_{ij} \leq 1 \quad \forall i \in I \tag{12d}$$

and optionally, assuming that all (12e) or specified (12f) facilities *j* or a given (minimum or exact) number of them (12g) is already located (opened, built, fixed) or that the specified minimum (un)coverage distance (travel time or cost) between facilities *j* and supply/demand points *i* has to be met (12h), subject to:

$$\sum_{i\in I} y_{ij} > 0 \quad \forall j \in J \tag{12e}$$

$$\sum_{i\in I} y_{ij} > 0 \quad \exists j \in J \tag{12f}$$

$$\sum_{j\in J}\left[\sum_{i\in I} y_{ij}\Big/\left(1+\sum_{i\in I} y_{ij}\right)\right] = or \geq P \tag{12g}$$

and/or

$$y_{ij} = 0 \leftrightarrow d_{ij}(x_j) < D \quad \forall i \in I, \forall j \in J \tag{12h}$$

Minimized number of supply/demand points *i* (13a) **covered** (partially covered not applicable) subject to specified maximum coverage distance (travel time or cost) between facilities *j* and supply/demand points *i*, both located within a limited space and corresponding to the Minimum Covering Problem [Boonmee et al., 2017; Berman et al., 1996].

$$\min_{x_j \in X, y_{ij} \in Y} \sum_{i\in I}\left[\sum_{j\leftrightarrow d_{ij}(x_j)\leq D}\sum_{i\in I} y_{ij}\Big/\left(1+\sum_{j\leftrightarrow d_{ij}(x_j)\leq D}\sum_{i\in I} y_{ij}\right)\right] \tag{13a}$$

subject to:

$$\sum_{j\in J} y_{ij} \geq 1 \quad \forall i \in I \tag{13b}$$

and optionally to:

$$\sum_{j\in J}\left[\sum_{i\in I} y_{ij}\Big/\left(1+\sum_{i\in I} y_{ij}\right)\right] = P \tag{13c}$$

Maximized number of supply/demand points *i* (14a) **uncovered,** (partially uncovered not applicable) subject to specified minimum (un)coverage distance (travel time or cost) between facilities *j* and supply/demand points *i*, both located within limited space and corresponding to the Minimum Covering Problem. To the best of the author's knowledge, it is a new one FLP model, not reported in the literature so far.

$$\max_{x_j \in X, y_{ij} \in Y} \quad |I| - \sum_{i \in I} \left[\sum_{j \leftrightarrow d_{ij}(x_j) \le D} \sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{j \leftrightarrow d_{ij}(x_j) \le D} \sum_{i \in I} y_{ij} \right) \right] \tag{14a}$$

subject to:

$$\sum_{j \in J} y_{ij} \ge 1 \quad \forall i \in I \tag{14b}$$

and optionally to:

$$\sum_{j \in J} \left[\sum_{i \in I} y_{ij} \Big/ \left(1 + \sum_{i \in I} y_{ij} \right) \right] = P \tag{14c}$$

And finally, there are also two other ways of formulating objective functions in the FLP mentioned by Boloori Arabani and Farahani [2012] in their survey paper. They are the profit maximization and risk minimization ones. But even these authors comment on profit maximization, that it is an objective that has received less attention in the literature, and give no examples of its applications. Whereas risk minimization if mentioned, the only context is the multicriteria formulation of the FLP. In the multicriteria context according to Farahani, Steadie Seifi and Asgari [2010], environmental and social objectives based on energy cost, land use and construction cost, congestion, noise, quality of life, pollution, fossil fuel crisis, and tourism are becoming customary. But, as the authors plainly expressed, one of the most important difficulties in tackling these problems is to find a way to measure such criteria.

For some further surveys of the FLP models see, for example, Turkoglu and Genevois [2020], Mangiaracina et al. [2015], Bruno et al. [2014], Farahani et al. [2014], Hale and Moberg [2003] or Eiselt and Laporte [1995].

Regarding supply chains and storage facilities, for example, warehouses, distribution and logistics centers location many, specific criteria are taken into account, such as technical (including road network density, number of potential contractors, storage infrastructure) and economic (including capital expenditures, annual operating costs) ones [Kauf and Laskowska-Rutkowska, 2019]. And, as the cited authors conclude, usually, however, the choice of location for a logistics center boils down to finding the lowest costs; thus, economic factors are given priority.

FLP MODELS CLASSIFICATION AND COMPARISON

Based on the survey of FLP models presented in the previous section, the following classification of a generic objective function utilized in a majority of the FLP formulations is proposed. The notation is based on two standard mathematical symbols: $x <\cdot y$ meaning that x is covered by y and $\neg$ a logical negation, meaning that it does not, does not exist, and also assuming that i represents supply/demand points and j, j' ($j \neq j'$) represent facilities:

- **Min *Dij*** – Minimized distance *D* (or time, cost) between *i* and *j* (Eqs. 5a–e).
- **Max *Dij*** – Maximized distance *D* (or time, cost) between *i* and *j* (Eqs. 11a–g).
- **Min *Djj'*** – Minimized pairwise distance *D* (or time, cost) between *j* and *j'* (Eqs. 6a–h).

- **Max *Djj'*** – Maximized pairwise distance *D* (or time, cost) between *j* and *j'* (Eqs. 12a–h).
- **Min *Ni<·j*** – Minimized number *N* of *i* covered by *j* (Eqs. 13a–c).
- **Max *Ni<·j*** – Maximized number *N* of *i* covered by *j* (Eqs. 8a–g).
- **Min *N¬i<·j*** – Minimized number *N* of *i* uncovered by *j* (Eqs. 9a–g).
- **Max *N¬i<·j*** – Maximized number *N* of *i* uncovered by *j* (Eqs. 14a–c).
- **Min *Nj·>i*** – Minimized number *N* of *j* required to cover *i* (Eqs. 7a–d).
- **Min *Cij*** – Minimized (total) cost *C* of serving *i* by *j* (Eqs. 10a–c).

The above objective function types have been confronted with possible fields of their applications, or in the other words, types of facilities to be located, identified based on the cited in this paper literature, that are:

- **C** – Correctional (e.g. prisons, jails).
- **CI** – Chemical Industry (e.g. chemical plants).
- **E** – Educational (e.g. kindergartens, primary schools, high schools, libraries).
- **EDS** – Express Delivery Services (e.g. courier deliveries, food deliveries).
- **EI** – Energy Industry (e.g. nuclear reactors / power plants, oil storage tanks, filling stations).
- **ES** – Emergency Services (e.g. fire stations, EMS – Emergency Medical Service / EMT – Emergency Medical Technician centers, EMS/EMT ambulances/vehicles).
- **F** – Financial Services (e.g. banks).
- **HS/RF** – Humanitarian Services / Relief Facilities (e.g. shelters, storage of emergency food, water, medicine and other supplies).
- **HSC** – Health Care Services (e.g. infirmaries, clinics, hospitals).
- **M** – Military (e.g. ammunition dumps).
- **PDT** – Production-Distribution-Trade (e.g. factories/plants, warehouses / distribution centers, retail shops / outlets – demand side).
- **T** – Telecommunication (e.g. concentrators, routers, terminals).
- **TH** – Transport Hubs (e.g. airports).
- **T&S** – Trade and Services (e.g. retail shops / outlets, franchises, restaurants – supply side).
- **WD** – Waste Disposal (e.g. landfills / garbage dumps, recycling sites, waste disposal plants, wastewater treatment plants).

However, given that the above examples of facilities are solely those mentioned directly in the cited literature, the applications of the analyzed objective functions can concern many more similar facilities as well.

Tab. 1 presents typical fields of application (types of facilities) of particular objective function types utilized in the FLP models, distinguishing the Desirable (alluring), the Neutral (ambivalent), and the Undesirable (obnoxious) facilities. The Desirable and Undesirable facilities are more or less intuitively comprehensible. The exception could be filling stations classified into the EI group of facilities, thus the group of the Undesirable ones, whereas this particular type of the EI facilities is more similar to the Neutral ones. In turn, the Neutral (ambivalent) ones need some explanation. This type of facility, on the one hand, should be located close to supply, and especially demand points if only to shorten delivery time (e.g., in the light of the Same Day Delivery strategies) or to increase accessibility (access time); on the other hand, some features or side effects of their presence in a given location (e.g. any form of pollution, as air, noise, light, thermal, visual, …, but also increased traffic, especially of heavy duty vehicles), weigh in favor of locating them relatively far away from demand points (especially individual customers – B2C). The most problematic seem to be transport hubs (the TH facilities), covering mentioned above airports [Owen and Daskin, 1998], but also a variety of other facilities such as train or bus stations (or just stops) as well. Moreover, not only passenger transport facilities should be taken into consideration, but also freight transport including, for example, intermodal hubs or harbors too (or other similar to the PDT facilities).

Table 1. Types of facilities vs. types of objective functions utilized in the FLP models

	Facilities	Objective functions Min *Dij*	Max *Dij*	Min *Djj'*	Max *Djj'*	Min *Ni<·j*	Max *Ni<·j*	Min *N¬i<·j*	Max *N¬i<·j*	Min *Nj·>i*	Min *Cij*	Elasticity ≈
Desirable (alluring)	E	×					×	(×)		(×)	×	50%
	F	×					×	(×)		(×)	×	50%
	HS/RF	×					(×)	(×)		×		40%
Neutral (ambivalent)	EDS	(×)			(×)		(×)	(×)		(×)	×	60%
	ES	×					×	(×)		×		40%
	HCS	×					×	(×)		(×)	×	50%
	PDT	×			(×)		×	(×)		(×)	×	60%
	T		(×)		(×)		(×)	(×)		(×)	×	60%
	TH	×	×		(×)	×	(×)		(×)	(×)	(×)	80%
	T&S	(×)		×			×	(×)		(×)	(×)	60%
Undesirable (obnoxious)	C		×			×			(×)			30%
	CI		×			×			(×)			30%
	EI		×			×			(×)			30%
	M		×			×			(×)			30%
	WD		×			(×)			(×)			30%
Applicability ≈		60%	50%	10%	30%	40%	70%	60%	40%	70%	50%	

(×) applications possible according to the author, but not reported directly in the cited literature

Source: Own work based on [Boonmee et al., 2017; Farahani et al., 2014; Boloori Arabani and Farahani, 2012; Jayaraman, 1998; Owen and Daskin, 1998; Berman et al., 1996; Sridharan, 1995; Kuby, 1987].

It can be observed in Tab. 1 that the objective functions utilized to locate the Desirable and the Undesirable facilities are fully disjunctive. However, this is not the case, when the neutral facilities are considered. Here, many different objective functions are utilized in research and practice. Presented in Tab. 1 elasticity and applicability indicate if locating given type of facilities many or just very selected objective functions can be applied and, on the other hand, if a given objective function type can be utilized for many or just very selected types of facilities, respectively. Moreover, some of the objective functions are mutually opposing, thus divergent, or unanimous, thus convergent (see Tab. 2). Should such objective functions aim at completely different or exactly the same results, solutions of the FLP? It cannot be generally stated. However, the only criterion not positively or negatively connected with the other is the cost (Min *Cij*).

Tab. 3 presents a comparison of particular objective function types utilized in the FLP models being a component of such models and their other fundamental components such as decision variables and constraints. It can be observed that, on the one hand, independently of an objective function utilized, the models are very similar, on the other hand, they can concern, cover different versions of the FLP, including the continuous (ConFLP) and discrete (DisFLP), the single-sourcing (SSFLP) and multi-sourcing (MSFLP), and also the uncapacitated (UFLP) and capacitated (CFLP) ones.

Table 2. Cross-comparison of the types of objective functions utilized in the FLP models

	Objective functions									
Objective functions	Min *Dij*	Max *Dij*	Min *Djj'*	Max *Djj'*	Min *Ni<·j*	Max *Ni<·j*	Min *N¬i<·j*	Max *N¬i<·j*	Min *Nj·>i*	Min *Cij*
Min *Dij*	=	≠				≈	≈			
Max *Dij*	≠	=			≈			≈		
Min *Djj'*			=	≠	≈			≈		
Max *Djj'*			≠	=						
Min *Ni<·j*		≈	≈		=	≠		=		
Max *Ni<·j*	≈				≠	=			=	
Min *N¬i<·j*	≈						=	≠	=	
Max *N¬i<·j*		≈	≈		=		≠	=		
Min *Nj·>i*						=	=		=	
Min *Cij*										=

= unanimous (identical) ≈ unanimous (similar) ≠ opposite

Source: Own work based on [Boonmee et al., 2017; Farahani et al., 2014; Boloori Arabani and Farahani, 2012; Owen and Daskin, 1998; Jayaraman, 1998; Berman et al., 1996; Sridharan, 1995; Kuby, 1987].

Table 3. Components of FLP mathematical models vs. types of objective functions utilized in them

	Objective functions									
Other components of FLP models	Min *Dij*	Max *Dij*	Min *Djj'*	Max *Djj'*	Min *Ni<·j*	Max *Ni<·j*	Min *N¬i<·j*	Max *N¬i<·j*	Min *Nj·>i*	Min *Cij*
Location decision variables – x_j	D/C	D/C	D/C	D/C	D/C	D/C	D/C	D/C	D/C	D/C
Assignment decision variables – y_{ij}	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B	ℝ/B
Capacity constraints	×/–	×/–	×/–	×/–	×/–	×/–	×/–	×/–	×/–	×/–
Demand satisfaction constraints	×	–	×	×	×	×	×	×	×	×
$\sum_{j\in J} y_{ij}$	= 1	–	= 1	≤ 1	= 1	≤ 1	≤ 1	= 1	= 1	= 1
Number of facilities constraints	×/–	×/–	×/–	×/–	×/–	×/–	×/–	×/–	–	×/–
in relation to P	≤/=	=/≥	≤/=	=/≥	=	≤/=	≤/=	=	–	≤/=

D discrete C continuous ℝ real B binary × obligatory – not applicable ×/– optional

Source: Own work based on [Boonmee et al., 2017; Farahani et al., 2014; Boloori Arabani and Farahani, 2012; Jayaraman, 1998; Owen and Daskin, 1998; Berman et al., 1996; Sridharan, 1995; Kuby, 1987].

The analytical elements of the FLP mathematical models can be distinguished as obligatory and optional components. The obligatory one is the assumption that a set of demand points, their number, location, and demand are known a priori. Whereas the optional four are: the location space (continuous or discrete), the number of facilities to be located (given or any), their capacity (constrained or not) and fixed costs (included or not). It is fundamental to know how these components (or lack of them in a model) influence the solution of the FLP defined by the number of facilities located. Based on Tab. 4 it can be concluded that the crucial is either the number of facilities to be located or their fixed costs, whereas the capacity and the location space are the secondary ones and thus can be optional. It makes the facility fixed costs a very important parameter of the FLP models, but still optional. The number of facilities to be located has to be flexible (given or maximal), whereas, their capacity limitations remain optional.

Table 4. Passible solutions of the FLP (i.e., the number of facilities located) as a result of its model's components

Facility capacity	**Number of facilities to be located**						**Facility fixed costs**
	given (exact)		given (max)		any		
constrained (capacitated)	all	all	all	all	as many as demand points	as many as demand points	not included
constrained (capacitated)	all	all	many (one to all)	many (one to all)	many (one to the number of demand points)	many (one to the number of demand points)	included (fixed charged)
unconstrained (uncapacitated)	all	all	all	all	as many as demand points	as many as demand points	not included
unconstrained (uncapacitated)	all	all	many (one to all)	many (one to all)	many (one to the number of demand points)	many (one to the number of demand points)	included (fixed charged)
	continuous	discrete	continuous	discrete	continuous	discrete	
	Facility location space						

Source: Own work.

CONCLUSIVE REMARKS

The FLP mathematical models are relatively similar as far as their components are considered (see Tab. 3). However, the small differences in the models are important and strongly influence possible results (solutions of the FLP) and to some extent a solution process as well. The most crucial element of the FLP models influencing their applicability is an objective function. It decides mostly if the model can be applied to locate the Desirable, the Neutral, or the Undesirable facilities. The less unanimous or similar to the other of analyzed objective functions analyzed turned out to be the Minimized (total) cost *C* of serving (supply/demand points) *i* by (facilities) *j* function (Min *Cij*). Moreover, this type of objective functions can be formulated in many different ways taking into account specific features of a particular FLPs, which is important in case of an optimal logistics or physical distribution networks design. And to locate an appropriate number of facilities, not all allowed or as many as demand points by assumption, it is necessary to include facility fixed costs into the FLP model. The other model components are much more flexible here, i.e. facilities can be capacitated or not, and the location space can be discrete or continues.

Comparing this research with the three most similar, recent and comprehensive ones [Turkoglu and Genevois, 2020; Mangiaracina et al., 2015; Farahani et al., 2014] it can observed that the FLP models are mostly quantitative, including optimization, but also simulation ones. The remaining ones, that is, 1/7 are conceptual or empirical, including case studies. The same proportion can be observed concerning the types of objective functions. Here, mostly used is the minimized cost being the most flexible criterion, as pointed out in this research. The remaining ones, as previously 1/7, are profit maximization, that is, still economic criterion including in fact costs, also service level and the other mixed quantitative and qualitative criteria when Multi-objective functions are applied, however, they are applied quite rarely. And, however, the facility fixed costs are not always directly pointed out, the most crucial factor affecting distribution costs is the demand level influencing capacity of facilities. In turn the capacity affects the total distribution costs by the economies of scale, that is, nothing but the fixed cost. Among the most widely considered objectives, in addition to costs and profits, are also maximization of the demand coverage or number of customers served, the minimization of customer travel distance, waiting time or cost, and also the minimization of the number of facilities. It can be concluded that the results of this research stay in accordance with the three compared ones; however, this research is both more mathematically oriented and wider, i.e., covers much more higher number of facility types. And thus, it makes the comparison done in this research, that is, the survey of models not the literature, much more comprehensive and, hopefully, useful.

The passible solutions of the FLP were only qualitatively analyzed in this research. Thus, further works will be to implement selected of the discussed in this paper FLP mathematical models in the OPL (Optimization Programming Language) and analyze them qualitatively by solving with the use of the IBM ILOG CPLEX Optimization Studio software for a selected, different, and specific instances of the FLP. All this still concerning supply chain perspective.

ACKNOWLEDGMENTS

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

REFERENCES

Ahmadi-Javid, A., Seyedi, P., Syam, S.S., 2017. A survey of health care facility location. Computers & Operations Research, 79, 223–263. https://doi.org/10.1016/j.cor.2016.05.018

Ambrosino, D., Scutellà, M.G., 2005. Distribution network design: New problems and related models. European Journal of Operational Research, 165(3), 610–624. https://doi.org/10.1016/j.ejor.2003.04.009

Berman, O., Drezner, Z., Wesolowsky, G.O., 1996. Minimum covering criterion for obnoxious facility location on a network. Networks, 28(1), 1–5. https://doi.org/10.1002/(SICI)1097-0037(199608)28:1%3C1::AID-NET1%3E3.0.CO;2-J

Boloori Arabani, A., Farahani, R.Z., 2012. Facility location dynamics: An overview of classifications and applications. Computers & Industrial Engineering, 62, 408–420. https://doi.org/10.1016/j.cie.2011.09.018

Boonmee, Ch., Arimura, M., Asada, T., 2017. Facility location optimization model for emergency humanitarian logistics. International Journal of Disaster Risk Reduction, 24, 485–498. https://doi.org/10.1016/j.ijdrr.2017.01.017

Brimberg, J., 1995. The Fermat–Weber location problem revisited. Mathematical Programming, 71(1, Ser. A), 71–76, MR 1362958. https://doi.org/10.1007/BF01592245

Bruno, G., Genovese, A., Improta, G., 2014. A historical perspective on location problems. BSHM Bulletin: Journal of the British Society for the History of Mathematics, 29(2), 83–97. https://doi.org/10.1080/17498430.2014.889266

Church, R.L., Garfinkel, R.S., 1978. Locating an obnoxious facility on a network. Transportation Science, 12(2), 107–118. https://doi.org/10.1287/trsc.12.2.107

Conceição, S., Pedrosa, L., Neto, A., Vinagre, M., Wolff, E., 2012. The facility location problem in the steel industry: A case study in Latin America. Production Planning & Control, 23(1), 26–46. https://doi.org/10.1080/09537287.2010.539991

Cornuejols, G., Nemhauser, G.L., Wolsey, L.A., 1990. The uncapacitated facility location problem. In: Francis, R.L., Mirchandani, P.B. (Eds.): Discrete location theory. New York, Wiley-Interscience, 119–171.

Dorrie, H., 1965. 100 great problems of elementary mathematics: their history and solution. New York, NY: Dover Publications.

Drezner, Z., Hamacher, H.W., 2004. Facility location: applications and theory. Springer.

Drezner, Z., Wesolowsky, G.O., 1980. A maximin location problem with maximum distance constraints. AIIE Transactions, 12(3), 249–252. https://doi.org/10.1080/05695558008974513

Eiselt, H., Marianov, V., 2011. Foundations of location analysis. International Series in Operations Research & Management Science. New York, Springer.

Eiselt, H.A., Laporte, G., 1995. Facility location: A survey of application and methods. Springer, New York.

Farahani, R.Z., Hekmatfar, M., Fahimnia, B., Kazemzadeh, N., 2014. Hierarchical facility location problem: Models, classifications, techniques, and applications. Computers & Industrial Engineering, 68, 104–117. https://doi.org/10.1016/j.cie.2013.12.005

Farahani, R.Z., Steadie Seifi, M., Asgari, N., 2010. Multiple criteria facility location problems: A survey. Applied Mathematical Modelling, 34, 1689–1709. https://doi.org/10.1016/j.apm.2009.10.005

Galli, L., Letchford, A.N., Miller, S., 2018. New valid inequalities and facets for the simple plant location problem. European Journal of Operational Research, 269(3), 824–833. https://doi.org/10.1016/j.ejor.2018.03.009

Guastaroba, G., Speranza, M.G., 2014. A heuristic for BILP problems. The single source capacitated facility location problem. European Journal of Operational Research, 238, 438–450. https://doi.org/10.1016/j.ejor.2014.04.007

Gupta, A., Könemann, J., 2011. Approximation algorithms for network design: A survey. Surveys in Operations Research and Management Science, 16, 3–20. https://doi.org/10.1016/j.sorms.2010.06.001

Hakimi, S.L., 1964. Optimum locations of switching centers and the absolute centers and medians of a graph. Operations Research, 12(3), 450–459. https://doi.org/10.1287/opre.12.3.450

Hale, T.S., Moberg, C.R., 2003. Location science research: A review. Annals of Operations Research, 123, 21–35. https://doi.org/10.1023/A:1026110926707

Jayaraman, V., 1998. Transportation, facility location and inventory issues in distribution network design: An investigation. International Journal of Operations & Production Management, 18(5), 471–494. https://doi.org/10.1108/01443579810206299

Kauf, S., Laskowska-Rutkowska, A., 2019. The location of an international logistics center in Poland as a part of the One Belt One Road initiative. LogForum, 15(1). 71–83. http://dx.doi.org/10.17270/J.LOG.2019.311

Klamroth, K., 2002. Single facility location problems with barriers. Berlin, Springer Verlag Telos.

Klose, A., Drexl, A., 2005. Facility location models for distribution system design. European Journal of Operational Research, 162(1), 4–29. https://doi.org/10.1016/j.ejor.2003.10.031

Korupolu, M.R., Plaxton, C.G., Rajaraman, R., 2000. Analysis of a local search heuristic for facility location problems. Journal of Algorithms, 37(1), 146–188. https://doi.org/10.1006/jagm.2000.1100

Kuby, M.J., 1987. Programming models for facility dispersion: The p-dispersion and maxisum dispersion problems. Geographical Analysis, 19(4), 315–329. https://doi.org/10.1111/j.1538-4632.1987.tb00133.x

Kuhn, H.W., Kuenne, R.E., 1962. An efficient algorithm for the numerical solution of the generalized Weber problem in spatial economics. Journal of Regional Science, 4, 21–34. https://doi.org/10.1111/j.1467-9787.1962.tb00902.x

Lin, C.-C., Lin, C.-C., 2018. The *p*-Center flow-refueling facility location problem. Transportation Research Part B, 118, 124–142. https://doi.org/10.1016/j.trb.2018.10.008

Magnanti, T.L., Wong, R.T., 1984. Network design and transportation planning: Models and algorithms. Transportation Science, 18(1), 1–55. https://doi.org/10.1287/trsc.18.1.1

Mangiaracina, R., Song, G., Perego, A., 2015. Distribution network design: A literature review and a research agenda. International Journal of Physical Distribution & Logistics Management, 45(5), 506–531. https://doi.org/10.1108/IJPDLM-02-2014-0035

Or, I., Pierskalla, W.P., 1979. A transportation location-allocation model for regional blood banking. AIIE Transactions, 11(2), 86–95. https://doi.org/10.1080/05695557908974447

Owen, S.H., Daskin, M.S., 1998. Strategic facility location: A review. European Journal of Operational Research, 111(3), 423–447. https://doi.org/10.1016/S0377-2217(98)00186-6

Rahmani, R., MirHassani, S.A., 2014. A hybrid Firefly-Genetic algorithm for the capacitated facility location problem. Information Sciences, 283, 70–78. https://doi.org/10.1016/j.ins.2014.06.002

Sliva, F., Serra, D., 2007. A capacitated facility location problem with constrained backlogging probabilities. International Journal of Production Research, 45(21), 5117–5134. https://doi.org/10.1080/00207540600823195

Sridharan, R., 1995. The capacitated plant location problem. European Journal of Operational Research, 87, 203–213. https://doi.org/10.1016/0377-2217(95)00042-O

Turkoglu, D.C., Genevois, M.E., 2020. A comparative survey of service facility location problems. Annals of Operations Research, 292, 399–468. https://doi.org/10.1007/s10479-019-03385-x

Vygen, J., 2005. Approximation algorithms for facility location problems. Bonn, Germany: Research Institute for Discrete Mathematics.

Weiszfeld, E., 1937. Sur le point par lequel la somme des distances den points donnés est minimum. Tohoku Mathematics Journal, 43, 355–386.

Adam Redmer ORCID ID: https://orcid.org/0000-0003-2154-232X
Faculty of Civil and Transport Engineering,
Poznan University of Technology, Poznan, **Poland**
e-mail: adam.redmer@put.poznan.pl