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ORIGINAL PAPER

A COMPREHENSIVE STUDY OF CLASSICAL HEURISTIC ALGORITHMS USED IN THE PROCESS OF SOLVING TRANSPORTATION PROBLEM

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ABSTRACT. **Background:** Transportation Problem (TP) is a special case of integer programming, characterised by indisputable practical significance (in particular in the area of logistics). For this reason, many techniques have been proposed to solve the problem both in optimum and approximate manner. The problem of selecting an effective technique for determining a suboptimal solution for TP was addressed by many researchers, however the implementation of only certain heuristics, 'test bed' applied, as well as non-performance of statistical tests make it impossible to clearly identify the recommended approach to application of heuristics in TP, leaving a research gap which determined the writing of this article. The additional purpose of this paper is to provide a summary of selected approximate methods, taking into consideration the number of iterations necessary to design the optimal solution by means of Modified Distribution (MODI) method and to demonstrate potential correlations between the parameters describing a problem instance and the efficiency of the methods.

Methods: This paper presents a comparative study of four classic techniques (NWC, LCM, VAM and RAM). The tests were performed on three sets of 2,500 pseudo-randomly generated tasks and the observations were also checked by means of the Wilcoxon Signed-Rank Test and Pearson correlation coefficient.

Results: The results confirms that VAM is characterised by a significant quality of the determined results, whereas NWC develops solutions of low efficiency. However, contrary to the observations made for small TP instances, RAM was characterised by a higher error value than LCM for huge set, demonstrating the impossibility to generalise results obtained for small problems (presented e.g. in literature), in order to determine their efficiency for higher instances.

Conclusions: It is recommended to apply VAM both for the determination of initial solution in MODI method and for performing allocation of resources, using only heuristics. However, taking into consideration the utilitarian approach and possible occurrence of the necessity to solve TP instances without using the appropriate software, it is recommended to use LCM for solving large instances of TP. The presence of strong correlation between the number of nodes describing the TP instance and the number of iterations necessary to determine the optimal solution by MODI method has been identified.

Key words: Transportation Problem, Best Initial Feasible Solution, MODI, VAM , RAM, LCM.

INTRODUCTION

Transportation Problem (TP) is a special case of integer programming [Reeb and Leavengood, 2002], formulated by Hitchcock [1941] and solved by Dantzig [1951]. The indisputable practical significance of the problem (Sharma et al. [2012] relied on TP when optimising transport processes at Albert David Company, Liu and Trung [2013] used them for physical flow management in flower delivery process, whereas Salami [2014] applied them at Nigeria Soft Drinks Industry) resulted in development of many techniques intended for solving the problem both in

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optimum and approximate manner. The methods belonging to the first of the listed groups are especially characterised by different time and memory complexity, while heuristics should also be assessed in terms of value of objective function of the obtained results.

The problem of selecting an effective technique for determining a suboptimal solution for TP was addressed e.g. by Ali and Mustapha [2013], Deshpande [2009], Khan et al. [2016], Liu and Trung [2013], Soomro et al. [2014]; however, the implementation of only certain heuristics (in some of the listed papers), 'test bed' applied (characterised by appearance of few test cases, additionally described by a small number of suppliers and customers), as well as non-performance of statistical tests make it impossible to clearly identify the recommended approach to application of heuristics in the process of designating the assignment of load transported between suppliers and customers, leaving a research gap which determined the writing of this article. The additional purpose of this paper is to provide a summary of selected approximate methods, taking into consideration the number of iterations necessary to design the optimal solution by means of Modified Distribution (MODI) method, which uses specific techniques in order to determine the basic results, with the effect of formulating the recommended approach to application of MODI algorithm in business practice. It has been assumed that the article is also intended to demonstrate potential correlations between the parameters describing a problem instance and the error determined by the methods, as well as the number of iterations necessary to define the optimal result.

While analyzing the selected problems occurring in business practice, a significantly higher number of participants can be noticed than the size of TP, which was discussed and verified empirically in the source literature (e.g. [Ali and Mustapha 2013, Deshpande 2009, Khan et al. 2016, Liu and Trung 2013, Soomro et al. 2014]), indicating also utilitarian need of research containing the larger instances of TP. Typical reliance on heuristic methods, connected with conclusions presented in works containing 'test bed', consisting of problems of other sizes than required, may cause application of faulty algorithms during optimization of flows and consequently lead to decreased efficiency of the entire network, hence authors of this work focused on the analysis of three collections of tasks of various sizes, which are to allow the choice of adequate methods, subject to the problem size and make it possible to formulate general conclusions, which justify exclusion of application of inefficient algorithms.

The article was divided into six parts. Following the introduction to the raised topics, the Transportation Problem was formulated, the algorithms used in the process of its solution were presented, the methodology of empirical research was described and the results of conducted research were discussed. The paper ends with discussion, conclusions and further work planned.

FORMULATION OF TRANSPORTATION PROBLEM

This article is based on the formulation of Transportation Problem which was presented in the paper by Shenoy et al. [1986]. It assumes the occurrence of homogeneous goods which are to be transported from a specific number of suppliers m to customers n. Each of the distributors is characterised by a set supply (its volume for supplier i is determined with variable a_i), whereas the customers are characterised by demand (represented by variable b_j for customer j). A model in which the total demand volume is equal to the supply is referred to as Balanced Transportation Problem (BTP), whereas a variant which does not fulfil the described condition is referred to as Unbalanced Transportation Problem - it requires introduction of a fictitious supplier or customer in order to adjust the problem to BTP. The cost of transporting a product unit between the source of origin i and the target destination j is represented by c_{ij} (where matrix c is referred to as the cost matrix). The problem assumes determination of variable values x_{ij} $(\forall i, j)$, representing the volume of load transported between the supplier i and customer j in such manner as to enable minimisation of the total transport cost while meeting customers' needs at the same time.

The mathematical problem formulation assumes the occurrence of the following objective function:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} \to min. \quad (1)$$

The constraints were determined as:

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, \dots, m, \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, \dots, n, \qquad (3)$$

$$x_{ij} \geq 0, \qquad (4)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j. (5)$$

The visualisation of example of BTP instance (for *m*=3 and *n*=3) was presented in Table 1.

Table 1. Visualisation of BTP instance solution

	b_1	b_2	b_3
a_1	x_{11}	x_{12}	x_{13}
a_2	x_{21}	x_{22}	x_{23}
a_3	x_{31}	x_{32}	x_{33}

Source: own work

ALGORITHMS USED IN THE PROCESS OF SOLVING AN INSTANCE OF TRANSPORTATION PROBLEM

North-West Corner Method

North-West Corner Method (NWC) implies iterative assignment of permitted transport value, beginning with element x_{11} (based on the representation of problem solution that was presented in Table 1). The technique pseudocode was presented in Figure 1.

NWC pseudocode

```
i = 0
j = 0
while i < m and j < n do
    if supply[i] < demand[j] then
        allocation[i, j] = supply[i]
        demand[j] = demand[j] − supply[i]
        supply[i] = 0
        i + +
    else if supply[i] > demand[j] then
        allocation[i, j] = demand[j]
        supply[i] = supply[i] − demand[j]
        demand[j] = 0
        j + +
    else
        allocation[i, j] = supply[i]
        supply[i] = 0
        demand[j] = 0
        i + +
        j + +
    end if
end while
return allocation
```

Source: own work

Fig. 1. NWC pseudocode

Least Cost Method

Least Cost Method (LCM) is based on the greedy approach and implies assigning the maximum permitted transport value to subsequent elements, choosing iteratively subsequent cost matrix cells, characterised by the lowest value.

Vogel Approximation Method

Vogel Approximation Method (VAM) was presented by Reinfeld and Vogel [1958]. The results determined by this method are usually characterised by the value of objective function similar to optimal or optimal [Gujjula et al. 2011]. It implies iterative performance of the following four steps until the entire transport has been planned:

1. Determine the cost difference between two lowest available values for each available row and column (not present in the tabu list).
2. Choose the column or row with the largest cost difference.
3. Assign the maximum available transport volume for the cell with the lowest cost, located in the indicated column or row.
4. Add the column and/or row for which the required needs have been met to the *tabu* list.

Russell Approximation Method

The Russell Approximation Method (RAM) proposed by Russell [1969] is based on iterative fulfilment of the following five steps until the entire transport has been planned:

1. For each available row i (not belonging to the *tabu* list) determine u_i, representing the largest unit cost cij present in the cost matrix in row i.
2. For each available column j (not belonging to the *tabu* list) determine v_j, representing the largest unit cost c_{ij} present in the cost matrix in column j.
3. For each available variable x_{ij} determine $\Delta_{ij}=c_{ij}-u_i-v_j$.
4. Assign the maximum available transport volume for the cell represented by the lowest value Δ.
5. Add the column and/or row for which the required needs have been met to the *tabu* list.

Modified Distribution Method

MODI (also known under the name of U-V Method [Tiwari and Shandilya 2006]). Method enables to determine the optimal solution of TP instance [Shenoy et al. 1986]. It uses an initial solution which was determined by the selected heuristics, whereas its procedure assumes iterative performance of the following steps:

1. Determine the initial solution using any heuristics.
2. If the number of occupied cells in the solution is lower than $m+n-1$, there is degeneracy, which must be removed (e.g. using the ε-perturbation method).
3. Solve the equation $u_i+v_j=c_{ij}$ for each occupied cell in the result, beginning from $u_i=0$ or $v_j=0$, determining the subsequent values iteratively.
4. Calculate $Z_{ij}=c_{ij}-(u_i+v_j)$ for each unoccupied cell.
5. If each unoccupied cell contains value Z_{ij} higher or equal to 0, the solution is optimal - finish executing the algorithm. Otherwise, go to step 6.
6. Choose the cell with the lowest value Z_{ij} (largest negative value) and assign an unknown value θ to it.
7. Identify the closed loop which begins and ends in the cell determined in step 6, connecting the occupied cells. Next, add and subtract the value θ alternately from the value of cells.
8. Determine the value θ in such manner that all cells should contain the value ≥0 as a result of performing the operation from step 7. If more than one cell is zeroed, mark only one cell as unoccupied.
9. Perform step 3.

```
MODI pseudocode
Generate the initial solution allocation
if number of occupied cells in allocation is lower than (m + n − 1) then
    Execute ε−perturbation method based on solution allocation
end if
do
    cellsList = prepare a list of occupied cells in allocation
    i = cellsList[0].supplierIndex
    j = cellsList[0].customerIndex
    potentialsU[i] = 0
    potentialsV[j] = costMatrix[i, j]
    Calculate the values in the potentialsU and potentialsV tables, according to potentialsU[i] + potentialsV[j] = costMatrix[i, j]
    For each unoccupied cell occurring in allocation calculate Z[i, j] = costMatrix[i, j] − (potentialsU[i] + potentialsV[j])
    if every calculated elements Z ⩾ 0 then
        optimal = true
    else
        optimal = false
        cell = select the cell with the smallest determined value Z[i, j] (∀i, j)
        Create a closed loop that starts and ends in cell, combining occupied cells that occurred in allocation
        θ = determine the minimum value in cells that belong to a loop with even position (second, fourth, . . . element of the list)
        Interchangeably add and subtract value θ from cell values in allocation belonging to the created loop
    end if
while optimal = false
return allocation
```

Source: own work

Fig. 2. MODI pseudocode

The ε -perturbation method quoted in step 2 is based on three steps:

1. Add for each customer a negligibly small number ε.
2. Add for the last supplier the value ε multiplied by the number of customers.
3. Solve the TP instance using any method.

The MODI method pseudocode (based on Shenoy et al. [1986] and Tiwari and Shandilya [2006]), using the ε-perturbation method, was presented in Figure 2.

METHODOLOGY OF RESEARCH

The algorithms were implemented in C# language, whereas the research was conducted on laptop Lenovo Y520 whose parameters were presented in Table 2. In MODI Method, the ε-perturbation method, using heuristics applied in order to create the initial solution, was employed.

Table 2. Parameters of laptop Lenovo Y520 which was used for conducting research

No.	Parameter	Value
1	Processor	Intel Core i7-7700HQ (4 cores, from 2.8 GHz to 3.8 GHz, 6 MB cache)
2	RAM	32 GB (SO-DIMM DDR4, 2400MHz)
3	HDD	1000 GB SATA 7200 RPM, 240 GB SSD M.2 PCIe
4	OS	Windows 10 Home 64-bit

Source: own work

The quality assessment of results determined by particular algorithms was performed on the 'test bed' consisting of three sets described with the appearance of 2,500 pseudo-randomly generated instances of BTP, whose characteristics is similar to the features of the problems occurring in business practice (generally set S1 represents the issues faced by small enterprises, S2 is intended for middle-sized companies, whereas S3 is intended for the largest entities). The parameter values for the original generator of tasks (which are independent from the analysed set) were presented in Table 3. The specific parameter values for S1, S2 and S3 were presented in Table 4. Demand and supply are represented by integers.

Table 3. Parameter values for generator BTP instances, describing all sets of tasks

Parameter	Value
Number of instances	2,500
Minimum demand at point	1
Maximum demand at point	300
Minimum supply at point	1
Maximum supply at point	300
Minimum cost for unit	0.2
Maximum cost for unit	20.2

Source: own work

Table 4. Parameter values for generator BTP instances, with division into sets of tasks

Parameter	Value		
	S1	S2	S3
Minimal number of customers	5	50	150
Maximal number of customers	25	100	250
Minimal number of suppliers	5	50	150
Maximal number of suppliers	25	100	250

Source: own work

As a result, the generated sets of tasks were characterised by the values shown in Tables 5, 6 and 7. The actual maximum demand and supply exceed the assumed values (shown in Table 3), because the balancing of issues was performed in the last step, which determined the difference between the accumulated demand and supply, and divided it evenly between all suppliers/customers (in a situation preventing equal division, the surplus was assigned to the last supplier or customer).

Table 5. Characteristics of tasks from the set S1

Measure\Parameter	Total demand	No. of suppliers	No. of customers	Supply at point	Demand at point	Unit cost
Minimum	622	5	5	1	1	0.2
Maximum	5147	25	25	988	977	20.2
Average	2802.7	14.87	15.02	188.46	186.54	10.2
Standard deviation	828.19	6.08	6.07	116.75	114.54	5.78
Coefficient of variation	29.55%	40.89%	40.42%	61.95%	61.4%	56.64%

Source: own work

Table 6. Characteristics of tasks from the set S2

Measure\Parameter	Total demand	No. of suppliers	No. of customers	Supply at point	Demand at point	Unit cost
Minimum	6618	50	50	1	1	0.2
Maximum	17804	100	100	507	479	20.2
Average	12574.82	74.83	75.09	168.03	167.47	10.2
Standard deviation	1954.62	14.76	14.74	91.19	90.72	5.77
Coefficient of variation	15.54%	19.73%	19.63%	54.28%	54.32%	56.57%

Source: own work

Table 7. Characteristics of tasks from the set S3

Measure\Parameter	Total demand	No. of suppliers	No. of customers	Supply at point	Demand at point	Unit cost
Minimum	22420	150	150	1	1	0.2
Maximum	41598	250	250	532	523	20.2
Average	32463.63	199.45	198.8	162.77	163.29	10.2
Standard deviation	3776.63	28.97	29.09	88.7	88.83	5.77
Coefficient of variation	11.63%	14.53%	14.63%	54.49%	54.4%	56.6%

Source: own work

The following were used as meters for quality assessment of algorithm results: number of iterations after which MODI determined the optimal solution *it* (their average number was marked as $\overline{it}$, whereas the standard deviation was marked as σ_{it}), percentage surplus of the value of objective function in comparison with optimum *e* (their average value was determined as $\overline{e}$, whereas the standard deviation was determined as σ_e) and number of optimal results b. The value of the second meter was determined using the following formula:

$$e=(result-optimum)/optimum \cdot 100\%.$$

The values e and it were subject to Wilcoxon Signed-Rank Test, using R. The following pairs of methods (marked as M1 and M2) were involved in the process, whereas the value of 0.05 was adapted as the level of test significance (the obtained p-values which were characterised by lower results were distinguished and they indicate that an alternative hypothesis was accepted, according to which M1 obtained lower results than M2).

RESULTS OF CONDUCTED RESEARCH

Results for set S1

Table 8 presents the summary of results for tasks in set S1, obtained by particular heuristics. NWC did not determine any optimal solution from the set 2,500 of problem instances, whereas LCM created four best allocations, and techniques based on approximation obtained 11 and 20 optimal solutions (for RAM and VAM respectively). The above-mentioned correlations were also maintained for other values describing the results, implying that the best quality of assignment was obtained by VAM method, whereas the worst quality was achieved by NWC. The results of Wilcoxon Signed-Rank Test (presented in Table 9) made it impossible to deny the above-mentioned observations.

Table 8. Results of heuristic methods for set S1

Indicator\Method	NWC	LCM	VAM	RAM
Minimum *e*	13.35%	0%	0%	0%
Maximum *e*	866.65%	133.62%	106.69%	106.26%
$\overline{e}$	272.26%	31.03%	15.76%	26.59%
σ_e	117.86%	17.88%	12.25%	16.24%
b	0	4	20	11

Source: own work

Table 9. Results of Wilcoxon Signed-Rank Test for *e* determined by heuristic methods for set S1

M1\M2	NWC	LCM	VAM	RAM
NWC	N/A	1	1	1
LCM	**0**	N/A	1	1
VAM	**0**	**2E-285**	N/A	**4.2E-189**
RAM	**0**	**1.61E-43**	1	N/A

Source: own work

Table 10 presents the summary of obtained results, taking into consideration their impact on MODI method operation. On their basis, the observations resulting from the analysis of heuristic methods were confirmed, with the conclusion that it is advantageous to apply VAM for determination of basic solution, whereas the use of NWC causes extension of the procedure of generating the optimal result. Particular attention should be drawn to the higher value of σ_{it} for MODI with RAM in comparison with MODI using LCM, which indicates a lower predictability of the number of iterations necessary in order to determine the optimal result by MODI with RAM. The results of Wilcoxon Signed-Rank Test (presented in Table 11) made it impossible to deny the above-mentioned observations.

Table 10. Results of MODI method for set S1

Indicator\Method	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
Minimum *it*	4	1	1	1
Maximum *it*	122	45	39	44
$\overline{it}$	42.88	14.82	10.54	14.73
σ_{it}	20.43	7.07	5.88	7.36

Source: own work

Table 11. Results of Wilcoxon Signed-Rank Test for the results of MODI method for set S1

M1\M2	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
MODI + NWC	N/A	1	1	1
MODI + LCM	**0**	N/A	1	0.950109
MODI + VAM	**0**	**6E-276**	N/A	**1.5E-264**
MODI + RAM	**0**	**0.049891**	1	N/A

Source: own work

Results for set S2

The obtained results for heuristic methods and average set of tasks (S2) were presented in Table 12. None of the analysed techniques determined an optimal solution, whereas the average error exceeded 1,200% for NWC. Among the other methods, the results created by VAM were characterised by the notably lowest value $\overline{e}$ (and its standard deviation). The results of Wilcoxon Signed-Rank Test (presented in Table 13) confirmed the above-mentioned observations, which are consistent with the theses formulated for S1.

Table 12. Results of heuristic methods for set S2

Indicator\Method	NWC	LCM	VAM	RAM
Minimum *e*	670.51%	30.34%	8.78%	29.1%
Maximum *e*	1762%	115.15%	88.29%	120.5%
$\overline{e}$	1225.22%	69.67%	39.18%	67.72%
σ_e	177.04%	14.04%	11.85%	14.02%
b	0	0	0	0

Source: own work

Table 13. Results of Wilcoxon Signed-Rank Test for *e* determined by heuristic methods for set S2

M1\M2	NWC	LCM	VAM	RAM
NWC	N/A	1	1	1
LCM	**0**	N/A	1	1
VAM	**0**	**0**	N/A	**0**
RAM	**0**	**3.17E-09**	1	N/A

Source: own work

The summary of the number of iterations after which MODI method determined the optimal solution for S2 was presented in Table 14. According to it, the best technique was again VAM, whereas the worst technique was NWC. However, particular attention should be drawn to the observation according to which, contrary to set S1, LCM determined more advantageous results than RAM. The thesis was confirmed by the results of Wilcoxon Signed-Rank Test, which were presented in Table 15.

Table 14. Results of MODI methods for set S2

Indicator\Method	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
Minimum *it*	261	60	34	65
Maximum *it*	790	242	215	258
$\overline{it}$	498.25	137.61	107.39	145.79
σ_{it}	103.58	30.48	26.56	32.19

Source: own work

Table 15. Results of Wilcoxon Signed-Rank Test for the results of MODI methods for set S2

M1\M2	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
MODI + NWC	N/A	1	1	1
MODI + LCM	**0**	N/A	1	**2.48E-82**
MODI + VAM	**0**	**0**	N/A	**0**
MODI + RAM	**0**	1	1	N/A

Source: own work

Results for set S3

The summary of the results determined by heuristic methods for a large set of tasks (S3) was presented in Table 16. Once again, none of the analysed techniques determined an optimal solution, whereas the average error exceeded 2,300% for NWC. Particular attention should be drawn to the observation, according to which $\overline{e}$ for LCM, VAM and RAM did not change significantly in comparison with the results obtained for S2. Once again, the results created by VAM were characterised by the notably lowest value $\overline{e}$ (and its standard deviation). Contrary to the previous analyses, the results of RAM for S3 were described by a worse value $\overline{e}$ than the results of LCM (however, the standard error deviation was lower for RAM). The results of Wilcoxon Signed-Rank Test presented in Table 17 confirmed the above-mentioned observations.

Table 16. Results of heuristic methods for set S3

Indicator\Method	NWC	LCM	VAM	RAM
Minimum *e*	1775.7%	40.24%	15.5%	40.56%
Maximum *e*	2900.96%	111.81%	75.78%	113.18%
$\overline{e}$	2332.95%	69.45%	38.76%	69.63%
σ_e	174.85%	10.35%	8.76%	9.75%
b	0	0	0	0

Source: own work

Table 17. Results of Wilcoxon Signed-Rank Test for *e* determined by heuristic methods for set S3

M1\M2	NWC	LCM	VAM	RAM
NWC	N/A	1	1	1
LCM	**0**	N/A	1	0.193256
VAM	**0**	**0**	N/A	**0**
RAM	**0**	0.806744	1	N/A

Source: own work

The obtained number of iterations after which MODI method determined the optimal solution for set S3 was presented in Table 18. On this basis, it was confirmed that the best technique was again VAM, whereas the worst technique was NWC. As in the case of set S2, LCM determined more advantageous results than RAM. The thesis was confirmed by the results of Wilcoxon Signed-Rank Test, which were presented in Table 19.

Table 18. Results of MODI method for set S3

Indicator\Method	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
Minimum *it*	1258	309	216	340
Maximum *it*	2871	802	662	832
$\overline{it}$	2020.97	513.55	407.36	549.96
σ_{it}	300.5	82.1	69.91	87.43

Source: own work

Table 19. Results of Wilcoxon Signed-Rank Test for the results of MODI method for set S3

M1\M2	MODI + NWC	MODI + LCM	MODI + VAM	MODI + RAM
MODI + NWC	N/A	1	1	1
MODI + LCM	**0**	N/A	1	**4.095E-218**
MODI + VAM	**0**	**0**	N/A	**0**
MODI + RAM	**0**	1	1	N/A

Source: own work

Correlation analysis

The values of the Pearson correlation coefficient *p* were determined to identify the existence of a correlation between *e* and the number of nodes (*n*+*m*). The obtained results were presented in Table 20 and, in accordance with the interpretation presented by Ratner [2009], they indicate the occurrence of strong positive linear connection between the data for NWC, whereas for the other methods its value is moderate (taking into consideration the results of analysis of scatter diagrams, it was assumed that no significant correlation occurred in them).

Table 20. Value of the Pearson linear correlation between the number of nodes and the average error

Method	*p*
NWC	0.97784511
LCM	0.582538311
VAM	0.517584878
RAM	0.638177185

Source: own work

The correlation between the number of iterations of MODI method *it* and the number of nodes that describe TP instance, was subject to similar analysis. On the basis of the Pearson linear correlation value (Table 21), the possibility of presence of strong positive correlation between the examined data for all methods was identified.

Table 21. Value of the Pearson linear correlation coefficient between the number of nodes and the *it*

Method	*p*
MODI + NWC	0.992342
MODI + LCM	0.988593
MODI + VAM	0.984694
MODI + RAM	0.989044

Source: own work

DISCUSSION AND CONCLUSIONS

In this article, the assumed goals have been achieved and the observations made by Ali and Mustapha [2013], Deshpande [2009], Khan et al. [2016], Liu and Trung [2013], according to which VAM is characterised by a significant quality of the determined results, whereas NWC develops solutions of low efficiency, have been confirmed (through statistical analysis including the Wilcoxon Signed-Rank Test). This correlation has also been maintained for the number of iterations after which the MODI method created optimal allocation, applying the above-mentioned techniques for determination of the initial solution. However, contrary to the observations made for small TP instances, RAM was characterised by a higher value $\bar{e}$ than LCM for set S3, demonstrating the impossibility to generalise results obtained for small problems (presented e.g. by Deshpande [2009]), in order to determine their efficiency for higher instances.

When analysing only the quality of the determined results, it is recommended to apply VAM both for the determination of initial solution in MODI method and for performing allocation of resources, using only heuristics. However, taking into consideration the utilitarian approach and possible occurrence of the necessity to solve TP instances without using the appropriate software, it is recommended to use LCM for solving large instances of TP due to the fact that technique offers solutions better than NWC and comparable with RAM; furthermore, it is characterised by the simplicity of application.

In contrast to other identified works containing the comparison of heuristics applied in the process of TP solving [Ali and Mustapha 2013, Deshpande 2009, Khan et al. 2016, Liu and Trung 2013, Soomro et al. 2014], we also performed the analysis of dependability between efficiency of particular methods and parameters describing the tasks, which is a novel solution of a kind. The presence of strong correlation between the number of nodes describing the TP instance and the number of iterations necessary to determine the optimal solution by MODI method has been identified.

Further work covering the analysis of heuristics applied for solving TP may include examination of new and simultaneously less popular heuristics (e.g. Extremum Difference Method described by Kasana and Kumar [2013], Inverse Coefficient of Variation Method by Jude et al. [2017], Logical Development of Vogel's Approximation Method developed by Das et al. [2014] or Karagul-Sahin Approximation Method by Karagul and Sahin [2019]).

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PORÓWNANIE KLASYCZNYCH ALGORYTMÓW HEURYSTYCZNYCH STOSOWANYCH W PROCESIE ROZWIĄZYWANIA PROBLEMÓW TRANSPORTOWYCH

STRESZCZENIE. Wstęp: Zagadnienie transportowe (ZT) jest specjalnym przypadkiem programowania całkowitoliczbowego, charakteryzującym się niekwestionowanym znaczeniem praktycznym (w szczególności w obszarze logistyki). Z tego powodu powstało wiele technik przeznaczonych do rozwiązywania problemu zarówno w sposób optymalny, jak i przybliżony. Problem wyboru efektywnej metody konstruowania suboptymalnego rozwiązania dla ZT został poruszony przez wielu badaczy, jednakże zastosowanie przez nich tylko niektórych heurystyk, użyte "łoże testowe", a także brak przeprowadzenia testów statystycznych uniemożliwiają jednoznaczne określenie odpowiedniego podejścia do stosowania heurystyki w ZT, pozostawiając lukę badawczą, która stała się inspiracją do napisania niniejszego artykułu. Dodatkowym celem artykułu jest porównanie wybranych metod przybliżonych, z uwzględnieniem liczby iteracji niezbędnych do zaprojektowania optymalnego rozwiązania za pomocą metody Modified Distribution (MODI) oraz wykazanie potencjalnych korelacji pomiędzy parametrami opisującymi instancję problemu a skutecznością technik.

Metody: W pracy przedstawiono badania porównawcze czterech klasycznych heurystyk (NWC, LCM, VAM i RAM). Testy przeprowadzono na trzech zestawach zadań, składających się z 2500 pseudolosowo wygenerowanych instatacji problemu. Obserwacje potwierdzono za pomocą testu Wilcoxon Signed-Rank i współczynnika korelacji liniowej Pearsona.

Wyniki: Badania potwierdzają, że VAM charakteryzuje się znaczącą jakością wyznaczonych wyników, podczas gdy NWC konstruuje rezultaty o niskiej jakości. W przeciwieństwie do wyników sformułowanych dla niewielkich instatacji ZT, wyniki metody RAM dla dużego zbioru charakteryzowały się wyższą wartością błędu niż rezultaty LCM, wykazując brak możliwości uogólnienia wniosków prawdziwych dla małych problemów (przedstawionych np. w literaturze przedmiotu).

Wnioski: Zaleca się stosowanie VAM zarówno do określania bazowego rozwiązania w metodzie MODI, jak i do przygotowania alokacji zasobów, w przypadku korzystania wyłącznie z heurystyk. Biorąc jednak pod uwagę podejście utylitarne i możliwość wystąpienia konieczności rozwiązywania instancji ZT bez użycia odpowiedniego oprogramowania, zaleca się stosowanie LCM do rozwiązywania dużych instancji problemu. Zidentyfikowano także silną korelację pomiędzy liczbą węzłów opisujących instancję ZT a liczbą iteracji niezbędnych do określenia optymalnego rozwiązania za pomocą metody MODI.

Słowa kluczowe: zagadnienie transportowe, Best Initial Feasible Solution, MODI, VAM , RAM, LCM

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