



## MODEL OF THE IMPACT OF PARAMETERS CONTROLLING REPLENISHMENT IN THE BS (MIN-MAX) CONTINUOUS REVIEW SYSTEM ON THE ACTUAL INVENTORY AVAILABILITY

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**ABSTRACT. Background:** Due to random changes in demand, inventory management is still - despite the development of alternative goods flow management concepts - an important issue both in terms of costs of maintenance and replenishment as well as the level of service measured by inventory availability levels. There are a number of replenishment systems to be used in such conditions, but they are most often formed on the basis of two basic ones: a system based on the reorder point and based on periodic inspection. This paper refers to the former system, the BS system (min-max), in which an order is placed after reaching inventory level B (information level, reorder point) for a quantity allowing to reach level S. This system is very often used in business practice. Observations conducted under realistic conditions indicate the need to improve the classical models describing the system. This results, among other things, from the fact that the actual level of available inventory at the start of the replenishment cycle may be significantly lower than level B, resulting in lower than expected levels of customer service. Taking account of this phenomenon through model determination of the cumulative distribution function for the observed difference makes it possible to select the correct parameters to control the replenishment system in question and - therefore - to achieve the expected economic effects.

**Methods:** The object of the study is to create a mathematical model allowing the determination of the required inventory level B taking into account the difference  $\Delta$  between this level and the actual level of inventory at the start of the replenishment cycle. To determine the effect of various factors such as demand distribution parameters in the adopted unit of time and the difference between level S (max) and B (min), a dedicated tool (simulator in EXCEL spreadsheet) for determining the distribution of frequency of value  $\Delta$  has been developed. Then a mathematical model allowing the determination of the distribution and its parameters as a function of the difference  $r = S - B$  for virtually any distribution of demand has been developed and implemented in a separate EXCEL spreadsheet.

**Results:** It was found that there is the need to take into account the distribution of the difference between the information level B (the reaching or exceeding of which is a signal to place an order) and the actual level of inventory at the start of the replenishment cycle when determining the inventory replenishment control parameters in the BS system. A mathematical model allowing to determine the incidence and distribution of function of value depending on the demand distribution parameters and difference  $r$  between the S level (max) and B level (min) has been developed and used for calculations. High compatibility of results obtained from model calculations with the results obtained through simulation imitating real events has been shown.

**Conclusions:** The model described in this paper will allow a more accurate determination of parameters that control the BS system to safeguard the required level of service and conditions relating to the volume of deliveries. Further work is required to develop an effective model solution for a general formula presented in this paper used to calculate the B parameter as a function of the required service level and the S parameter depending on the designated (e.g. economic) average delivery.

**Key words:** inventory management, restocking in the BS system (min-max, up-to-level review continuous review replenishment model), level of service, modelling, simulation.

### INTRODUCTION

The discussed BS inventory replenishment system [ELA, 1994] consists in that an order

may be placed at any moment when available inventory achieves or drops below B-level, while order quantity is calculated as a difference between determined maximum level (S) and available inventory [ELA, 1994].

The BS system is widely used in practice, particularly in the case of a limited number of relatively big releases.

In literature, the system is often referred to as up-to-level continuous review (in opposition to fixed order continuous review, BQ-level according to ELA terminology). It should be noted that in numerous publications, the term "up-to-level" also refers to a periodical review, in the classical ST variant, and Ss. The subject of papers based on the discussed system (BS in continuous review) is different aspects of inventory replenishment. They include such elements as determining a system's optimum parameters considering cost-related criteria (e.g. [Babai., Jemai, Dallery, 2011], where demand distribution compatible with Poisson's distribution, characterising slowly-rotating entries, was adopted). Other papers present the effectiveness of various forecasting methods in implementing the BS-system (e.g. [Teunter, Sani, Li, Disney, Gaalman, 2014]. In most cases, the research is based on an assumption on the stochastic nature of changes to quantities influencing the execution of a system, quantities of deficiencies and scale of the so-called pent-up demand (e.g. [Taleizadeh, Taghi, Niaki, Meibodi, 2013]. In most papers, developed algorithms are used to simulate studied processes.

The works analysed usually assume that inventory replenishment commences upon the achievement of reorder point B. Practice and initial simulation tests carried out by the author of this paper show that in many cases it happens with a level of inventory much lower than the set B-level. Thus, the purpose of the paper is to determine the impact of different factors (particularly the distribution of frequency in which non-zero demand quantities and values of parameters B and S occur), and the impact of this phenomenon on inventory replenishment, particularly in relation to service level.

## CHARACTERISTICS OF THE BS SYSTEM

General rules governing the execution of the BS system include:

- 1) Determining the controlling parameters:
  - setting the required service level (method of factor definition and factor value),
  - calculating parameters of demand distribution in an adopted time unit (random changes exclusively),
  - determining LT - replenishment lead time
- 2) Specifying informational level (reorder point) B. In a classic approach, informational level B is specified on the basis of dependence

$$B = D \cdot LT + SS \quad (1)$$

where:

D – mean demand in an adopted time unit (e.g. mean daily/weekly demand)

LT - replenishment lead time between review and receipt of delivery.

SS - safety stock expressed in the following way:

$$SS = \omega \cdot \sigma_{DLT} \quad (2)$$

where:

$\omega$  – safety coefficient, dependent on adopted service level and type of distribution of demand occurrence frequency,

$\sigma_{DLT}$  - standard deviation of demand in replenishment cycle time

In general cases (random variability of demand and replenishment cycle time), the following formula applies:

$$\sigma_{DLT} = \sqrt{\sigma_D^2 \cdot LT + \sigma_{LT}^2 \cdot D^2} \quad (3)$$

where:

$\sigma_D$  – standard deviation of demand in an adopted time unit (the same as for D)

$\sigma_{LT}$  – standard deviation of replenishment lead time.

- 3) Determining level S, to which inventory will be replenished upon ordering. It should be borne in mind that difference between levels S and B will determine mean order quantity. In a simplified approach,  $S =$

$B+Q$ , where  $Q$  may be, for instance, economic order quantity ( $Q = EOQ$ ).

- 4) An inventory replenishment procedure compatible with set parameters, which consists in that in a specific moment, resulting from an adopted review cycle,
- current status of economic stock (available, disposed stock)  $S_e$  is determined

$$S_e = S_W + S_o + S_{er} - S_b \quad (4)$$

where:

$S_w$  stock physically available in the warehouse (on-hand),  
 $S_o$  orders placed, but not yet implemented,  
 $S_{er}$  stock en route,  
 $S_b$  stock already booked.

- and an order in the following quantity is made:

$$q = S - S_e \quad (5)$$

Figure 1 illustrates rules governing the implementation of the BS system.

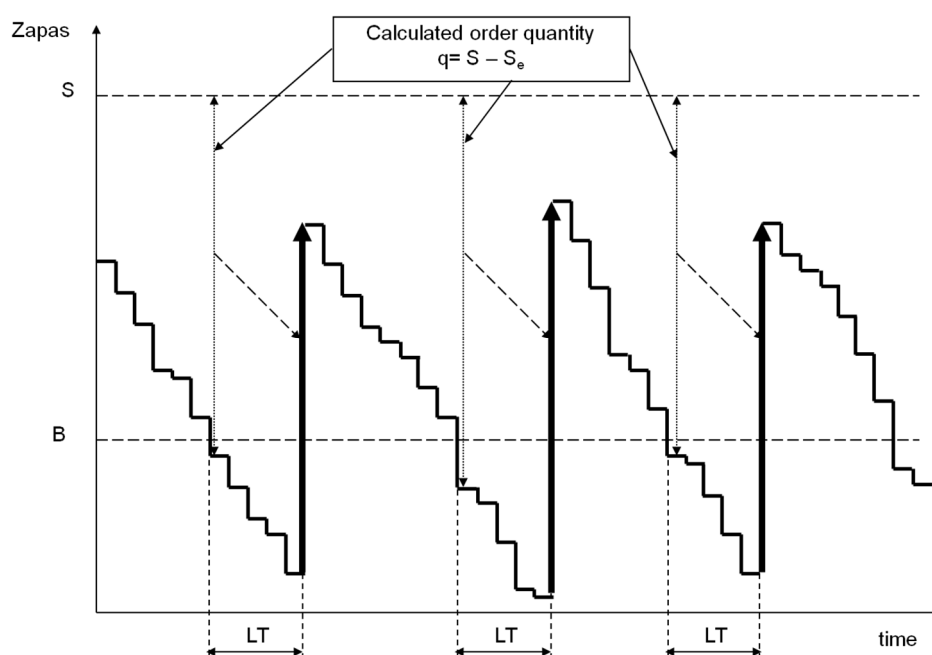


Fig. 1. Illustration of a rule for performing inventory replenishment in the BS system  
 Rys. 1. Ilustracja zasady realizacji odnawiania zapasu w systemie BS

Considerations presented in the paper assume invariability of replenishment cycle time  $T$  ( $\sigma_T \approx 0$ ). In such a case, dependence (1) will assume the following form:

$$B = D \cdot LT + \omega \cdot \sigma_D \sqrt{LT} \quad (6)$$

An additional assumption concerns a type of distribution of demand  $D$ . Normal distribution has been assumed as a characteristic one for fast-moving goods.

A premise underlying the considerations presented in the article was the fact that,

according to the assumption, upon making an order (more generally, upon commencing a replenishment cycle), available inventory is usually lower than B-level. Difference between B and actual level of economic stock  $S_e$  on commencing the cycle has been here defined as  $\Delta$ :

$$\Delta = B - S_e \quad (7)$$

For further considerations, the following abbreviations have been adopted:

- D** – arithmetic mean from all demand quantities
- $\sigma_D$  - standard deviation from all demand quantities
- D\*** - average (arithmetic mean) from non-zero demand quantities
- $\sigma_{D^*}$  - standard deviation calculated from non-zero demand quantities

Simulation tests of the BS inventory system carried out by the author have shown dependence between service level calculated as probability of servicing demand in a replenishment cycle ( $\alpha$  Service Level) and a difference between levels B and S ( $r=S-B$ ). An example of this dependence has been shown in Figure 2a. A simulation experiment described below was carried out to confirm these observations.

The tests were performed with the use of an authorship application made in an Excel spreadsheet.

To explain the phenomenon observed during the simulation, illustrated in Figure 2, distribution of frequency of quantity  $\Delta$  in function  $r$  was tested in detail. To this end, a tool (simulator) was configured in a way to additionally allow the measurement of actual inventory level on launching a replenishment cycle, thus upon the meeting of condition  $S_e \leq B$ ). The registration allowed determining the distribution of variable  $\Delta$  ( $\Delta = B - S_e$ ) and calculating its parameters.

Both in the simulation experiment and in further model calculations, a classic probabilistic definition defining service level as probability of non-occurrence of inventory

shortage in a period covering replenishment cycle  $LT$  has been adopted as a definition of service level influencing practical use of dependence (3). It is defined as probability of servicing demand  $\alpha$  Service Level [Tempelmeier H., 2000].

## SCOPE OF SIMULATION TESTS SERVING THE CONSTRUCTION AND VERIFICATION OF THE MODEL

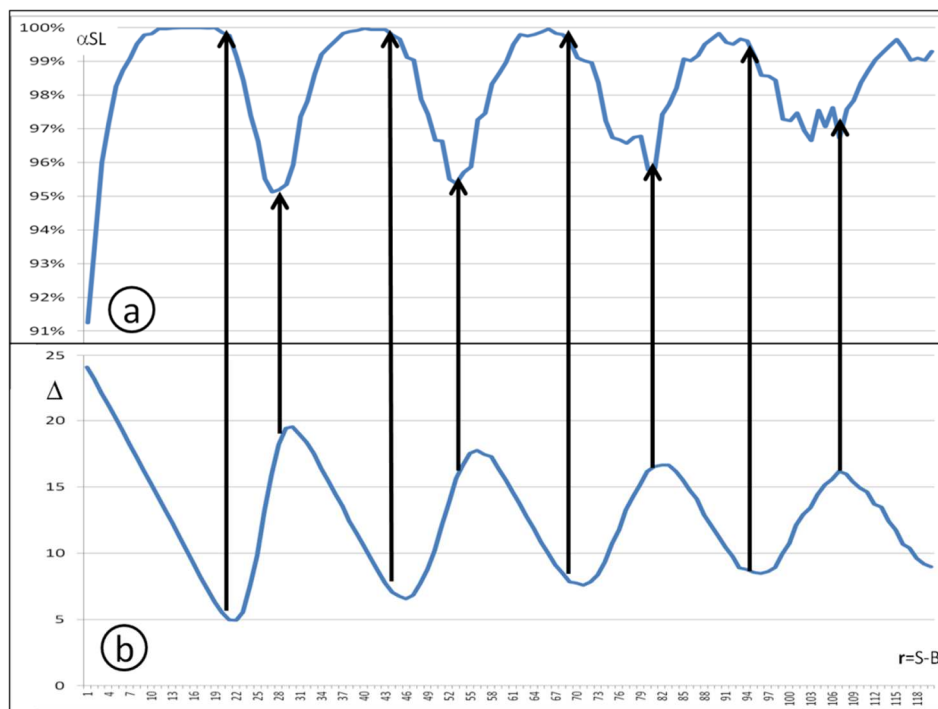
The main source of data for simulating inventory replenishment are randomly generated time series of demand for an adopted time unit (1 day was adopted). Mean parameters of generated distributions were: mean **D** = 25.06, standard deviation  $\sigma_D=2.50$ . The nature of changes in demand allows assuming that zero values do not occur in a time series of demand, thus: **D\***=**D**=25.06 and  $\sigma_{D^*}=\sigma_D=2.50$ . Generated distributions were compatible with a normal distribution, which was verified with the Kolmogorov–Smirnov test.

Simulation tests performed made it possible to specify (for an adopted demand distribution) dependence between quantities  $\Delta$  and  $r$ . It has been shown in Figure 2b.

The first step towards developing a mathematical model of the observed phenomenon was the definition of model dependence of the value of expected difference  $\Delta$  on difference  $r$ :  $E(\Delta)=f(r)$ .

The model's character is discreet. Studied values of difference  $r$  correspond to successive natural figures. It will correspond to the majority of actual cases which deal with a defined individual quantity of releases. Thus, for any value of index  $j$ :  $r_j = S_j - B$ . It was also assumed that  $r_1 = 1$  and  $r_{j+1} = r_j + 1$ .

Calculation of expected value  $E(\Delta)=f(r_j)$  is of recursive nature. While in the case when  $D_i^* \geq r_j$ , there is  $\Delta_{i,j} = D_i^* - r_j$ , for  $D_i^* < r_j$  there is  $\Delta_{i,j} = E(\Delta_{r_j - D_i^*})$ .



Source: own study

Fig. 2. The courses of dependence of an adopted  $\alpha SL$  - 2a service level indicator and difference  $\Delta (\Delta = B - S_e) - 2b$ , on the difference of levels  $r = S - B$

Rys. 2. Otrzymane drogą symulacji przebiegi zależności przyjętego wskaźnika poziomu obsługi POP - 2a, oraz różnicy  $\Delta (\Delta = B - Z_w) - 2b$ , od różnicy poziomów  $r = S - B$ .

Table 1. Rules of calculation of expected value  $E(\Delta) = f(r_j)$   
 Tabela 1. Zasady obliczania oczekiwanej wartości  $E(\Delta) = f(r_j)$

	$D_i^*$	$D_1^*=1$	$D_2^*=2$	$D_3^*=3$	$D_4^*=4$	$D_{max-1}^*$	$D_{max}^*$
$r$	$\sum f(D_i^*) = 1$	$f(D_1^*)$	$f(D_2^*)$	$f(D_3^*)$	$f(D_4^*)$	$f(D_{max-1}^*)$	$f(D_{max}^*)$
$r_1=1$	$\Delta(r_1)$	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	$D_{max-2}^*$	$D_{max-1}^*$
$r_2=2$	$\Delta(r_2)$	$\Delta(r_1)$	<b>0</b>	<b>1</b>	<b>2</b>	$D_{max-3}^*$	$D_{max-2}^*$
$r_3=3$	$\Delta(r_3)$	$\Delta(r_2)$	$\Delta(r_1)$	<b>0</b>	<b>1</b>	$D_{max-4}^*$	$D_{max-3}^*$
$r_4=4$	$\Delta(r_4)$	$\Delta(r_3)$	$\Delta(r_2)$	$\Delta(r_1)$	<b>0</b>	$D_{max-5}^*$	$D_{max-4}^*$
$r_{max-1}$	$\Delta(r_{max-1})$	$\Delta(r_{max-2})$	$\Delta(r_{max-3})$	$\Delta(r_{max-4})$	$\Delta(r_{max-5})$		
$r_{max}$	$\Delta(r_{max})$	$\Delta(r_{max-1})$	$\Delta(r_{max-2})$	$\Delta(r_{max-3})$	$\Delta(r_{max-4})$		
		$G[D_i^*; r_j] = 0$				$G[D_i^*; r_j] = 1$	

Source: own study

The above rules have been presented in Table 1. For  $D^*=4$  and  $r=3$ , for instance, the difference is:  $\Delta = 4 - 3 = 1$ , and this quantity (occurring with a probability equal to  $f(D^*=4)$ ) is included in the calculation of expected value  $E(\Delta) = \Delta(r=3)$ .

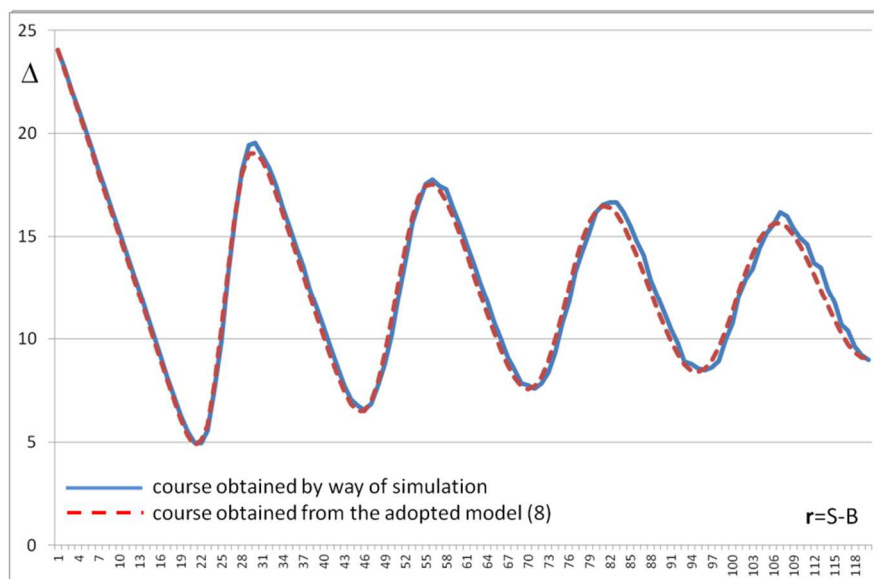
For  $D^*=3$  and  $r=4$ , on the other hand, the difference is:  $\Delta = 3 - 4 = -1$ . For calculating expected value  $E(\Delta) = \Delta(r=4)$ , in this case, value  $E(\Delta) = \Delta(r=1)$  will be calculated "earlier".

The above elements allow presenting the rule for calculating expected value  $E(\Delta_j)$  in the following manner:

$$E(\Delta_j) = \Delta(r_j) = \frac{\sum_{i=1}^{i_{\max}} G[D_i^*; r_j] \cdot (D_i^* - r_j) \cdot f(D_i^*) + \sum_{i=1}^{i_{\max}} \{1 - G[D_i^*; r_j]\} \cdot \Delta(r_j - D_i^*) \cdot f(D_i^*)}{f(D_i^*)} \quad (8)$$

$$\text{where: } G[D_i^*; r_j] = \begin{cases} D_i^* \geq r_j \rightarrow G[D_i^*; r_j] = 1 \\ D_i^* < r_j \rightarrow G[D_i^*; r_j] = 0 \end{cases}$$

Figure 3 illustrates compatibility of courses of dependence  $\Delta = f(\mathbf{r})$  obtained by way of simulation and with the use of the presented model (formula 8).



Source: own study

Fig. 3. Comparison of courses  $\Delta=f(\mathbf{r})$  for results obtained by way of simulation and with the use of the model (formula 8)

Rys. 3. Porównanie przebiegów  $\Delta=f(\mathbf{r})$  dla wyników otrzymanych drogą symulacji i przy zastosowaniu modelu (formula 8)

Using the model to determine the quantity of parameters  $B$  and  $S$ , which control the functioning of the system and guarantee maintaining required service level, and observing restrictions or requirements related to the volume of sales, requires modifying the classic formula to enable it to calculate reorder level  $B$  (formula 6).

In the first approximation, expected quantity  $E(\Delta)$  was added to the formula.

$$B = D \cdot LT + E(\Delta) + \omega \cdot \sigma_D \sqrt{LT} \quad (9)$$

As it was done previously, variation of replenishment cycle time was neglected.

Thus, for the adopted reorder level  $B$ , expected service level may be determined on the basis of dependence:

$$POP = \Phi[\omega] = \Phi \left[ \frac{B - D \cdot LT - E(\Delta)}{\sigma_D \cdot \sqrt{LT}} \right] \quad (10)$$

On the basis of courses presented in Figure 4a, it may be concluded that using the formula (9) would provide understated levels of reorder point  $B$  for specific values of difference  $\mathbf{r}$ , which in practice would mean lower service levels than expected.

To improve compatibility of both courses, standard deviation of difference  $\Delta$  ( $\sigma_\Delta$ ) was additionally introduced to calculations of safety stock. In calculations, a commonly-

known dependence on the calculation of a random variable  $Var(X) = E(X^2) - [E(X)]^2$  was adopted.

Previously specified quantity  $E(\Delta)$  was used, and expected value  $E(\Delta^2)$  was calculated in the same way at it has been presented above.

$$E(\Delta_j^2) = \sum_{i=1}^{i_{max}} G[D_i^*; r_j] \cdot (D_i^* - r_j)^2 \cdot f(D_i^*) + \sum_{i=1}^{i_{max}} \{1 - G[D_i^*; r_j]\} \cdot [\Delta(r_j - D_i^*)]^2 \cdot f(D_i^*) \quad (11)$$

Thus, with the use of results obtained from formulas (8) and (11), standard deviation of  $\Delta$  ( $\sigma_\Delta$ ): was calculated:

$$\sigma_{\Delta_j} = \sqrt{E(\Delta_j^2) - [E(\Delta_j)]^2} \quad (12)$$

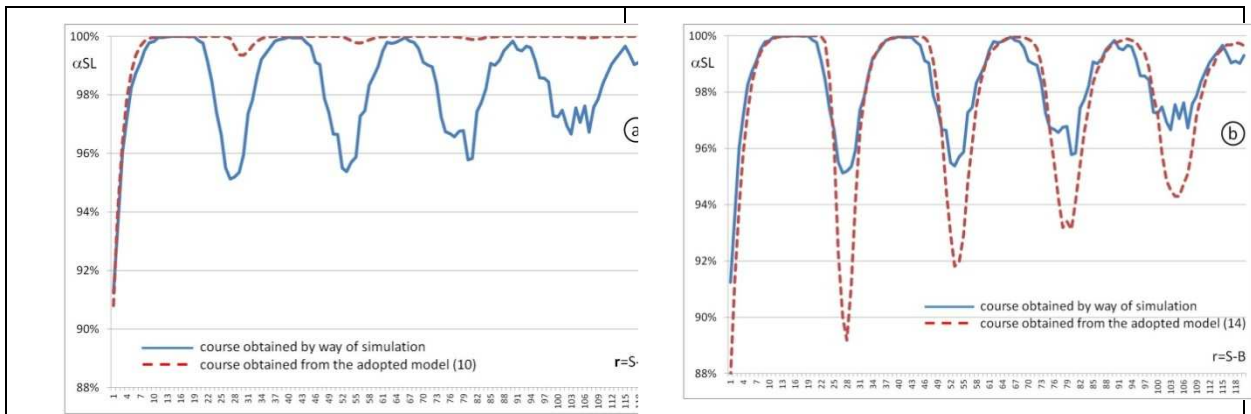
On the basis of these calculations, formula (12) was broadened to the following form:

$$B = D \cdot LT + E(\Delta) + \omega \cdot \sqrt{\sigma_D^2 \cdot LT + \sigma_\Delta^2} \quad (13)$$

As it can be seen, it was assumed here that distributions of both variables ( $D_T=D \cdot LT$  and  $\Delta$ ) are compatible with a regular distribution, which would allow using a common safety coefficient and simple determination of expected service level.

$$POP = \Phi[\omega] = \Phi \left[ \frac{B - D \cdot LT - E(\Delta)}{\sqrt{\sigma_D^2 \cdot LT + \sigma_\Delta^2}} \right] \quad (14)$$

Comparison of results of simulation tests and quantities obtained from the model once again pointed at a significant discrepancy (Fig. 4b), however of a different nature than in the case of formula (10) (Figure 4a).



Source: own study

Fig. 4. Comparison of courses  $\alpha SL=f(r)$  for results obtained by way of simulation and with the use of models described by formulas 10 (Fig. 4a) and 14 (Fig. 4b).

Rys. 4. Porównanie przebiegów  $POP=f(r)$  dla wyników otrzymanych drogą symulacji i przy zastosowaniu modeli opisanych formułami 10 (rys. 4a) oraz 14 (rys. 4b).

On the basis of courses presented in Figure 4, it may be concluded that using formula (9) would this time provide overstated levels of reorder B for specific values of difference  $r$ , which in practice would mean unnecessarily higher service levels than expected.

The above would mean that most probably, at least for certain scopes of difference  $r$ , the assumption regarding the compatibility of distribution of quantity  $\Delta$  with regular

distribution is incorrect. To this purpose, detailed simulation tests were carried out in order to determine the character of distribution, occurrence frequency and a course of distribution function of difference  $\Delta$ , for various levels of variable  $r$ .

It was stated that for  $r$  values with greatest deviations between simulation results and the model, distribution  $f(\Delta)$  is of complex nature and is indeed a mixture of two distributions

resulting from distribution  $f(D^*)$ . It has been illustrated in Figure 5.

Thus, an attempt at developing a model of distribution  $f(\Delta)$  for different levels of difference  $r$  was taken. As in the case of a model for an expected value and standard deviation  $\Delta$ , it is a complex recursive model possible to be implemented in a spreadsheet.

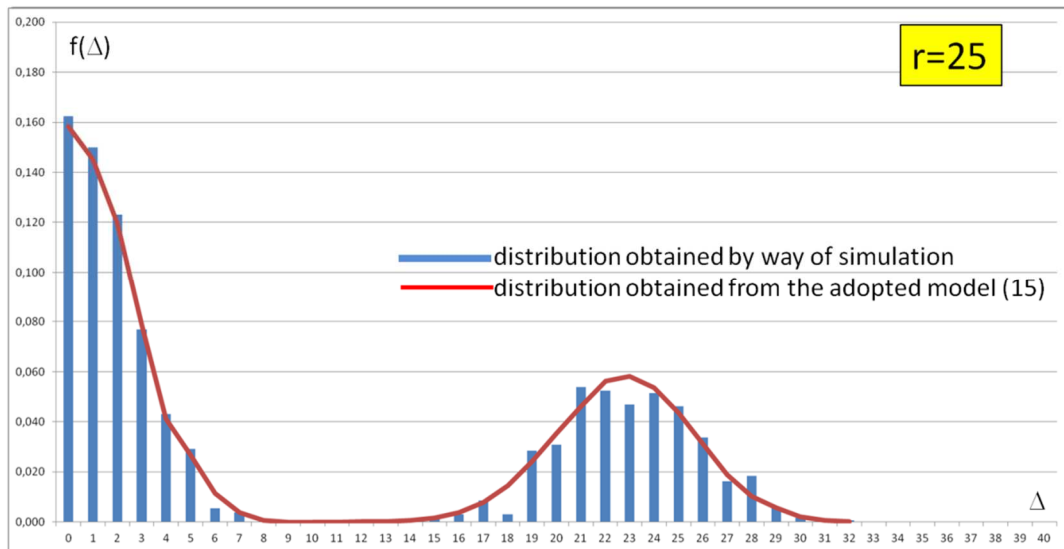
As it was in the case of determining  $E(\Delta)$  and  $\sigma_\Delta$ , the model was verified by creating a table ( $i_{max}=50$ ) x ( $j_{max}=500$ ).

$$f(\Delta_{i,j}) = G[i; j; i_{max}] \cdot f(D_{i+j}^*) + \sum_{k=0}^{i_{max}} H[D_{i+k}^*; r_{j-1-k}] \cdot f(\Delta_{i+k, j-1-k}) \cdot f(D_{i+k}^*) \quad (15)$$

$$\text{where: } G[i; j; i_{max}] = \begin{cases} i - j \leq i_{max} \rightarrow G[i; j; i_{max}] = 1 \\ i - j > i_{max} \rightarrow G[i; j; i_{max}] = 0 \end{cases}$$

$$\text{and } H[D_{i+k}^*; r_{j-1-k}] = \begin{cases} D_{i+k}^* \leq r_{j-1-k} \rightarrow H[D_{i+k}^*; r_{j-1-k}] = 1 \\ D_{i+k}^* > r_{j-1-k} \rightarrow H[D_{i+k}^*; r_{j-1-k}] = 0 \end{cases}$$

Figure 15 shows high compatibility of the model distribution with distribution obtained by way of simulation.



Source: own study

Fig. 5. An experimental (simulation result) and model distribution of frequency with which a quantity of difference  $\Delta$  -  $f(\Delta)$  occurs, for a case in which significant deviation of service level obtained during simulation from a quantity calculated with the use of formulas (10) and (14) was observed (see Fig. 4).

Rys. 5. Doświadczalny (jako wynik symulacji) i modelowy rozkład częstości występowania wielkości różnicy  $\Delta$  -  $f(\Delta)$ , dla przypadku, w którym obserwowano znaczące odchylenie poziomu obsługi otrzymanego w trakcie symulacji, od wielkości obliczonej przy wykorzystaniu formuł (10) i (14) (patrz rys. 4)

It allowed testing distribution for different  $r$ -values. Examples of calculation results have been shown in figures presented in Table 2: The consequence of diversified courses of density function  $f(\Delta)$  is the diversification of the course of distribution function  $F(\Delta)$ , also shown there.

On the basis of observations described above, another modification of formula was suggested in terms of calculating reorder level  $B$  by extending formula (12) to the following form:

$$B = D \cdot LT + E(\Delta) + \sqrt{\omega_{\alpha SL}^2 \cdot \sigma_D^2 \cdot LT + [\Delta_{F(\Delta)=\alpha SL} - E(\Delta)]^2} \quad (16)$$



where:

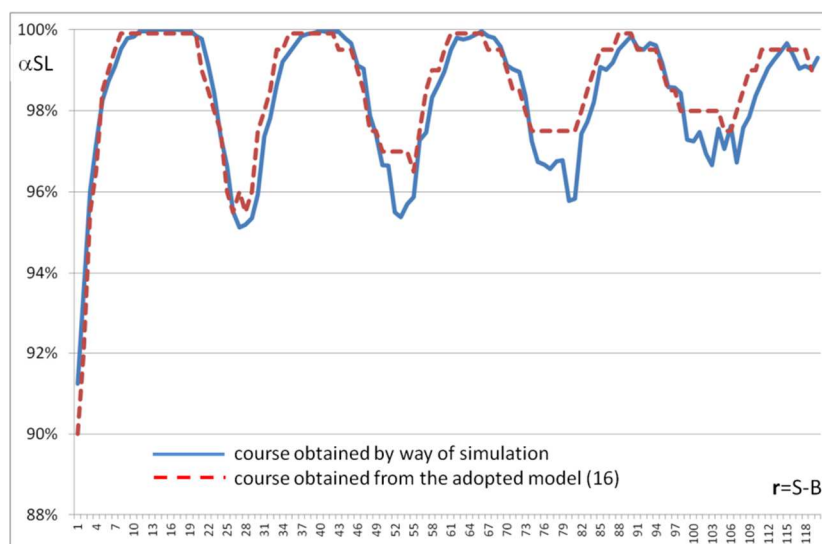
$\Delta_{F(\Delta)=\alpha SL}$  – value of  $\Delta$ , for which distribution function  $F(\Delta)$  equals the assumed service level  $\alpha SL$ .

$\omega_{\alpha SL}$  – safety coefficient in terms of variability of demand corresponding to an adopted service level  $\alpha SL$ , according to the type of demand distribution,

Table 2. Selected distributions of occurrence frequency and distribution function  $\Delta$  for various levels of difference  $r$  ( $r=S-B$ )  
 Tabela 2. Wybrane rozkłady częstości występowania oraz dystrybuanty różnicy  $\Delta$ , dla różnych poziomów różnicy  $r$  ( $r=S-B$ )

	$r = S - B = 10$	$r = S - B = 25$	$r = S - B = 32$	$r = S - B = 500$
$E(\Delta)=f(r)$ $\sigma_{\Delta}=f(r)$				
Distribution $f(\Delta)$				
Distribution function $F(\Delta)$				

Source: own study



Source: own study

Fig. 6. Comparison of courses  $\alpha SL=f(r)$  for results obtained by way of simulation and with the use of the model (on the basis of formula 16)

Rys. 6. Porównanie przebiegów  $POP=f(r)$  dla wyników otrzymanych drogą symulacji i przy zastosowaniu modelu (na podstawie formuły 16)

As it can be seen, it was assumed that both variables ( $D_{LT}$  and  $\Delta$ ) are treated independently

and may be subject to various distributions. Formula (16) does not allow simple

determination of dependences regarding an expected service level, nevertheless, however, with assumed level of reorder B, it is possible to approximately set service level  $\alpha$ SL which complies with dependence (16). Achieved results compared with courses obtained by way of simulation have been shown in Figure 6.

## CONCLUSIONS

A model (or rather three models with different degrees of complexity) of the impact of parameters controlling replenishment in the BS (MIN-MAX) continuous review model on the actual inventory availability has been presented. It was possible by determining mathematical models of dependences that allow setting expected value, standard deviation and distribution density of difference  $\Delta$  between specified level of reorder B and actual level of economic stock  $S_e$  upon commencing an inventory replenishment cycle.

Implementing these models in an Excel spreadsheet allowed specifying model dependences  $\Delta$ ,  $\sigma_\Delta$  and  $f(\Delta)$  as a function of variable  $\mathbf{r}$  (difference between levels S and B) for any non-zero distributions of the value of demand  $D^*$ . Obtained results showed high compatibility with results of simulation tests, both for dependence  $\Delta=f(\mathbf{r})$ , and  $\sigma_\Delta=f(\mathbf{r})$ .

The condition for achieving a model's compatibility with simulation results with reference to a service level  $\alpha$ SL indicator (probability of servicing demand in an inventory replenishment cycle) was the inclusion of distribution F ( $\Delta$ ) and a fact that it generally differs from a distribution function regarding the distribution of non-zero values of demand F( $D^*$ ) in the model.

The presented model may support the selection of parameters B and S of the discussed inventory replenishment system from the perspective of both expected service level and cost efficiency.

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# MODEL WPLYWU PARAMETRÓW STERUJĄCYCH ODNAWIANIEM ZAPASÓW W PRZEGLĄDZIE CIĄGŁYM TYPU MIN-MAX (BS) NA RZECZYWISTY POZIOM ICH DOSTĘPNOŚCI

**STRESZCZENIE. Wstęp:** Zarządzanie zapasami w warunkach losowych zmian popytu jest wciąż - mimo rozwoju alternatywnych koncepcji zarządzania przepływem dóbr - ważnym zagadnieniem zarówno z punktu widzenia kosztów utrzymania i uzupełniania zapasów, jak i poziomu obsługi mierzonego poziomem dostępności zapasu. Istnieje szereg systemów uzupełniania zapasu w takich warunkach, przy czym stanowią one najczęściej rozwinięcie dwóch podstawowych: systemu opartego na punkcie ponownego zamówienia oraz opartego na przeglądzie okresowym. Artykuł odnosi się do pierwszego z nich, systemu BS (min-max), w którym zamówienie składane jest po osiągnięciu przez dostępny zapas poziomu B (poziomu informacyjnego, punktu ponownego zamówienia), w wielkości stanowiącej uzupełnienie do poziomu S. System ten jest bardzo często stosowany w praktyce gospodarczej. Obserwacje prowadzone w rzeczywistych warunkach wskazują konieczność udoskonalania klasycznych modeli opisujących ten system. Wynika to m. in. z tego, że rzeczywisty poziom dostępnego zapasu w chwili rozpoczęcia cyklu uzupełnienia może być znacząco niższy od poziomu B, co skutkuje niższymi od oczekiwanych poziomami obsługi klienta. Uwzględnienie tego zjawiska poprzez modelowe wyznaczenie dystrybuanty obserwowanej różnicy pozwala na poprawny dobór parametrów sterujących omawianym systemem odnawiania zapasu i - tym samym - osiągnięcie oczekiwanych efektów ekonomicznych.

**Metody:** Przedmiotem prezentowanych badań było stworzenie modelu matematycznego pozwalającego na wyznaczenie poziomu informacyjnego w tzw. systemie BS (inaczej min-max) odnawiania zapasu uwzględniającego różnicę pomiędzy wyznaczonym poziomem B, a rzeczywistym poziomem zapasu w chwili rozpoczęcia cyklu uzupełnienia. Dla wyznaczenia wpływu różnych czynników, m. in. parametrów rozkładu popytu w przyjętej jednostce czasu oraz różnicy pomiędzy poziomami S (max) oraz B (min), opracowano dedykowane narzędzie (symulator w arkuszu kalkulacyjnym EXCEL) pozwalające na określenie rozkładu częstości występowania wartości. Następnie opracowano i zaimplementowano w odrębnym arkuszu EXCEL model matematyczny pozwalający na wyznaczenie tego rozkładu i jego parametrów, jako funkcji różnicy  $r=S-B$  dla praktycznie dowolnego rozkładu popytu.

**Wyniki:** Wykazano konieczność uwzględnienia przy wyznaczaniu parametrów sterujących odnawianiem zapasu w systemie BS rozkładu różnicy pomiędzy poziomem informacyjnym B (którego osiągnięcie lub zejście poniżej niego stanowi sygnał do złożenia zamówienia), a rzeczywistym poziomem zapasu w chwili rozpoczęcia cyklu uzupełnienia. Opracowano i wykorzystano do obliczeń model matematyczny pozwalający na wyznaczenie rozkładu częstości występowania i dystrybuanty wielkości w zależności od parametrów rozkładu popytu oraz różnicy  $r$  pomiędzy poziomami S (max) oraz B (min). Stwierdzono wysoką zgodność wyników otrzymanych z obliczeń modelowych z wynikami otrzymanymi w drodze symulacji, imitującej rzeczywiste zdarzenia.

**Wnioski:** Przedstawiony w artykule model pozwoli na bardziej precyzyjne wyznaczenie parametrów sterujących systemem BS, gwarantujących zachowanie wymaganego poziomu obsługi oraz uwarunkowań dotyczących wielkości dostaw. Dalszych prac wymaga opracowanie efektywnego modelowego rozwiązania przedstawionej w artykule ogólnej postaci formuły na obliczenie parametru B w funkcji wymaganego poziomu obsługi) oraz parametru S w zależności od wyznaczonej (np. ekonomicznej) średniej wielkości dostawy.

**Słowa kluczowe:** zarządzanie zapasami, odnawianie zapasu w systemie BS (min-max, up-to-level continous review replenishment model), poziom obsługi, modelowanie, symulacja.

## MODELL DER EINWIRKUNG DER PARAMETER ZUR STEUERUNG DER BESTANDSERNEUERUNG IM SYSTEM DER DAUERÜBERWACHUNG VOM TYP MIN-MAX (BS) AUF DIE TATSÄCHLICHE LIEFERBARKEIT DER BESTÄNDE

**ZUSAMMENFASSUNG. Einleitung:** Trotz der Entwicklung von alternativen Konzepten zur Materialfluss-Steuerung bleibt das Bestandsmanagement unter zufälligen Nachfrageveränderungen ein wichtiges Anliegen, sowohl in Hinsicht auf die Kosten der Unterhaltung und Ergänzung von Beständen, als auch in Hinsicht auf die Servicequalität, gemessen an der Lieferbarkeit der Bestände. Es gibt eine Reihe von Systemen zur Bestandsergänzung unter den o. g. Umständen, wobei es sich meistens um die Entwicklung von zwei grundlegenden Systemen handelt: gestützt auf den Punkt der Neubestellung und auf die periodischen Übersichten. Der Artikel bezieht sich auf das erste der beiden, auf das BS-System (Min.-Max.), bei dem die Bestellung erfolgt, nachdem der lieferbare Bestand das Niveau B erreicht hat (Informationsniveau, Punkt der Neubestellung), und zwar in einer Menge, die eine Ergänzung bis auf das Niveau S ermöglicht. Dieses System wird in der praktischen Wirtschaft sehr häufig angewendet. Beobachtungen in realistischen Zuständen weisen auf die

Notwendigkeit hin, die klassischen Modelle zur Beschreibung dieses Systems zu verbessern. Dies ergibt sich u. a. daraus, dass der tatsächlich lieferbare Bestand zum Zeitpunkt der Einleitung des Ergänzungsverfahrens wesentlich niedriger sein kann, als das B-Niveau, was zu einer Verschlechterung der Kundenbetreuung führt. Durch eine Berücksichtigung dieses Phänomens durch die modellhafte Festlegung der Verteilungsfunktion im Rahmen der beobachteten Diskrepanz können eine richtige Einstellung der Parameter zur Steuerung des erwähnten Systems der Bestandserneuerung und damit auch die Erreichung der erwünschten wirtschaftlichen Ziele erreicht werden.

**Methoden:** Gegenstand der dargestellten Untersuchungen war die Erstellung eines mathematischen Modells, mit dem das Informationsniveau für die Bestandserneuerung in dem sog. BS-System (anders "min-max" genannt) als der Unterschied zwischen dem festgelegten B-Niveau und dem tatsächlichen Bestand zum Zeitpunkt der Einleitung des Ergänzungsverfahrens festgelegt werden kann. Um den Einfluss unterschiedlicher Faktoren, etwa der Parameter der Nachfrageverteilung innerhalb des angenommenen Zeitraums, und der Unterschiede zwischen dem Bestand S (max) und B (min) zu berücksichtigen, wurde ein zielgerechtes Tool (ein Simulator im EXCEL-Kalkulationsbogen) entwickelt, mit dem die Verteilung der Auftrittshäufigkeit von Werten festgestellt werden kann. Anschließend wurde in einem gesonderten EXCEL-Bogen ein mathematisches Modell entwickelt und umgesetzt, mit dem diese Verteilung und deren Parameter als eine Funktion der Differenz  $r=S-B$  für praktisch jede Nachfrageverteilung festgelegt werden kann.

**Ergebnisse:** Es wurde aufgezeigt, dass bei der Festlegung der Parameter zur Steuerung der Bestandserneuerung im BS-System die Differenz zwischen dem Informationsniveau B, (dessen Erreichung oder Unterschreitung das Bestellungssignal darstellt) und dem tatsächlichen Bestand zum Zeitpunkt der Einleitung des Ergänzungsverfahrens berücksichtigt werden soll. Es wurde ein mathematisches Modell erstellt und bei Berechnungen eingesetzt, anhand dessen die Verteilung der Häufigkeit und die kumulative Verteilung der Größe festgelegt werden können, und zwar je nach Verteilungsparametern und der Differenz  $r$  zwischen den Bestandsniveaus S (max) und B (min). Es wurde eine hohe Vereinbarkeit zwischen den Ergebnissen der Modellberechnungen und den Ergebnissen einer Simulation festgelegt, die die tatsächlichen Vorgänge imitieren sollte.

**Fazit:** Mit dem im vorliegenden Beitrag dargestellten Modell können die Parameter zur Steuerung des BS-Systems detaillierter festgelegt werden, sodass die Einhaltung der erwünschten Qualität der Kundenbetreuung und der Liefermengen sichergestellt werden kann. Weiterer Arbeiten bedarf dennoch die Entwicklung einer effektiven, modellhaften Lösung der im Beitrag allgemein dargestellten Formel für die Berechnung des Parameters B in der Funktion des erforderlichen Niveaus der Kundenbetreuung und des Parameters S, je nach der festgelegten (z. B. wirtschaftlichen) mittleren Liefergröße.

**Codewörter:** Bestandsmanagement, Bestandserneuerung im BS-System (min-max, up-to-level continuous review replenishment model), Kundenbetreuung, Modellierung, Simulation

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