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AN INVENTORY MODEL FOR GENERALIZED WEIBULL DETERIORATING ITEMS WITH PRICE DEPENDENT DEMAND AND PERMISSIBLE DELAY IN PAYMENTS UNDER INFLATION

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ABSTRACT. This paper derives an inventory model is developed for items that deteriorates at a generalized Weibull distributed rate when demand for the items is dependent on the selling price. Shortages are not allowed and price inflation is taken into consideration over finite planning horizon. A brief analysis of the cost involved is carried out by theoretical analysis.

Key words: inventory, inflation, Weibull distribution, selling price dependent demand, delay in payment.

INTRODUCTION

Harris-Wilson's [1915] classical inventory model assumes that the depletion of inventory is due to a constant demand rate. However in real-life situations, there is inventory loss by deterioration. Certain products like food items, drugs, pharmaceuticals, radioactive substances deteriorate during their normal storage period and therefore how to control and maintain inventories of deteriorating items becomes an important problem for decision makers. In this connection, studies of many researchers like Ghare and Schrader [1963], Covert and Philip [1973], Shah and Jaiswal [1977], Cohen [Cohen 1973], Giri and Goyal [1985] are very important.

In the traditional inventory EOQ model, the purchaser must pay for the items as soon as the items are received. However in practice, the supplier may provide a permissible delay to their customers. Thus the delay payment to the supplier is a kind of price discount. Since paying later indirectly reduces the purchase cost, it can motivate customers to increase their order quantity. Goyal [1985] derived an EOQ model under the condition of permissible delay in payments. Aggrawal and Jaggi [1995] extended Goyal's model to allow for deteriorating items. In this connection several papers such as Davis and Gaither [1985], Liao et al. [2006], Teng [2002] are very important. These models were developed with the assumption that the inflation does not have significant role to play on the inventory model. However an inventory represents a capital investment and must complete with other assets for a firm's limited capital funds. Thus, it is important to consider the effects of inflation on inventory system. Buzacott [1975] has considered the effects of inflation on the inventory system by assuming a constant inflation rate. Mishra [1979] derived an inflation model for the EOQ in which the time value of money and different inflation rates were considered. In this connection the studies

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of many researchers like Brahmbhatt [1982], Chandra and Bahner [1985], Uthayakumar and Geetha [2009], Kyo-Lung Hou [2006] are very important. We have also studied the researchers like Mishra [2012], Prandhan [2012], Panda [2013] to develop this paper.

In the present paper, we drive an EOQ model for inventory of items that deteriorate at a generalized Weibull distributed rate with price dependent demand and permissible delay in payments under inflation.

NOTATIONS OF INVENTORY MODEL

The fundamental notations used in this paper are given as follows:

(a) L_H = Length of finite planning horizon.

(b) D[p(t)] : selling price dependent demand per unit time

$$=\lambda - \mu\, p(t)$$

Where λ is fixed demand, λ > 0, μ > 0 and λ >> μ.

(c) p (t)=pe^{rt} : selling price per unit at time

(d) c (t) = ce^{rt} : unit purchase cost at time t, where p is unit selling price at t = 0

(e) A(t) = Ae^{rt} : ordering cost per order at time t, where A is ordering cost at t=0,

(f) The deterioration rate function follows a two parameter generalized Weibull distribution given as

$$\theta(t) = \lambda\beta t^{\beta-1} e^{-\lambda t^{\beta}},\ t > 0$$

(g) h = Inventory holding cost per unit per year excluding interest charges.

(h) r : Constant rate of inflation per unit time.

(i) Q : Optimum order quantity.

(j) T : Optimum cycle time.

(k) IC : Interest charged per annum by the supplier

(l) IE : Interest earned per annum by the retailer (IC > IE)

(m) M_{DP} : Permissible delay period for setting accounts.

(n) I(t) : Instantaneous level of inventory

(o) TRC (p, T) : Total relevant cost over [0, L_H]

(p) NRP (p, T) : Total relevant profit over finite planning horizon

The components of total relevant cost (TRC) consists of

(i) Cost of placing order (CPO)

(ii) Cost of deterioration (COD)

(iii) Cost of carrying inventory excluding interest charges (CCI)

(iv) Cost of Interest charged (IC) for unsold items at the initial time or after permissible delay period M_{DP}

(v) Interest earned (IE) from sales revenue during the permissible delay period [0, M]

Hence,

TRC = CPO + COD + CCI + IC – IE

The net profit is the difference of Gross revenue and total relevant cost, where, Gross Revenue

$$(GR) = (pe^{rT} - ce^{rT}).D[p(t)]$$
$$= (pe^{rT} - ce^{rT}).\lambda\text{-}\mu pe^{rT}$$

ASSUMPTIONS OF INVENTORY MODEL

The fundamental assumptions used in this paper are given as follows:

a. Shortages are not allowed,
b. The Lead time is zero,
c. Inflation rate is constant,
d. The account is not settled during the permissible delay period,
e. During the permissible delay period the generated sales revenue is deposited in an interest bearing account,
f. At the end of credit period, the customer pay off all the unit ordered and starts paying for the interest charged on the items in stock,
g. There is no repair or replacement of the deteriorated units during the given cycle,
h. Deterioration rate follows Weibull distributed with two parameter α and β.

MATHEMATICAL FORMULATION

Let the length of planning horizon is divided into n parts, where n is the number of replenishments occur during the period L_H i.e. L_H = nT.

Due to reasons of market demand and deterioration of the items, the inventory level gradually diminishes during the period [0, T].

The differential equation that governs the variation of inventory with respect to time is

$$\frac{dI(t)}{dt} + \theta(t)\, I(t) = \mu p e^{rt} - \lambda \quad ; 0 \le t \le T \tag{1}$$

with boundary conditions

I (0) = Q and I (T) = 0

The solution under boundary condition is given by

$$I(t) = \frac{\lambda}{\beta+1}\left(\frac{T^{\beta+1}}{t^{\beta}} - t\right) - \mu p \sum_{k=0}^{\infty} \frac{r^k}{k!\,(\beta+k+1)}\left(\frac{T^{k+\beta+1}}{t^{\beta}} - t^{k+1}\right) \tag{2}$$

Since the lengths of time intervals are all the same, we have

$I(jT+t) = I(t)\ ; 0 \le j \le n-1,\ 0 \le t \le T$

$$\therefore I(jT+t) = \frac{\lambda}{\beta+1}\left(\frac{T^{\beta+1}}{t^{\beta}} - t\right) - \mu p \sum_{k=0}^{\infty} \frac{r^k}{k!\,(\beta+k+1)}\left(\frac{T^{k+\beta+1}}{t^{\beta}} - t^{k+1}\right) \tag{3}$$

The total relevant costs in [0, L_H] consists of the following elements

1. COST OF PLACING ORDERS (CPO)

CPO = A (0) + A (T) + A (2T) + …….. A{(n-1)T}

$$= A\left(\frac{e^{rL_H} - 1}{e^{rT} - 1}\right) \quad \text{where } L_H = nT \tag{4}$$

2. COST OF DETERIORATED UNITS (COD)

$$\text{COD} = \{C(0) + C(T) + C(2T) + ... + C[(n-1)T]\}.I(jT+t)$$

$$= C\left\{\frac{\lambda}{\beta+1}\left(\frac{T^{\beta+1}}{t^{\beta}} - t\right) - \mu p \sum_{k=0}^{\infty} \frac{r^k}{k!\,(\beta+k+1)}\left(\frac{T^{k+\beta+1}}{t^{\beta}} - t^{k+1}\right)\right\}\left(\frac{e^{rL_H} - 1}{e^{rT} - 1}\right) \tag{5}$$

3. COST OF CARRYING INVENTORY (CCI)

$$\text{CCI} = h\sum_{j=0}^{n-1} C(jT)\int_0^T I(jT+t)\,dt$$

$$= hC\left\{\frac{\lambda}{2(1-\beta)}T^2 - \mu p \sum_{k=0}^{\infty} \frac{r^k}{(1-\beta)(k+2)\,k!}T^{k+2}\right\}\left(\frac{e^{rL_H} - 1}{e^{rT} - 1}\right) \tag{6}$$

There are two possibilities based on the customer’s two choices.

Case I : $T \geq M_{DP}$

Since, optimal cycle length T is greater than the permissible delay time M_{DP}, the interest charged during the period $[M_{DP}, T]$ is given by

$$IC_1 = I_C \sum_{j=0}^{n-1} C(jT) \int_{M_{DP}}^{T} I(jT+t)\,dt$$

$$IC_1 = I_C C \left[\begin{array}{l} \dfrac{\lambda}{2(1-\beta)} T^2 + \dfrac{\lambda}{2(\beta+1)} M_{DP}^2 - \dfrac{\lambda}{(1-\beta^2)} T^{\beta+1} M_{DP}^{1-\beta} \\ -\mu p \displaystyle\sum_{k=0}^{\infty} \left\{ \dfrac{r^k}{(k+2)(1-\beta)k!} T^{k+2} + \dfrac{r^k}{(k+\beta+1)k!} \left(\dfrac{M_{DP}{}^{k+2}}{k+2} - \dfrac{T^{k+\beta+1} M_{DP}{}^{1-\beta}}{1-\beta} \right) \right\} \end{array} \right] \left(\dfrac{e^{rL_H}-1}{e^{rT}-1} \right) \tag{7}$$

Now Interest earned in $[0, L_H]$ is

$$IE_1 = I_e \sum_{j=0}^{n-1} p(jT) \int_0^{M_{DP}} \left(\lambda - \mu p e^{rt} \right) t\,dt$$

$$= I_e\, p \left[\lambda \frac{M_{DP}{}^2}{2} - \frac{\mu p}{r^2} \left\{ (rM_{DP}-1) e^{rM_{DP}} + 1 \right\} \right] \left(\frac{e^{rL_H}-1}{e^{rT}-1} \right) \tag{8}$$

Now, the total average cost per unit over $[0, L_H]$ is given by

$$TRC_1(p,T) = \frac{1}{T}\left(CPO + COD + CCI + IC_1 - IE_1 \right)$$

So using the equations (4), (5), (6), (7) and (8) in above equation, we have

$$TRC_1(p,T) = \frac{1}{T} \left[\begin{array}{l} A + C \left\{ \mu p \displaystyle\sum_{k=0}^{\infty} \dfrac{r^k}{(\beta+k+1)k!} \left(t^{k+1} - \dfrac{T^{k+\beta+1}}{t^\beta} \right) + \dfrac{\lambda}{\beta+1} \left(\dfrac{T^{\beta+1}}{t^\beta} - t \right) \right\} \\ + hC \left\{ \dfrac{\lambda}{2(1-\beta)} T^2 - \mu p \displaystyle\sum_{k=0}^{\infty} \dfrac{r^k}{(1-\beta)(k+2)k!} T^{k+2} \right\} \\ + I_C C \left[\begin{array}{l} \dfrac{\lambda}{2(1-\beta)} T^2 + \dfrac{\lambda}{2(\beta+1)} M_{DP}^2 - \dfrac{\lambda}{(1-\beta^2)} T^{\beta+1} M_{DP}^{1-\beta} \\ -\mu p \displaystyle\sum_{k=0}^{\infty} \left\{ \dfrac{r^k}{(k+2)(1-\beta)k!} T^{k+2} + \dfrac{r^k}{(k+\beta+1)k!} \left(\dfrac{M_{DP}{}^{k+2}}{k+2} - \dfrac{T^{k+\beta+1} M_{DP}{}^{1-\beta}}{1-\beta} \right) \right\} \end{array} \right] \\ - pI_e \left[\lambda \dfrac{M_{DP}{}^2}{2} - \dfrac{\mu p}{r^2} \left\{ (rM_{DP}-1) e^{rM_{DP}} + 1 \right\} \right] \end{array} \right] \left(\frac{e^{rL_H}-1}{e^{rT}-1} \right) \tag{9}$$

and net profit is given

$$NRP_1(p,T) = GR - TRC_1(p,T) = (pe^{rT} - Ce^{rT})(\lambda - \mu p e^{rT}) - TRC_1(p,T) \tag{10}$$

Case II : $T < M_{DP}$

In this case there is no interest charged.

The interest earned is

$$IE_2 = D[p(t)]I_e \sum_{k=0}^{n-1} p(jT)\left[\int_0^T t\,dt + T\left(M_{DP} - T\right)\right]$$
$$= pI_e\left[\left(\lambda - \mu p e^{rt}\right)\left(T\,M_{DP} - \frac{T^2}{2}\right)\right]\frac{(e^{rL_H}-1)}{(e^{rT}-1)}$$

Thus the total Relevant cost TRC_2 (p, T) per unit time is

$$TRC_2(p,t) = \frac{1}{T}\left(CPO + COD + CCI - IE_2\right)$$

$$= \frac{1}{T}\left[\begin{array}{l} A + C\left\{\mu p \sum_{k=0}^{\infty}\frac{r^k}{(\beta+k+1)k!}\left(t^{k+1} - \frac{T^{k+\beta+1}}{t^\beta}\right) + \frac{\lambda}{\beta+1}\left(\frac{T^{\beta+1}}{t^\beta} - t\right)\right\} \\ + hC\left\{\frac{\lambda}{2(1-\beta)}T^2 - \mu p \sum_{k=0}^{\infty}\frac{r^k}{(1-\beta)(k+2)k!}T^{k+2}\right\} \\ - pI_e\left\{\left(\lambda - \mu p e^{rt}\right)\left(T\,M_{DP} - \frac{T^2}{2}\right)\right\}\end{array}\right]\left(\frac{e^{rL_H}-1}{e^{rT}-1}\right) \quad (11)$$

and the net profit is given by

$$NRP_2\,(p, T) = (pe^{rT} - ce^{rT})\,(\lambda - \mu p e^{rT}) - TRC_2\,(p,T) \quad (12)$$

THEORETICAL RESULT OF THE MODEL

To obtain the optimum values the first order condition for NRP_1 (p,T) to be maximum is given by

$$\frac{\partial\, NRP_1(p,T)}{\partial p} = 0 \quad \text{and} \quad \frac{\partial\, NRP_1(p,T)}{\partial T} = 0$$

Now

$$\frac{\partial\, NRP_1(p_, T)}{\partial T} = 0 \; gives$$

$$\frac{\partial TRC_1}{\partial T} = \lambda rp\, e^{rT} - \lambda Cr e^{rT} - 2r\mu p^2 e^{2rT} + 2r\mu\, pCe^{2rT}$$

$$\frac{\partial\, NRC_1(p_1 T)}{\partial p} = 0 \; gives$$

$$\frac{\partial\, TRC_1}{\partial p} = \lambda e^{rT} - 2\mu\, p\, e^{2rT} + \mu C\, e^{2rT}$$

Which on solving gives the solution say T_1.

It maximizes the net profit because $\frac{\partial^2 NRP_1(p_, T)}{\partial T^2} < 0$, Similarly

$\frac{\partial\, NRP_2(p_, T)}{\partial p} = 0$ can be solved for T say this solution to be T_2

The cycle time $T = T_2$ obtained by solving equation NRP_2 (p,T)

Also it maximizes the net profit because

$$\frac{\partial^2 NRp_2(p_, T)}{\partial T^2} < 0$$

Hence,

The Optimum Cycle time

$$T = \begin{cases} T_1 & \text{if } M_{DP} \le T \\ T_2 & \text{if } M_{DP} > T \end{cases}$$

So,

$$Q = \begin{cases} Q(T_1) & \text{if } M_{DP} \le T \\ Q(T_2) & \text{if } M_{DP} > T \end{cases}$$

And the net profit of the inventory system is given by

$$NRP = \begin{cases} NRp_1 & \text{if } M_{DP} \le T \\ NRp_2 & \text{if } M_{DP} > T \end{cases}$$

CONCLUSIONS

The paper studies an inventory model for deteriorating items. The demand for the item is dependent on the selling price. The replenishment source allows the inventory manager a certain fixed period of time to settle his accounts. No interest is charged during this period, but beyond it the manager has to pay an interest. The effect of inflation on various costs is also taken into consideration. The optimum ordering policy is determined by maximizing the profit over the planning horizon.

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MODEL ZARZĄDZANIA ZAPASEM WEIBULLA DLA POPYTU ZALEŻNEGO OD CEN ORAZ OPÓŹNIEŃ W PŁATNOŚCI W WARUNKACH INFLACJI

STRESZCZENIE. Praca przedstawia opracowanie modelu zarządzania zapasem dla towarów, które ulegają zużyciu zgodnie z modelem Weibulla, i których popyt zależny jest od ceny sprzedaży. Braki nie są dozwolone. Czynnik inflacji został uwzględniony dla określonego horyzontu czasowego. Została przeprowadzona krótka teoretyczna analiza kosztów.

Słowa kluczowe: zapasy, inflacja, dystrybucja Weibulla, popyt zależny od ceny zbytu, opóźnienie w płatności

MODELL DES MANAGEMENTS FÜR DIE WEIBULL-VORRATSHALTUNG FÜR DIE DURCH PREISE BEDINGTE NACHFRAGE UND ZAHLUNGSVERZÖGERUNG BEI INFLATION

ZUSAMMENFASSUNG. Die Arbeit projiziert die Erstellung eines Modells für das Management von Vorräten an den Waren, die gemäß dem Weibull-Modell verbraucht werden und deren Nachfrage durch ihren Preis bedingt ist. Fehlmengen sind nicht zulässig. Der Inflations-Faktor wurde für einen bestimmten Zeithorizont berücksichtigt. Es wurde auch eine kurze theoretische Kostenanalyse durchgeführt.

Codewörter: Vorräte, Inflation, Weibull-Distribution, die durch den Verkaufspreis bedingte Nachfrage, Zahlungsverzug

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