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APPLYING ROBUST RANKING METHOD IN TWO PHASE FUZZY OPTIMIZATION LINEAR PROGRAMMING PROBLEMS (FOLPP)

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ABSTRACT. Background: This paper explores the solutions to the fuzzy optimization linear program problems (FOLPP) where some parameters are fuzzy numbers. In practice, there are many problems in which all decision parameters are fuzzy numbers, and such problems are usually solved by either probabilistic programming or multi-objective programming methods.

Methods: In this paper, using the concept of comparison of fuzzy numbers, a very effective method is introduced for solving these problems. This paper extends linear programming based problem in fuzzy environment. With the problem assumptions, the optimal solution can still be theoretically solved using the two phase simplex based method in fuzzy environment. To handle the fuzzy decision variables can be initially generated and then solved and improved sequentially using the fuzzy decision approach by introducing robust ranking technique.

Results and conclusions: The model is illustrated with an application and a post optimal analysis approach is obtained. The proposed procedure was programmed with MATLAB (R2009a) version software for plotting the four dimensional slice diagram to the application. Finally, numerical example is presented to illustrate the effectiveness of the theoretical results, and to gain additional managerial insights.

Key words: Decision making, Fuzzy Optimization Linear programming (FLOP), Two phase method, Post optimal analysis.

INTRODUCTION

Over the last few years, more and more manufacturers had applied the optimization technique most frequently in linear programming to solve real-world problems and there it is important to introduce new tools in the approach that allow the model to fit into the real world as much as possible. Any linear programming model representing real-world situations involves a lot of parameters whose values are assigned by experts' opinion, and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not precisely know the value of those parameters. If exact values are suggested these are only statistical inference from past data and their stability is doubtful, so the parameters of the problem are usually defined by the decision maker in an uncertain space. Therefore, it is useful to consider the knowledge of experts' opinion about the parameters as fuzzy data. Fuzzy data helps the decision maker to take the decision in open ended space; since the market is volatile it is very difficult to take the optimum decision of the decision parameters. In the mean time fuzzy related data helps for obtaining the optimal solution and then the post optimal solution gives the managerial implications for the given problem.

Two significant questions may be found in these kinds of problems: how to handle the

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relationship between the fuzzy parameters, and how to find the optimal values for the fuzzy multi-objective function. The answer is related to the problem of ranking fuzzy numbers.

In fuzzy decision making problems, the concept of optimizing the decision was introduced by Bellman and Zadeh [1970]. Zimmerman [1978] presented a fuzzy approach to multi-objective linear programming problems in his classical paper. Lai and Hwang [1992] considered the situations where all parameters are in fuzzy number. Lai and Huang [1992] assume that the parameters have a triangular possibility distribution. Gani et al. [2009] introduce fuzzy linear programming problem based on L-R fuzzy number. Jimenez et al. [2005] propose a method for solving linear programming problems where all coefficients are, in general, fuzzy numbers and using linear ranking technique. Bazaar et al. [1990] and Nasseri et al. [2005] define linear programming problems with fuzzy numbers and simplex method is used for finding the optimal solution of the fuzzy problem. Rangarajan and Solairaju [2010] compute improved fuzzy optimal Hungarian assignment problems with fuzzy numbers by applying robust ranking techniques to transform the fuzzy assignment problem to a crisp one. Pattnaik [2012] presented a fuzzy approach to several linear and nonlinear inventory models. Swarup et al. [2006] explain the method to obtain sensitivity analysis or post optimality analysis of the different parameters in the linear programming problems.

In fact, in order to make linear programming more effective, the uncertainties that happen in the real world cannot be neglected. Those uncertainties are usually associated with per unit cost of the product, product supply, customer demand and so on. Looking at the property of representing the preference relationship in fuzzy terms, ranking methods can be classified into two approaches. One of them associates, by means of different functions, each fuzzy number to a single of the real line and then a total crisp order relationship between fuzzy numbers is established. The other approach ranks fuzzy numbers by means of a fuzzy relationship. It allows decision maker to present his preference in a gradual way, which in a linear programming problem allows it to be handled with different degrees of satisfaction of constraints. This paper considers fuzzy multi-objective linear programming problems whose parameters are fuzzy numbers but whose decision variables are crisp. The aim of this paper is to introduce robust ranking technique for defuzzifying the fuzzy parameters and then sensitivity analysis for the requirement vector in the constraint function is also performed that permits the interactive participation of decision maker in all steps of decision process, expressing his opinions in linguistic terms for managerial insights. The major techniques used in the above research articles are summarized in Table 1. In the present scenario the market is totally uncertain so it is very difficult to take the decision for the exact cost of the product. For avoiding this type of unusual difficulties the optimum solution can be obtained in fuzzy decision space.

Table 1. Major Characteristics of Fuzzy Optimization Linear Programming (FOLP) Models on Selected Researches
Tabela 1. Cechy charakterystyczne modeli optymalizacji liniowej w wybranych opracowaniach

Author(s) and Published Year	Structure of the Model	Fuzzy Number	Objective Model	Model Type	Ranking Function	Sensitivity Study
Zimmermann (1978) [16]	Fuzzy	Triangular	Single	Cost	Linear	No
Maleki et al. (2000) [9]	Fuzzy	Trapezoidal	Multi	Profit	Linear	No
Jimenez et al. (2005) [5]	Fuzzy	Triangular	Multi	Cost	Linear	No
Nasseri et al. (2005) [10]	Fuzzy	Trapezoidal	Multi	Profit	Linear	No
Buckly et al. (2000) [3]	Fuzzy	Triangular	Multi	Profit	Linear	No
Present Paper (2014)	Fuzzy	Trapezoidal	Multi	Profit	Robust	Yes

The remainder of this paper is organized as follows. In Section 2, it is introduced fuzzy numbers and some of the results of applying arithmetic on them. Assumptions, notations and definitions are provided for the development of the model. In Section 3, it is introduced Robust' ranking technique for solving fuzzy number linear programming problems. In Section 4, a linear programming problem with fuzzy variables is proposed and in Section 5 it is explained a fuzzy version of the simplex algorithm of two phase method for solving this problem. An application is presented to illustrate the development of the model in Section 6. The sensitivity analysis is carried out in Section 7 to observe the changes in the optimal solution. Finally Section 8 deals with the summary and the concluding remarks.

PRELIMINARIES

It is reviewed that the fundamental notation of fuzzy set theory initiated by Bellman and Zadeh [2]. Below it is given definitions taken from Zimmerman [16].

Definition 2.1 *Fuzzy sets*

If X is a collection of objects denoted generally by x, then a fuzzy set $\tilde{A}$ in X is defined as a set of ordered pairs $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))/x\epsilon X\}$, where $\mu_{\tilde{A}}(x)$ is called the membership function for the fuzzy set$\tilde{A}$. The membership function maps each element of X to a membership value between 0 and 1.

Definition 2.2 *Support of a fuzzy set*

The support of a fuzzy set $\tilde{A}$ is the set of all points x in X such that $\mu_{\tilde{A}}(x) > 0$. That is $support\ (\tilde{A}) = \{x/\mu_{\tilde{A}}(x) > 0\}$.

Definition 2.3 $\alpha -$ *level of fuzzy set*

The $\alpha -$ cut (or) $\alpha -$ level set of a fuzzy set $\tilde{A}$ is a set consisting of those elements of the universe X whose membership values exceed the threshold level α. That is $\tilde{A}_\alpha = \{x/\mu_{\tilde{A}}(x) \geq \alpha\}$.

Definition 2.4 *Convex fuzzy set*

A fuzzy set $\tilde{A}$ is convex if, $\mu_{\tilde{A}}\big(\lambda x_1 + (1-\lambda x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big), x_1, x_2 \epsilon X$ and $\lambda\epsilon[0,1]$. Alternatively, a fuzzy set is convex, if all $\alpha -$level sets are convex.

Definition 2.5 *Convex normalized fuzzy set*

A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that it exists at least one $x_0\epsilon R$ with $\mu_{\tilde{A}}(x_0) = 1$ and $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.6 *Trapezoidal fuzzy numbers*

Among the various fuzzy numbers, triangular and trapezoidal fuzzy numbers are of the most important. Note that, in this study only trapezoidal fuzzy numbers are considered. A fuzzy number is a trapezoidal fuzzy number if the membership function of its be in the following function of it being in the following form:

Any trapezoidal fuzzy number by $\tilde{a} = (a^L, a^U, \alpha, \beta)$, where the support of $\tilde{a}$ is $(a^L - \alpha, a^U + \beta)$ and the modal set of $\tilde{a}$ is $[a^L, a^U]$. Let *F(R)* is the set of trapezoidal fuzzy numbers.

Definition 2.7 *Arithmetic on fuzzy numbers*

Let $\tilde{a} = (a^L, a^U, \alpha, \beta)$ and $\tilde{b} = (b^L, b^U, \gamma, \theta)$ be two trapezoidal fuzzy numbers and $x\epsilon R$. Then, the results of applying fuzzy arithmetic on the trapezoidal fuzzy numbers as shown in the following:
Image of $\tilde{a}$: $-\tilde{a} = (-a^U, -a^L, \beta, \alpha)$
Addition: $\tilde{a} + \tilde{b} = (a^L + b^L, a^U + b^U, \alpha + \gamma, \beta + \theta)$
Scalar Multiplication: $x > 0,\ x\tilde{a} = (xa^L, xa^U, x\alpha, x\beta)$ and $x < 0,\ x\tilde{a} = (xa^U, xa^L, -x\alpha, -x\beta)$

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Scalar Multiplication: $x > 0,\ x\widetilde{a} = (xa^L, xa^U, x\alpha, x\beta)$ and $x < 0,\ x\widetilde{a} = (xa^U, xa^L, -x\alpha, -x\beta)$

RANKING FUNCTION

A convenient method for comparing of the fuzzy numbers is by use of ranking functions. A ranking function is a map from *F(R)* into the real line. The orders on *F(R)* are:

$\tilde{a} \geq \tilde{b}$ if and only if $\Re(\tilde{a}) \geq \Re(\tilde{b})$

$\tilde{a} > \tilde{b}$ if and only if $\Re(\tilde{a}) > \Re(\tilde{b})$

$\tilde{a} = \tilde{b}$ if and only if $\Re(\tilde{a}) = \Re(\tilde{b})$

Where, $\tilde{a}$ *and* $\tilde{b}$ are in *F (R).* It is obvious that $\tilde{a} \leq \tilde{b}$ if and only if $\tilde{b} \geq \tilde{a}$. Since there are many ranking function for comparing fuzzy numbers but robust ranking function is applied. Robust's ranking technique satisfies compensation, linearity and additive properties and provides results which are consistent with human intuition. Give a convex fuzzy number $\tilde{a}$, the Robust's Ranking index is defined by

$$\Re(\tilde{a}) = \int_0^1 0.5(a_\alpha^L, a_\alpha^U)d\alpha,$$

where (a^L, a^U) is the α- level cut of the fuzzy number $\tilde{a}$.

In this paper this method for ranking the objective values. The robust ranking index $\Re(\tilde{a})$ gives the representative value of the fuzzy number $\tilde{a}$. It satisfies the linearity and additive property.

FUZZY OPTIMIZATION LINEAR PROGRAMMING PROBLEMS (FOLPP)

However, when formulating a mathematical programming problem which closely describes and represents a real-world decision situation, various factors of the real world system should be reflected in the description of objective functions and constraints involve many parameters whose possible values may assigned by experts. In the conventional approaches, such parameters are required to be fixed at some values in an experimental and subjective manner through the experts' understanding of the nature of the parameters in the problem-formulation process.

It must be observed that, in most real-world situations, the possible values of these parameters are often only imprecisely known to the experts. With this observation in mind, it would be certainly more appropriate to interpret the experts' understanding of the parameters as fuzzy numerical data which can be represented by means of fuzzy sets of the real line known as fuzzy numbers.

Definition 4.1 *Linear programming*

A linear programming (LP) problem is defined as:
$Max\ z = cx$
$s.t.\ Ax = b$
$x \geq 0$
Where, $c = (c_{1,} c_2, \ldots \ldots . . c_n)$, $b = (b_1, b_2, \ldots \ldots b_m)^T$, and $A = [a_{ij}]_{m \times n}$.

In the above problem, all of the parameters are crisp. Now, if some of the parameters be fuzzy numbers then fuzzy linear programming is obtained which is defined in the next section.

Definition 4.2 *Fuzzy linear programming*

Suppose that in the linear programming problem some parameters be fuzzy numbers. Hence, it is possible that some coefficients of the problem in the objective function, technical coefficients the right hand side coefficients or decision making variables be fuzzy number Maleki [2002], Maleki et al. [2000], Rommelfanger et al. [1989] and Verdegay [1984]. Here, the linear programming problems with fuzzy numbers in the objective function.

Definition 4.3 *Fuzzy number linear programming*

A fuzzy number linear programming (FLP) problem is defined as follows:

$Max\ \tilde{z} = \tilde{c}x$

$s.t.\ Ax = b$

$x \geq 0$

where, $b\epsilon R^m$, $x\epsilon R^n$, $A\epsilon R^{m\times n}$, $\tilde{c}^T\epsilon((F(R))^n$, and $\Re$ is a Robust ranking function.

Definition 4.4 *Fuzzy feasible solution*

The vector $x\epsilon R^n$ is a feasible solution to FLP if and only if x satisfies the constraints of the problem.

Definition 4.5 *Fuzzy optimal solution*

A feasible solution x^* is an optimal solution for FLP, if for all feasible solution x for FLP, then $\tilde{c}x^* \geq \tilde{c}x$.

Definition 4.6 *Fuzzy basic feasible solution*

The basic feasible solution for FLP problems is defined as: Consider the system $Ax = b$ and $x \geq 0$, where A is an $m \times n$ matrix and b is an m vector. Now, suppose that $rank(A, b) = rank(A) = m.$ Partition after possibly rearranging the columns of A as $[B, N]$ where $B,\ m \times m,$ is nonsingular. It is obvious that $rank(B) = m$. The point $x = (x_B^T, x_N^T)^T$ where, $x_B = B^{-1}b$, $x_N = 0$ is called a basic solution of the system. If $x_B \geq 0$, then x is called a basic feasible solution (BFS) of the system. Here *B* is called the basic matrix and *N* is called the non basic matrix.

A FUZZY VERSION OF SIMPLEX ALGORITHM FOR TWO PHASE METHOD

In the first phase of this method, the sum of the artificial variables is minimized subject to the given constraints (known as auxiliary fuzzy linear programming problem, FLP) to get the fuzzy basic feasible solution to the original FLP. Second phase then optimizes the original objective function starting with the fuzzy basic feasible solution obtained at the end of phase I. The iterative procedure of the fuzzy algorithm may be summarize as follows.

Step-1 Express the problem in the standard form and check whether there exists a starting fuzzy basic feasible solution.

- If there is a ready starting fuzzy basic feasible solution, go to Phase II.
- If there does not exist a ready starting fuzzy basic feasible solution, go on to the next step.

Phase-I

Step-2 Add the artificial variable to the left side of each of the equations corresponding to constraints of the type $\geq$ or $=$. However addition of these artificial variables causes violation of the corresponding constraints. Therefore, we would like to get rid of these variables and would not allow them to appear in the final solution. This is achieved by assigning (-1) in the objective function.

Step-3 Solve the modified FLP by simplex method, until any one of the three cases may arise.

1. No artificial variable appears in the basis and the optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.
2. At least one artificial variable is present in the basis with zero value. In such a case the current optimum basic feasible solution is degenerate.
3. At least one artificial variable is present in the basis with a positive value. In such a case, the given FLP does not possess a fuzzy optimal basic feasible solution. The given problem is said to have a fuzzy pseudo-optimum basic feasible solution.

Phase-II

Step-4 Consider the optimum basic feasible solution of *Phase-I* as a starting basic feasible solution for the original FLPP. Assign actual coefficients to the variables in the objective function and a value zero to the artificial variables that appear at zero value in the final simplex table of *Phase-I*.

APPLICATION

In this section the application of fuzzy version simplex algorithm solution to FLP has been presented. This application is the diet problem for pigs.

Diet Problem

A farmer has three products $P_1, P_2\ and\ P_3$ which he plans to mix together to feed his pigs. He knows the pigs require a certain amount of food $F_1\ and\ F_2$ available, per gram of $P_1, P_2\ and\ P_3$. The approximate time, in hours, each P_i spends in each F_j is given in Table 2.

Also, each pig should have approximately at least 54 units of F_1 *and* approximately at least 60 units of F_2, per day. The costs of $P_1, P_2\ and\ P_3$ vary slightly from day to day but the average costs are: (1) 8¢ per gram of P_1; (2) P_2 is 9¢ per gram; and (3) 10¢ per gram for P_3.

The farmer wants to know how many grams of $P_1, P_2\ and\ P_3$he should mix together each day, so his pigs will get the approximate minimum, to minimize his costs.

Table 2. Times of product P_i is in department D_j
Tabela 2. Okres pobytu produktu P_i w dziale D_j

Product	D1	D2
P1	2.5	5
P2	4.5	3
P3	5	10

Since all selling price numbers given are uncertain, the FLP model is formulated. The Trapezoidal fuzzy number for each value given is obtained. So, the FLP is given by:

Min $\tilde{z} = (6,8,9,10)x_1 + (6,9,10,11)x_2 + (9,10,12,13)x_3$

Such that
$2.5x_1 + 4.5x_2 + 5x_3 \geq 54$
$5x_1 + 3x_2 + 10x_3 \geq 60$
$\forall\ x_1, x_2, x_3\ \geq 0$

Solution
$\min(-\tilde{z}) = max\tilde{z}'$
Max $\tilde{z}' = -(6,8,9,10)x_1 - (6,9,10,11)x_2 - (9,10,12,13)x_3$
$2.5x_1 + 4.5x_2 + 5x_3 - x_4 + A_1 = 54$
$5x_1 + 3x_2 + 10x_3 - x_5 + A_2 = 60$

Table 3. Optimal Values for the Proposed Fuzzy Linear Programming Model (Phase-I)
Tabela 3. Optymalne wartości proponowanego modelu programowania liniowego (Faza I)

$\tilde{C}_j$			(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-1	**-1**	**Min ratio**
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	$\tilde{y}_7$	
-1	$\tilde{y}_6$	54	2.5	4.5	5	-1	0	1	0	10.8
-1	$\tilde{y}_7$	60	5	3	[10]	0	-1	0	1	6→
$\tilde{z}_j$		-114	-7.5	-7.5	-15↑	1	1	(0,0,0,0)	(0,0,0,0)	$\tilde{\Delta}_j$
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	$\tilde{y}_7$	**Min ratio**
-1	$\tilde{y}_6$	24	0	[3]	0	-1	5/10	1	-5/10	8→
(0,0,0,0)	$\tilde{y}_3$	6	5/10	3/10	1	0	-1/10	0	1/10	20
$\tilde{z}_j$		-24	(0,0,0,0)	-3↑	(0,0,0,0)	1	-5/10	(0,0,0,0,)	-1/2	$\tilde{\Delta}_j$
$\tilde{C}_B$	$\tilde{Y}_B$	$\tilde{X}_B$	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$	$\tilde{y}_6$	$\tilde{y}_7$	**Min ratio**
(0,0,0,0)	$\tilde{y}_2$	8	0	1	0	$\frac{-1}{3}$	$\frac{5}{30}$	1/3	-5/30	-
(0,0,0,0)	$\tilde{y}_3$	36/10	5/10	0	1	$\frac{1}{10}$	$\frac{-3}{20}$	-1/10	3/20	-
$\tilde{z}_j$		(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	1	1	$\tilde{\Delta}_j \geq 0$

Table 4. Optimal Solution of the LPP (Phase-II)

Tabela 4. Optymalne rozwiązanie LPP (Faza II)

$\widetilde{C}_j$			(-10,-9,-8,-6)	(-11,-10,-9,-6)	(-13,-12,-10,-9)	(0,0,0,0)	(0,0,0,0)	**Min ratio**
$\widetilde{C}_B$	$\widetilde{Y}_B$	$\widetilde{X}_B$	$\widetilde{y}_1$	$\widetilde{y}_2$	$\widetilde{y}_3$	$\widetilde{y}_4$	$\widetilde{y}_5$	
(-11,-10,-9,-6)	$\tilde{y}_2$	8	0	1	0	$\frac{-1}{3}$	$\frac{5}{30}$	-
(-13,-12,-10,-9)	$\tilde{y}_3$	36/10	5/10	0	1	$\frac{1}{10}$	$\frac{-3}{20}$	-
$\tilde{z}_j$		8(-11,-10,-9,-6) +36/10 (-13,-12.-10, -9)≈ $-312/10$ ≈-31.2	5/10(-13,-12,-10,-9)+(6,8,9,10) ≈ 2.75	(0,0,0,0)	(0,0,0,0)	-1/3(-11,-10,-9,-6)+1/10(-13,-12,-10,-9) ≈ 27.17	5/30(-11,-10,-9,-6)-3/20(-13,-12,-10,-9) ≈ 0.15	$\tilde{\Delta}_j \geq 0$

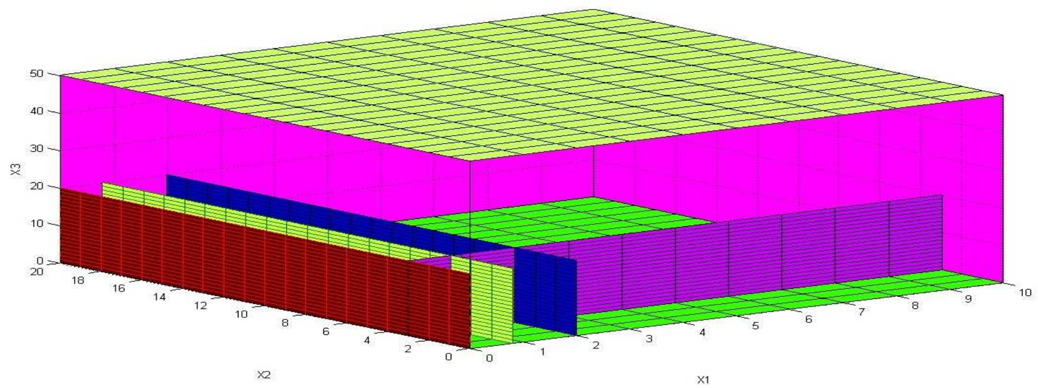

Fig. 1. Dimensional Slice and Mesh plot of Fuzzy Total Cost $z(x_{1,}x_2,x_3)$, $x_1, x_2 and\ x_3$.
Rys. 1. Warstwy przestrzenne oraz siatka kosztu całkowitego $z(x_{1,}x_2,x_3)$, $x_1, x_2\ i\ x_3$.

Table 3 and Table 4 derive the fuzzy optimal solution of the given problem using simplex method of two phase of FOLP. In the solution procedure the robust ranking method is used for defuzzifying the fuzzy numbers but other procedures are remaining same as for the crisp method for obtaining the optimal solution by Two Phase method of LP. In crisp LP model the optimal total cost is Rs. 108 but in FOLP model the optimal total cost is Rs. 31.2 with identical solution of the decision variables both for crisp LP and fuzzy OLP. The optimal total cost in fuzzy space is less than that of the crisp LP model so it is advisable and acceptable to apply fuzzy logic for obtaining the optimum decision in an uncertain market.

From Table 3 and Table 4 it is found that the fuzzy optimal solutions are $\tilde{x}_1 = 0, \tilde{x}_2 = 8, \tilde{x}_3 = 3.6\ and\ \tilde{z} = 31.2$. Fig. 1 shows the four dimensional slice and mesh plot of fuzzy total profit $\tilde{z}(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3), \tilde{x}_1,\ \tilde{x}_2\ and\ \tilde{x}_3$.

SENSITIVITY ANALYSIS

Discrete variation in b

The investigations that deal with changes in the optimum solutions due to discrete variations in the parameter b_i is called sensitivity analysis. Consider the fuzzy linear programming problem

Let the component b_k of the vector **b** be changed to $b_k + \Delta b_k$, hence range of Δb_k, so that the optimum solution $\widetilde{\boldsymbol{X}}_B^*$ also remains feasible is $\underset{b_{ik}>0}{\text{Max}} \left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\} \leq \Delta b_k \leq \underset{b_{jk}<0}{\text{Min}} \left\{\frac{-\tilde{x}_{Bj}}{b_{jk}}\right\}$

From the Table 3 we observe $\tilde{\boldsymbol{x}}_B = [\,8{,}3.6\,]$ and $\boldsymbol{B}^{-1} = [\tilde{y}_6, \tilde{y}_7] = \begin{bmatrix} \frac{1}{3} & \frac{-5}{30} \\ \frac{-1}{10} & \frac{3}{20} \end{bmatrix}$. The individual effects of changes in $b_1 and\ b_2$ where $\boldsymbol{b} = [\,b_1\ b_2\,]$ such that the optimality of the basic feasible solution is not violated, are given by $\underset{b_{ik}>0}{\text{Max}}\left\{\frac{-\tilde{x}_{Bi}}{b_{ik}}\right\} \le \Delta b_k \le \underset{b_{jk}<0}{\text{Min}}\left\{\frac{-\tilde{x}_{Bj}}{b_{jk}}\right\}$. $\underset{b_{11}>0}{\text{Max}}\left\{\frac{-8}{\frac{1}{3}}\right\} \le \Delta b_1 \le \underset{b_{21}<0}{\text{Min}}\left\{\frac{-3.6}{\frac{-1}{10}}\right\} = -24 \le \Delta b_1 \le 36$ and $\underset{b_{22}>0}{\text{Max}}\left\{\frac{-3.6}{\frac{3}{20}}\right\} \le \Delta b_2 \le \underset{b_{12}<0}{\text{Min}}\left\{\frac{-8}{\frac{-5}{30}}\right\} = -24 \le \Delta b_2 \le 48$.

Hence, $-24 \le \Delta b_1 \le 36$ and $-24 \le \Delta b_2 \le 48$. Now, since $b_1 = 6\ and\ b_2 = 10,$ the required range of variation is $30 \le b_1 \le 90$ and $36 \le b_2 \le 108$. It implies that in fuzzy decision space the requirement value has the limit 30 units to 90 units for the first constraint and has the limit 36 units to 108 units for the second constraint for obtaining the optimal solution of the given LP model. It gives the managerial implications for taking the optimum decision with the bounded values of the requirement vector of the FLP model.

CONCLUSIONS

The main contribution of this paper is to formulate a linear programming problem with fuzzy parameters and defuzzifying the fuzzy parameters by using robust ranking technique. Based on the optimal solution it allows taking a decision interactively with the decision maker in fuzzy decision space. The decision maker also has additional information about the availability of the violation of requirement vector in the constraints, and about the compatibility of the cost of the solution with his wishes for the values of the objective function which extend the classical LP models with no sensitivity analysis in the past. By using fuzzy theory and robust ranking approach, individual firm's cost strategies are examined and the optimal solutions with sensitivity analysis for obtaining the managerial implications in fuzzy environment are derived. These analysis of the results are established which present a number of managerial insights into the economic behavior of the firms, and can serve as the basis for empirical study in the future study.

Thus, there are possible extensions to improve this present model. The decision maker can intervene in all the steps of the decision process which makes this approach very useful to be applied in a lot of real-world problems where the information is uncertain with nonrandom, like general management, project management, marketing and production management.

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ZASTOSOWANIE ODPORNOŚCIOWEJ METODY RANKINGOWEJ W PROBLEMACH 2-FAZOWYCH ROZMYTEJ OPTYMALIZACJI LINIOWEJ (FOLPP)

STRESZCZENIE. **Wstęp:** Praca analizuje rozwiązanie problemów rozmytej optymalizacji liniowej (FOLPP) w przypadku, gdy niektóre parametry to liczby rozmyte. W praktyce, istnieje wiele problemów, w których wszystkie parametry decyzyjne są liczbami rozmytymi. Takie problemy są rozwiązywane zazwyczaj przy pomocy programów probalistycznych lub wieloobiektowych metod programistycznych.
Metody: W pracy, poprzez zastosowanie koncepcji porównania liczb rozmytych, przedstawiono efektywną metodę rozwiązywania omawianych problemów. Problem programowania linowego został oparty na środowisku rozmytym. Przy przyjętych założeniach, optymalne rozwiązanie może być teoretycznie osiągnięte poprzez zastosowanie 2-fazowej metody simplex w środowisku rozmytym. W celu podjęcia decyzji rozmytej, zmienne mogą być wpierw wygenerowane, następnie rozwiązane i poprawione sekwencyjnie poprzez zastosowanie podejścia decyzji rozmytej i techniki odpornościowej metody rankingowej.
Wyniki i wnioski: Wypracowany model został przedstawiony za pomocą aplikacji, zastosowano analizę optymalizacyjną. Proponowana procedura została zaprogramowana przy pomocy MATLAB (R2009a) w celu otrzymania 4-wymiarowego wykresu. Następnie zaprezentowano przykład liczbowy w celu przybliżenia efektywności teoretycznych rezultatów pracy oraz uzyskania dodatkowego spojrzenia na problem.

Słowa kluczowe: podejmowanie decyzji, rozmyta optymalizacja liniowa (FOLP), metoda 2-fazowa, analiza optymalizacyjna.

ANWENDUNG VON WIDERSTANDSFÄHIGER RANGREIHEN-METHODE IN DEN 2-PHASEN-FRAGESTELLUNGEN DER UNSCHARFEN LINEAREN OPTIMIERUNG (FOLPP)

ZUSAMMENFASSUNG. Einleitung: Die Arbeit setzt sich mit Lösungen der unscharfen linearen Optimierung im Falle, wenn manche Parameter unscharfe Mengen darstellen, auseinander. In Wirklichkeit gibt es viele Problemstellungen, in denen alle entscheidungstragenden Parameter unscharfe Mengen sind. Solche Problemstellungen werden gewöhnlich anhand proballistischer Programme oder mithilfe von objektorientierten Programmierungsmethoden gelöst.

Methoden: In der Arbeit stellte man eine effektive Methode für die Lösung der betreffenden Probleme durch die Anwendung eines auf den Vergleich von unscharfen Mengen hinzielenden Konzeptes dar. Die Problemstellung der linearen Programmierung stützte man auf das Fuzzy-Medium. In den angenommenen Fällen kann theoretisch eine optimale Lösung durch die Anwendung der 2-Phasen-Simplex-Methode im unscharfen Medium erzielt werden. Für das Treffen einer unscharfen Entscheidung können die Variablen erst einmal generiert, ferner gelöst und dann sequenziell verbessert werden, insbesondere durch die Inanspruchnahme der an der unscharfen Entscheidung, sowie an der widerstandsfähigen Rangreihenmethode orientierten Verfahren.

Ergebnisse und Fazit: Das ausgearbeitete Modell wurde anhand eines Anwendungskonzeptes, in dem eine Optimierungsanalyse zur Geltung kam, dargestellt. Das vorgeschlagene Anwendungsverfahren wurde anhand von MATLAB (R2009a) zwecks Erzielung eines 4D-Diagramms programmiert. Ferner präsentierte man ein zahlenmäßiges Beispiel zwecks Projizierung der Effektivität der theoretischen Ergebnisse der Arbeit, sowie zwecks einer zusätzlichen Wahrnehmung des betreffenden Problems.

Codewörter: Entscheidungstreffen, unscharfe lineare Optimierung (FOLP), 2-Phasen-Methode, Optimierungsanalyse.

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