



MODELLING OF CONTACT PROBLEMS INVOLVED IN ENSURING THE SAFETY OF RAIL TRANSPORT

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ABSTRACT. Background: Mathematical modelling aids diagnostics the track and rolling stock, as it often for technical reasons it is not possible to obtain a complete set of measurement data required to diagnose the rail and wheel deformation caused by the impact of a rail vehicle on the track. The important issue in a railway diagnostics is to study the effects of contact wheel and rail. Diagnostics investigations of track and rolling stock have a fundamental role in ensuring the safety of transport of passengers and goods.

The aim of the study presented in the paper was to develop simulation methods of mathematical modelling of the wheel-rail system useful in the diagnostics of the track and a railway vehicle.

Methods: In the paper two ways of modelling were presented and discussed. One of these ways is the method which consists in reducing the contact issue to field issue and solving the identification of the field source in 2-D system. Also presented a different method designed on the basis of the methods using one period energy concept. This method is adapted for modelling the dynamics of the contact wheel-rail for the normal force. It has been shown that the developed modelling methods to effectively support the study on the effects of mechanical and thermal of contact wheel-rail and contribute to the safety of operations.

Results and conclusions: In the case of field sources identifications two specific issues were examined: the issue of rail torsion and the identification of heat sources in the rail due to exposure the rolling contact wheel-rail. In the case of the method using one period energy concept it was demonstrated the usefulness of this method to the study of energy processes in the contact wheel-rail under the normal periodic force. The future direction of research is to establish cooperation with research teams entrusted with the diagnostic measurements of track and rolling stock.

Key words: contact wheel-rail, diagnostics of tract and rolling stock, numerical methods, one period energy.

INTRODUCTION

Diagnostics investigations of track and rolling stock have a fundamental role in ensuring the safety of transport of passengers and goods. The important issue in a railway diagnostics is to study the effects of contact wheel and rail [Strzyżakowski 2007]. Because often due to technical reasons it is not possible to obtain a complete set of measurement data required to diagnose the rail and wheel deformation caused by the impact of a rail vehicle on the track, the mathematical

modelling is needed to support experimental research. [Rydygier and Strzyżakowski 2009b]. One of the methods of mathematical modelling of the wheel-rail is the Simulation Method of identification which consists in reducing the contact issue to field issue and solving the identification of the field source in 2-D system. Another way of modelling the contact wheel-rail is the method based on the methods used in the study of electrical circuits using one period energy concept. This method was adapted for modelling the dynamics of the contact wheel-rail on the assumption that in the vertical plane of the rail the normal force has

a character of periodic signal. The aim of the study described in the paper was to develop simulation methods of mathematical modelling of the wheel-rail system useful in the diagnostics of the track and a railway vehicle.

THE SIMULATION METHOD

Simulation Method of identifying the sources of the field is a numerical method that uses computational tools taken from the combinatorial analysis [Rydygier and Strzyżakowski 2009a]. In the design of algorithms used computational tools in the form of monic power polynomials $T_n(q)$ and $P_n(q)$ and in software procedures used modified numerical triangles generating power polynomials. The investigated system was described by Poisson equation with known boundary conditions $u|_{\Gamma}$ at the edge of the area Γ in the form [Potter 1980]

$$\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = f(x, y), \quad (1)$$

where $x \in (0, l_x)$, $y \in (0, l_y)$, $u = u(x, y) \in R^2$ is a field function, and $f = f(x, y) \in R^2$ is a function of field sources distribution (a sources' function).

The task of identifying the sources of the field in the system (1) consists in determining the sources' function $f(x, y)$. In order to solve this task in a numerical way a continuous description of the system was approximated by discrete model. After replacing continuous variables x and y to discrete variables using the formula $x = ih$, $i = 0, 1, 2, \dots, M$, $y = jh$, $j = 0, 1, 2, \dots, N$, $M = l_x/h$, $N = l_y/h$, h is a length of a step of discretization for a rectangular grid on the area of dimensions $l_x \times l_y$ and next approximating the differential equation (1) using a finite difference scheme, the algebraic equations system was obtained which maintains values of the field function u with values of the source function f in nodes of a rectangular grid as follows [Potter 1980]

$$u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + u_{i,j+1} - 2u_{i,j} + u_{i,j-1} = h^2 f(i, j) = q_{i,j}$$

$$i = 1, 2, \dots, M, j = 1, \dots, N, \quad (2)$$

where $u_{i,j} = u(i, j)$, $q_{i,j} = q(i, j)$.

Border conditions for equation (2) have the form

$$u(0, j) = U_0(j), \quad u(M, j) = U_M(j), \quad j = 0, 1, \dots, N,$$

$$u(i, 0) = U_0(i), \quad u(i, N) = U_N(i), \quad i = 0, 1, \dots, M.$$

For a discrete model (2) the solution of the identification problem based on the evaluation which involves the values of source function $q(i, j)$ in the nodes of the grid. Field function as well as the sources' function were approximated by a discrete Fourier series [Potter 1980]

$$f_{m,n} = \sqrt{2} \sum_{k=1}^{N-1} F_m(k) \sin \frac{k\pi n}{N},$$

$$f_{m,0} = f_{m,N} = 0, \quad m = 0, 1, 2, \dots, M,$$

$$u_{m,n} = \sqrt{2} \sum_{k=1}^{N-1} U_m(k) \sin \frac{k\pi n}{N},$$

$$u_{0,n} = u_{M,n} = 0, \quad n = 0, 1, \dots, N, \quad (3)$$

where $F(k)$ and $U(k)$ means coefficients for $k = 1, 2, \dots, N-1$.

Equation (2) can be transformed to the form

$$\frac{1}{h^2} \left[(U_{m+1}(k) - 2U_m(k) + U_{m-1}(k)) - (4\sin^2 \frac{k\pi}{2N}) U_m(k) \right] =$$

$$= F_m(k), \quad m = 1, 2, \dots, M, \quad (4)$$

with boundary conditions defined by $U_0(k) = 0$ and values $U_M(k)$ from a equation $u_{M,n} = 0$, $n = 1, \dots, N$.

After substituting the monic power polynomial $P(q_k)$ for $q_k = 4 \cdot \sin^2 \frac{k\pi}{2N}$ to the equation (4), a solution of direct problem takes the form

$$U_m(k) = P_m(q_k) U_1(k) + \sum_{l=1}^{m-1} P_{m-l}(q_k) h^2 F_l(k), \quad m = 2,$$

$$3, \dots, M-1. \quad (5)$$

The values $U_1(k)$ in the equations (4) can be determined from the boundary conditions. For

$N = M$ from system of $M-1$ equations the set of coefficients $U_M(k)$, $k = 1, 2, \dots, M-1$ can be determined. Next substituting these coefficients to the equation (4) for $m = M$, the set of coefficients $U_1(k)$ for $k = 1, 2, \dots, M-1$ can be determined. The field function u can be obtained from formula (3). Solving the inverse problem the source function is obtained in the following form

$$F_i(k) = \frac{U_{l+1}(k) - P_{l+1}(q_k)U_1(k) - \sum_{i=1}^{l-1} P_{l+1-i}(q_k)h^2 F_i(k)}{P_l(q_k)h^2} \quad (6)$$

The sources function f can be obtained by formula (3). The issue of identification the sources of the field is the inverse problem and therefore its solution requires the use of stabilization procedures [Tikhonov et al. 1995]. Therefore Simulation Method has been extended by a specific numerical approximation procedure developed on the bases of a inverse distance method for smoothing data in the 2-D systems. This method of stabilization is a kind of self regularization procedure [Rydygier and Strzyżakowski 2012a]. It should be noted that computer simulations had an important contribution in testing the effectiveness, accuracy and stability of the Simulation Method as well as they were used to improve the processing of the measurement data and testing some regularization procedures and fixing various regularization coefficients.

RESULTS

With the help of the Simulation Method the problem of torsion of a rail can be solved. This problem is the contact issue because the wheel of railway vehicle causes a deformation of a rail [Rydygier and Strzyżakowski 2010]. Rail was described by the Timoshenko model. Results of calculations are shown in the Figure 2 whereas in the Figure 1 there are shown the input data which correspond to the auxiliary function $\psi(x, y)$ associated with a twisting angle in a cross-sectional plane.

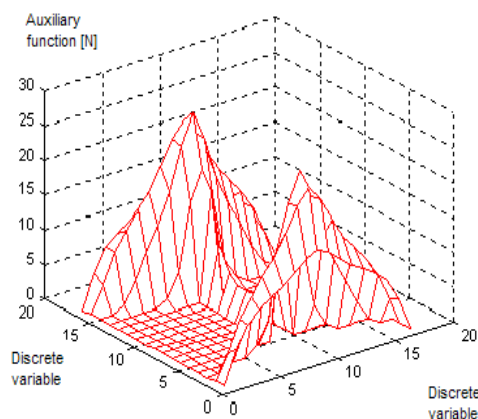


Fig. 1. Input data
Rys. 1. Dane wejściowe

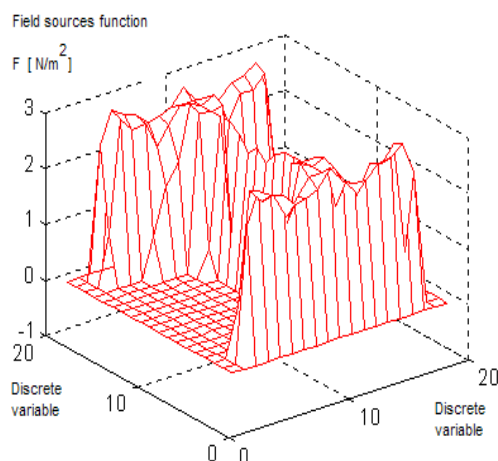


Fig. 2. The sources function for a torsion of rail
Rys. 2. Funkcja źródłowa dla skręcenia szyny

It is noticed that the form of computed source function corresponds with the analytical solution of a inverse problem described by the Poisson equation with constant source function.

The established Simulation Method was also used to identify heat sources caused by a rolling contact wheel-rail [Rydygier and Strzyżakowski 2010]. The temperature distribution in a heat trace of the rail as the input data is shown in the Figure 3. Results of calculations are illustrated in the Figure 4 in a form of heat density distribution.

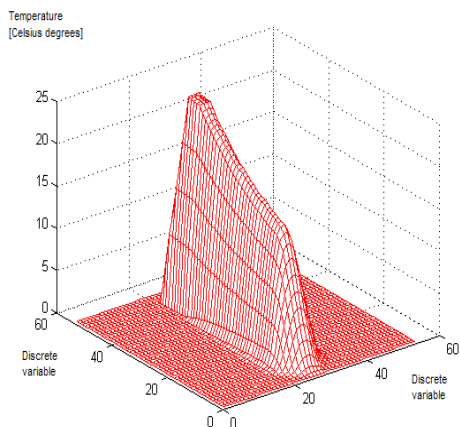


Fig. 3. Temperature distribution in a heat trace
Rys. 3. Rozkład temperatury w śladzie cieplnym

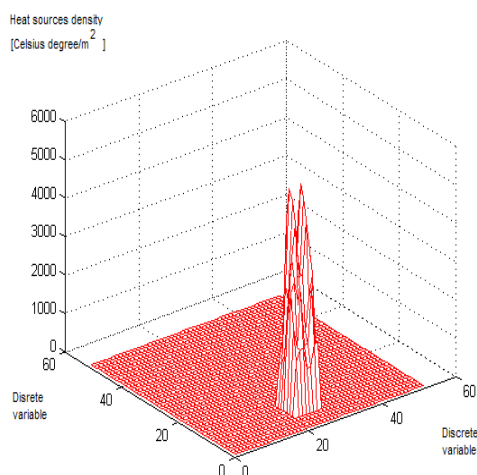


Fig. 4. The calculated sources function
Rys. 4. Wyznaczona funkcja źródłowa

It should be pointed that the identification of sources in the heat trace can be used to identify the dynamical parameters of the track-vehicle. These results allow the verification of the mechanical properties of the contact and next the appointment of local stress, slip, friction power density. Since the rolling contact wheel-rail produces thermal effects on the surface of the rail, so the mechanical quantities causing the temperature field can be determined [Szolc et al. 2002].

ONE PERIOD ENERGY METHOD

To modelling of a wheel-rail system is also used the method using the concept of one period energy [Rydygier and Strzyżakowski

2011]. The one period energy concept is used in the analysis of real-time energy processes in electric circuits in periodically non sinusoid states. Energy process can then be examined for the energy phase plane and evaluated by a change in the instantaneous voltage and current of the circuit in the one period T . Considering the dynamical circuit operating in a non sinusoidal state for the excitation signal as a voltage $v(t) = v(t + T)$ and the response as a current $i(t) = i(t + T)$, the energy transferred from the source $v(t)$ to the receiver in the time interval $\Delta t = nT$, $n \in N$, can be defined by the following term [Trzaska 2008]

$$W(\Delta t) = nW_T, \quad (7)$$

where W_T means the one period energy, i.e. the energy supplied to the receiver during one period of excitation and response.

For the examined circuit the one period energy W_T can be written as

$$\begin{aligned} W_T &= \int_0^T v(t)i(t)dt = \int_0^T v(t) \frac{d}{dt} \left(\int i(\tau)d\tau \right) dt = \\ &= \int_{q(0)}^{q(T)} v(t)dq(t) = \int_{\psi(0)}^{\psi(T)} i(t)d\psi(t), \quad (8) \end{aligned}$$

where $q(t) = \int i(t)dt$ means a load and $\psi(t) = \int v(t)dt$ means a magnetic flux.

The form of expression (8) shows that one period energy W_T collected by the receiver from the source defines a limited area in the phase loop in the phase plane of the coordinates $(v(t), q(t))$ or equivalently $(\psi(t), i(t))$. Considering the contact wheel-rail it is assumed that the studied system is in a steady state and that the vertical force which acts from a wheel on a rail has the nature of a periodic signal [Rydygier and Strzyżakowski 2012b]

$$F(t) = f(t + T), \quad (9)$$

where the period T corresponds to the arrival time for a wheel in the next carriage, $T = \Delta x/v$, v - velocity along the track.

In the vertical plane perpendicular to the railway track the rolling contact dynamical system can be described by the following set of equations

$$m \frac{d^2 y_1}{dt^2} + b_2 \frac{d(y_1 - y_2)}{dt} + k_2(y_1 - y_2) + b_1 \frac{dy_1}{dt} + k_1 y_1 = 0 \quad (10)$$

$$M \frac{d^2 y_2}{dt^2} + b_2 \frac{d(y_2 - y_1)}{dt} + k_2(y_2 - y_1) = F(t),$$

where $y_1 = y_1(t)$ and $y_2 = y_2(t)$ mean displacements, m - the replacement mass of the track, b_1, k_1 - dynamic parameters of the track, while the mass M and the dynamic parameters b_2, k_2 correspond to the rail.

The one period energy in the case of the study effects of acting normal force on the rail can be presented on the basis of the equation (8) by the following formula

$$W_T = \int_0^T F w dt = \int_{y(0)}^{y(T)} F dy_2, \quad w = \frac{dy_2}{dt} \quad (11)$$

The form of the expression (11) shows that the the energy W_T transferred in one period can be defined by the area enclosed by a loop in power phase plane with coordinates $(y_2(t), F(t))$. Basing on the electrical circuits analogy, the force $F(t)$ corresponds to source voltage $v(t)$, and the displacement $y(t)$ corresponds to an instantaneous load $q(t)$. After transforming the second order set of differential equations with two variables (10) to the first order set of differential equations with four variables with the use of following substitution

$$\frac{dy_1}{dt} = y_3(t), \quad \frac{dy_2}{dt} = y_4(t), \quad (12)$$

and then using the *ode23* function from MATLAB library for numerical solution of ordinary differential equations, the displacement $y_2(t)$ was determined. The data for masses and dynamic parameters of

damping and elasticity were taken from [Kisilowski et al. 1991]. Timing displacement $y_2(t)$ and the normal force $F(t)$ in one period are shown in the Figure 5 and the Figure 6. It is noted that the Figure 5 shows a solution for a zero value of the normal force, while the Figure 6 shows a solution for a fixed value of strength. For the period $T = 1$ s it can be estimated the time of acting the normal force from the equation $\Delta x = v\Delta t$. Assuming the vehicle speed $v = 80$ km/h and estimating the length of contact for 10^{-2} m it was obtained the time of duration of signal as $\Delta t = 5$ ms.

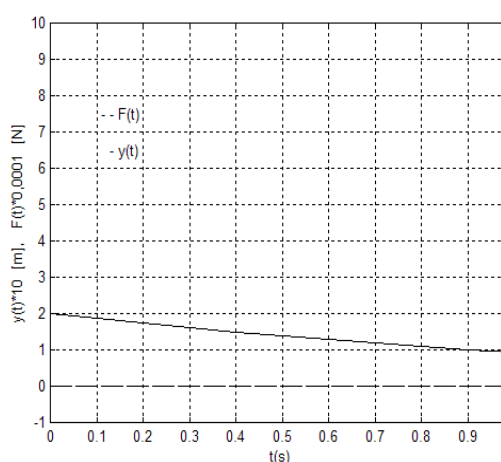


Fig. 5. Displacement and force for $F(t) = 0$
Rys. 5. Przemieszczenie i siła gdy $F(t) = 0$

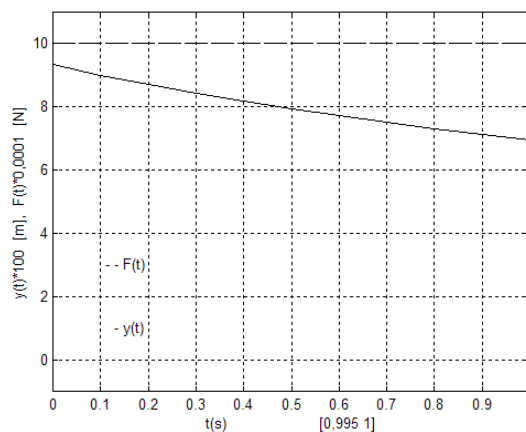


Fig. 6. Displacement and force for $F(t) = 100$ kN
Rys. 6. Przemieszczenie i siła gdy $F(t) = 100$ kN

In the system of coordinates $(y_2(t), F(t))$ the one period energy loop takes the form shown in the Figure 7.

From a plot of the loop shown in the Figure 7, it can determine the value of one period energy as the area covered by this loop.

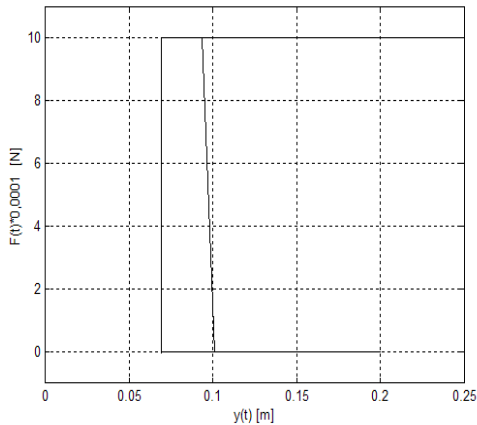


Fig. 7. One period energy loop
Rys. 7. Pętla energii jednookresowej

CONCLUSIONS

In the paper it was showed that two computer simulation methods, one using field sources identification and the second using one period energy concept, are useful in modelling the wheel-rail system and can support the diagnostics of a rail vehicle and the track. The future direction of research is to apply established methods by a cooperation with teams engaged the diagnostic measurements of track and rolling stock.

Modelling studies of track and rolling stock not only support the diagnostic measurements, but they serve to ensure the transport safety of passengers and goods. Without adequate research model may be appeared the case that the wheels of new trains in the old rails will be deformed, which not only causes discomfort of transport and will be a source of noise nuisance to the environment, but it can also be the cause of the train crash. This situation appeared at the beginning of this year in Poland according wagons manufactured by 'Pesa' company and purchased by the Warsaw Access Railway Company (WKD). As a result passengers must still ride the old, rickety warehouses trains that were produced in the 70's of last century.

Board of the WKD does not preclude the charging of penalties for the manufacturer that purchased the trains are not suitable for use. Similar incident occurred 10 years ago when the Warsaw underground trains were renewing. Italian manufacturer, the 'Alstom' company, after some months of disputes finally agreed to replace the defective wheel sets.

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UDZIAŁ MODELOWANIA ZAGADNIEŃ KONTAKTOWYCH W ZAPEWNIENIU BEZPIECZEŃSTWA TRANSPORTU SZYNOWEGO

STRESZCZENIE. Wstęp: Modelowanie matematyczne wspomaga diagnostykę toru i taboru kolejowego, gdyż często z przyczyn technicznych nie jest możliwe uzyskanie pełnego zestawu danych pomiarowych wymaganych do zdiagnozowania deformacji szyny i koła powstałych w wyniku oddziaływania pojazdu szynowego na tor. W diagnostyce toru i taboru kolejowego ważnym zagadnieniem jest badanie skutków kontaktu koła pojazdu i szyny. Badania diagnostyczne toru i taboru kolejowego odgrywają istotną rolę w zapewnieniu bezpieczeństwa przewozów pasażerów i towarów.

Celem pracy było opracowanie symulacyjnych sposobów modelowania matematycznego układu koło - szyna kolejowa użytecznych w diagnostyce toru i pojazdu kolejowego.

Metody: Przedstawiono i przedyskutowano dwa sposoby modelowania układu koło - szyna kolejowa. Jednym z tych sposobów jest metoda polegająca na sprowadzeniu badanego zagadnienia kontaktowego do zagadnienia połowego i rozwiązaniu zadania identyfikacji źródeł pola w układzie 2-D. Drugim sposobem jest metoda wykorzystująca koncepcję energii jednookresowej. Ten sposób obliczeń został zaadaptowany do modelowania dynamiki kontaktu koło - szyna dla siły normalnej. Wykazano, że opracowane metody modelowania efektywnie wspomagają badania skutków mechanicznych i cieplnych kontaktu koło - szyna oraz przyczyniają się do zapewnienia bezpieczeństwa przewozów.

Wyniki i wnioski: W przypadku metody identyfikacji źródeł pola rozpatrzono dwa zagadnienia szczegółowe: zagadnienie skręcania szyny kolejowej oraz dokonano identyfikacji źródeł ciepła w szynie kolejowej spowodowanego kontaktem tocznym koło - szyna. Natomiast w przypadku metody wykorzystującej koncepcję energii jednookresowej wykazano przydatność tej metody do badania procesów energetycznych kontaktu koło - szyna kolejowa pod działaniem siły normalnej o charakterze periodycznym. Przyszłym kierunkiem badań jest praktyczne wykorzystanie opracowanych metod przez nawiązanie współpracy z zespołami badawczymi dokonującymi pomiarów diagnostycznych toru i taboru kolejowego.

Słowa kluczowe: kontakt koło - szyna kolejowa, diagnostyka toru i taboru kolejowego, metody numeryczne, energia jednookresowa

BEITRAG DER MODELLIERUNG VON KONTAKTPROBLEMEN RAD-SCHIENE ZUR GEWÄHRLEISTUNG DER SICHERHEIT IM SCHIENENVERKEHR

ZUSAMMENFASSUNG. Einleitung: Die mathematische Modellierung hilft, die Gleise und Schienenfahrzeuge zu diagnostizieren, zumal es aus technischen Gründen oft nicht möglich ist, einen vollständigen Satz von Messdaten, die für die Feststellung der wegen der Einwirkung des Schienenfahrzeuges auf die Gleise verursachten Verformung von Schiene und Rad erforderlich sind, zu gewinnen. Die Diagnostik von Gleis- und Rollmaterial ist ein wichtiges Thema, um die Auswirkungen des Kontakts zwischen den Rädern und Schienen zu studieren. Untersuchungen des Gleis- und Rollmaterials spielt eine große Rolle bei der Gewährleistung der Sicherheit der Beförderung von Personen und Gütern.

Das Ziel dieser Studie war es, eine Simulations-Methode für die mathematische Modellierung des Rad-Schiene-Systems zu konstruieren, die bei der Beurteilung von Bahnstrecken und Schienenfahrzeugen brauchbar wäre.

Methoden: Es wurden zwei Möglichkeiten der Modellierung des Rad-Schiene präsentiert und diskutiert. Eine der möglichen Herangehensweisen ist die Methode, wonach das Kontaktproblem auf die Feldkontaktfrage zurückgeführt und die Lösung der Identifikation der Feld-Quelle im 2D- System erreicht wird. Die andere Methode ist eine Methode, die das Single-Cycle-Energie-Konzept in Anspruch nimmt. Diese Berechnungsmethode wurde an die Modellierung der Dynamik des Kontaktes zwischen dem Rad und der Schiene bei normaler Kraft angepasst. Es hat sich gezeigt, dass die entwickelten Modellierungsmethoden zur effektiven Unterstützung der Studie über die Auswirkungen vom mechanischen und thermischen Kontakt innerhalb des Rad-Schiene-Zusammenstoßes dienen und zur Gewährleistung der Sicherheit im Güter-Eisenbahntransport beitragen können.

Ergebnisse und Fazit: Im Falle der Methode für die Ermittlung der Feld-Quellen wurden zwei detaillierte Fragen untersucht: die Frage der Schienenverdrehung und die Frage der Identifizierung von Wärmequellen innerhalb des rollenden Kontaktes, die zwischen der Rad- und Schienenoberfläche entsteht. Dagegen im Falle des Verfahrens mit Anwendung des Single-Cycle-Energie-Konzeptes zeigte man die Nützlichkeit dieser Methode für die Untersuchung von energetischen Prozessen im Kontakt innerhalb des Rad-Schiene-Systems unter Einwirkung der Kraft vom periodischen Charakter auf. Die künftige Ausrichtung der Forschung ist die Aufnahme einer engen Zusammenarbeit mit Forschungsteams, die sich mit den diagnostischen Messungen von Gleisen und rollenden Schienenfahrzeugen beschäftigen.

Codewörter: Kontakt Rad-Schiene, Diagnostik von Schienen und Schienenfahrzeugen, numerische Methoden, Single-Cycle-Energie

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