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PRODUCTION-INVENTORY MANAGEMENT MODEL FOR A WEIBULL DETERIORATING ITEM WITH LINEAR DEMAND AND SHORTAGES

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ABSTRACT. Background: Physical decay or deterioration of goods in stock is an important feature of real inventory systems.

Material and methods: In the present paper, we discuss an production inventory model for a Weibull deteriorating item over a finite planning horizon with a linearly time-varying demand rate and a uniform production rate, allowing shortages, which are completely backlogged.

Results and conclusions: A production inventory model is developed for a Weibull deteriorating item over a finite planning horizon with a linear time varying demand, finite production rate and shortages. The optimal number of production cycles that minimizes the average system cost is determined.

Key words: Production, Shortage, Deterioration, Inventory, Demand.

INTRODUCTION

Physical decay or deterioration of goods in stock is another important feature of real inventory systems. Ghare and Schrader [Ghare, Schrader 1963] were the first to develop an EOQ model for an item with exponential decay and constant demand. Dave and Patel [Dave, Patel 1981] extended the model of Donaldson [Donaldson 1977] to incorporate deterioration. Their model was further extended by Sachan [Sachan 1984], which permitted backlogging of items. Both these models assumed the successive cycles to be of the same duration. Keeping the reorder points fixed and increasing the order size only rather than adjusting both should result in a higher inventory cost. Bakhari-Kashani [Bahari-Kashani 1989] developed a heuristic model by relaxing the restriction of equal replenishment cycles. Xu and wang [Xu, Wang 1990] discussed the same model in discrete time. Chung and Ting [Chung, Ting 1993] developed a more general heuristic model for both finite and infinite horizons. Giri and Chaudhuri [Giri, Choudhuri 1997 discussed two heuristic models, for both finite and infinite planning horizons, in which the demand rate, deterioration rate, ordering cost, holding cost and shortage cost all vary with time; also shortages in inventory are allowed and are completely backlogged. Literature in inventory modeling for deteriorating items is extensive. All the models referred to above are economic lot size (ELS) models dealing with inventory replenishment situations only. In an ELS model, the main aim is to determine an EOQ for a cycle; the cycle starts with the EOQ as the initial stock. An economic production lot size (EPLS) model deals with an inventory-cum-production system in which procurement of inventory occurs through production within

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the cycle itself, the cycle does not start with an EOQ as in an ELS model.

The classical EPLS model assumes both the demand rate and the machine production rate to be predetermined and inflexible [Hax, Candea 1984]. However, it is usually observed in the market that the sale of many consumer goods increases rapidly after gaining consumer acceptance. It is, therefore, quite appropriate to consider time time-varying demands in EPLS models. Hong et al. [Hong, Sandrapaty, Hayya, 1990] discussed an EPLS model with a linearly trended demand and a uniform production rate. Goswami and Chaudhuri [Goswami, Chaudhuri 1991] developed such a model considering inventory shortages. Schweitzer and Seidmainn [Schweitzer, Seidmann 1991] questioned the assumption of uniform production rate and pointed out that the machine production rate should be flexible to adjust itself with the variability in the market demand.

Giri and Chaudhuri [Giri, Chaudhuri 1999] discussed a production inventory model in which the demand varies linearly with time, unit production cost is taken to be a function of the production rate and shortages in inventory are allowed and fully backlogged. Also the machine production rate, assumed to be flexible, is treated as a decision variable. Yan and Cheng [Yan, Cheng 1998] studied a perishable single item in a single period production-inventory system in which the production rate, the demand rate and the deterioration rate are all considered as a function of time; the model also allows a shortages which is partially back-logged. Several improvements to the model of Yan and Cheng [Yan, Cheng 1998] were suggested by Balkhi [Balkhi 2000], Balkhi et al. [Balkhi, Goyal, Giri 2001] and Goyal [Goyal 2001].

In the present paper, we discuss an production inventory model for a Weibull deteriorating item over a finite planning horizon with a linearly time-varying demand rate and a uniform production rate, allowing shortages, which are completely backlogged. The production inventory system in each cycle consists of four stages. The initial stock in each cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to build up continuously after meeting demand and deterioration. Production is stopped at a certain time. The accumulated inventory is then gradually depleted and ultimately becomes zero due to demand and deterioration. Production does not restart at this stage and inventory shortages continue to accumulate for some time. Thereafter, production starts and shortages are gradually cleared after meeting current demands. The cycle ends with zero inventory.

For every machine, there exists a critical design production rate (CDPR), which is taken as a production rate in the present model. The demand is taken to be linearly time varying and we consider case of increasing demand. The number of cycles over a finite planning horizon is determined optimally. The results are illustrated with two numerical examples with increasing demands.

ASSUMPTIONS

1. Production rate is finite and constant.
2. Weibull deterioration rate is considered.
3. Shortages are allowed and completely backlogged.
4. Time horizon is finite.
5. The finite time horizon is divided into a finite number of replenishment cycles, each of equal duration.
6. Demand rate is linear in time.

NOTATIONS

$f(t)$ $a + bt$ is the demand rate at time t where $a \geq 0$, $b \neq 0$. Here a is the initial demand rate and b is the rate at which the demand rate itself changes.

$\alpha\beta\, t^{\beta-1}$ Deterioration rate which is Weibull where $0 < \alpha < 1, \beta \geq 1$

P Production rate, $P > a + bt$, $t \in [0, H]$.

C_h Inventory holding cost per item per unit time

C_s Shortage cost per item per unit time

A_s Set-up cost per cycle

C_p Production cost per item

H Time horizon
n Number of cycles in $[0,H]$

THE MODEL AND ITS SOLUTION

The initial stock of the i[th] cycle (I=1,2…n) is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. Production stops at time t_i'. The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_i', t_i]$. Shortages accumulate over $[t_i, S_i]$. Production restarts at time S_i. The accumulated shortages are fully supplied during $[S_i, T_i]$ after meeting current demands. The cycle ends with zero inventory. It repeats itself. Our problem may be precisely stated as follows. "Given a linear trend in demand (either positive or negative), a uniform production rate (P) and a finite time horizon (H), divide the time horizon (H) into a number (n) of production cycles of equal periods (H/n) and within each period, determine when to produce products in order to minimize the average cost of the inventory system."

Here t_i', t_i, S_i and T_i are connected by the following relations

$$\left.\begin{aligned} t_i' &= k_i t_i + (1-k_i)T_{i-1}, \\ t_i &= rT_1 + (1-r)T_{i-1}, \\ S_i &= d_i T_i + (1-d_i)t_i, \\ T_i &= \frac{H}{n} i \end{aligned}\right\} \quad \text{(A)}$$

for i =1, 2,….., n.,
$0 < r < 1, 0 < k_i < 1, 0 < d_i < 1.$

The instantaneous inventory level $I(t)$ at any time $t\varepsilon[T_{i-1}, t_i]$ is governed by the following differential equations:

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = P - f(t), \; T_{i-1} \le t \le t_i', \tag{1}$$

with $I(T_{i-1}) = 0$;

$$\frac{dI(t)}{dt} + \alpha\beta t^{\beta-1} I(t) = -f(t), \; t_i' \le t \le t_i, \tag{2}$$

with $I(t_i) = 0$;

$$\frac{dI(t)}{dt} = -f(t), \; t_i \le t \le S_i, \tag{3}$$

with $I(t_i) = 0$;

$$\frac{dI(t)}{dt} = P - f(t), \; S_i \le t \le T_i, \tag{4}$$

with $I(T_i) = 0$;

The solution of equation (1) is

$$I(t) = (P-a)\left(t + \frac{\alpha t^{\beta+1}}{\beta+1} - \alpha t^{\beta+1}\right) - b\left(\frac{t^2}{2} + \frac{\alpha t^{\beta+2}}{\beta+2} - \frac{\alpha t^{\beta+2}}{2}\right) + c\left(1 - \alpha t^{\beta}\right)$$

By putting the initial condition, we get

$$I(t) = (P-a)\begin{bmatrix} (t - T_{i-1}) + \dfrac{\alpha}{\beta+1}\left(t^{\beta+1} - T_{i-1}^{\beta+1}\right) \\ -\alpha\left(t^{\beta+1} - t^{\beta}T_{i-1}\right) \end{bmatrix} - b\begin{bmatrix} \dfrac{t^2}{2} + \alpha\dfrac{t^{\beta+2}}{\beta+2} - \alpha\dfrac{t^{\beta+2}}{2} - \dfrac{T_{i-1}^2}{2} \\ + \dfrac{\alpha\beta}{2(\beta+2)}T_{i-1}^{\beta+2} + \dfrac{\alpha}{2}T_1^2 t^{\beta} - \dfrac{\alpha}{2}T_{i-1}^{\beta+2} \end{bmatrix} \tag{5}$$

The solution of equation (2) is

$$I(t) = -a\left[t + \frac{\alpha}{\beta+1}t^{\beta+1} - \alpha t^{\beta+1}\right] - b\left[\frac{t^2}{2} + \frac{\alpha\beta}{2(\beta+2)}t^{\beta+2} - \frac{\alpha}{2}t^{\beta+2}\right] + c\left(1 - \alpha t^{\beta}\right)$$

Substituting the value of $I(t_1')$ from equation (5), we get

$$\begin{aligned} I(t) &= -a\left[t + \frac{\alpha}{\beta+1}t^{\beta+1} - \alpha t^{\beta+1}\right] \\ &+ a\left[T_{i-1} + \frac{\alpha}{\beta+1}T_{i-1}^{\beta+1} - \alpha t^{\beta}T_{i-1}\right] \\ &- b\left[\frac{t^2}{2} + \frac{\alpha\beta}{2(\beta+2)}t^{\beta+2} - \frac{\alpha}{2}t^{\beta+2}\right] \\ &+ b\left[\frac{T_{i-1}^2}{2} - \frac{\alpha\beta}{2(\beta+2)}T_{i-1}^{\beta+2} + \frac{\alpha}{2}T_{i-1}^{\beta+2} - \frac{\alpha t^{\beta}T_{i-1}^2}{2}\right] \\ &+ P\left[t_i' - T_{i-1} + \frac{\alpha}{\beta+1}\left(t_i'^{\beta+2} - T_{i-1}^{\beta+1}\right) - \alpha t^{\beta}t_i' + \alpha t^{\beta}T_{i-1}\right] \end{aligned} \tag{6}$$

The solution of equation (3) is

$$I(t) = -at - \frac{bt^2}{2} + c$$

By using initial condition $I(t_i)=0$, we get

$$I(t)=-(t-t_i)\left\{a+\frac{b}{2}(t_i+t)\right\} \tag{7}$$

The solution of equation (4) is

$$I=(P-a)t-\frac{bt^2}{2}+c$$

$$I(t)-I(S_i)=(P-a)(t-S_i^2)-\frac{b}{2}(t^2-S_i^2)$$

Substituting the value of $I(S_i)$ from equation (7), we get

$$I(t)=-a(t-t_i)+P(t-S_i)-\frac{b}{2}(t^2-S_i^2) \tag{8}$$

Using the initial condition $I(T_i)$=0,in equation (8), we get

$$b(T_i+t_i)+2a-2p\left(\frac{T_i-S_i}{T_i-t_i}\right)=0, T_i\neq t_i,$$

Then using (A), we get

$$d_i=1-\frac{a}{P}-\frac{bH}{2nP}(2i-1+r) \tag{9}$$

The inventory in $[T_{i-1},t_i']$ is

$$I_{i1}=\int_{T_{i-1}}^{t_i'} I(t)dt$$

Now using (5), we get

$$\begin{aligned} I_{i1}=(P-a)&\left[\begin{aligned}&\frac{T_{i-1}^2}{2}+\frac{t_i'^2}{2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}T_{i-1}^{\beta+2}\\ &-\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_i'^{\beta+2}-T_{i-1}t_i'\\ &-\frac{\alpha T_{i-1}^{\beta+1}t_i'}{(\beta+1)}+\frac{\alpha T_{i-1}t_i'^{\beta+1}}{(\beta+1)}\end{aligned}\right]\\ &-b\left[\begin{aligned}&\frac{t_i'^3}{6}+\frac{T_{i-1}^3}{3}+\frac{\alpha\beta}{(\beta+1)(\beta+3)}T_{i-1}^{\beta+3}-\frac{\alpha\beta}{2(\beta+3)(\beta+2)}t_i'^{\beta+3}\\ &-\frac{T_{i-1}^2t_i'}{2}-\frac{\alpha T_{i-1}^{\beta+2}t_i'}{(\beta+2)}+\frac{\alpha T_{i-1}^2t_i'^{\beta+1}}{2(\beta+1)}\end{aligned}\right]\end{aligned} \tag{10}$$

Now using (6), we get the inventory in the time interval $[t_i',t_i]$

$$I_{i2}=\int_{t_i'}^{t_i} I(t)dt$$

$$\begin{aligned}=-a&\left[\begin{aligned}&\frac{t_i^2}{2}-\frac{t_i'^2}{2}-\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_i^{\beta+2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_i'^{\beta+2}\\ &+T_{i-1}t_i'-T_{i-1}t_i-\frac{\alpha T_{i-1}^{\beta+1}t_i}{(\beta+1)}+\frac{\alpha T_{i-1}t_i'^{\beta+1}}{(\beta+1)}\\ &+\frac{\alpha}{\beta+1}t_i^{\beta+1}T_{i-1}-\frac{\alpha}{\beta+1}t_i'^{\beta+1}T_{i-1}\end{aligned}\right]\\ &-b\left[\frac{t_i^3}{6}-\frac{t_i'^2}{6}-\frac{\alpha\beta}{2(\beta+3)(\beta+2)}t_i^{\beta+3}+\frac{\alpha\beta}{2(\beta+3)(\beta+2)}t_i'^{\beta+3}\right.\\ &-\frac{T_{i-1}^2t_i}{2}+\frac{T_{i-1}^2t_i'}{2}-\frac{\alpha}{\beta+2}T_{i-1}^{\beta+2}t_i\\ &\left.+\frac{\alpha}{\beta+2}T_{i-1}^{\beta+2}t_i'-b\left[\frac{\alpha T_{i-1}^2t_i^{\beta+1}}{2(\beta+1)}-\frac{\alpha T_{i-1}^2t_i'^{\beta+1}}{2(\beta+1)}\right]\right]\\ &+P\left[\begin{aligned}&t_i't_i-t_i'^2-T_{i-1}t_i+T_{i-1}t_i'+\frac{\alpha}{\beta+1}t_i'^{\beta+1}t_i\\ &-\frac{\alpha}{\beta+1}T_{i-1}^{\beta+1}t_i+\frac{\alpha}{\beta+1}T_{i-1}^{\beta+1}t_i'-\frac{\alpha}{\beta+1}t_i't_i^{\beta+1}\\ &+\frac{\alpha}{\beta+1}T_{i-1}t_i^{\beta+1}-\frac{\alpha}{\beta+1}T_{i-1}t_i'^{\beta+1}\end{aligned}\right]\end{aligned} \tag{11}$$

Using (7), the shortage during $[t_i,S_i]$ is

$$I_{i3}=\int_{t_i}^{S_i}[-I(t)]dt=\left\{\begin{aligned}&\frac{a}{2}(S_i-t_i)^2+\frac{b}{6}S_i^3\\ &-\frac{b}{2}t_i^2S_i+\frac{b}{3}t_i^3\end{aligned}\right\} \tag{12}$$

Using (8), the shortage during the time interval $[S_i,T_i]$ is

$$\begin{aligned}I_{i4}&=\int_{S_i}^{T_i}[-I(t)]dt\\ &=\frac{a}{2}(S_i-t_i)^2-\frac{a}{2}(S_i-t_i)^2-\frac{P}{2}(T_i-S_i)^2\\ &\quad+\frac{b}{6}(T_i^3-S_i^3)-\frac{b}{2}t_i^2(T_i-S_i),\end{aligned} \tag{13}$$

From the relations (A), we have

$$\begin{cases} t_i'=\dfrac{H}{n}(rk_i+i-1)\\ t_i=\dfrac{H}{n}(r+i-1)\\ S_i=\dfrac{H}{n}\{i-(1-d_i)(1-r)\}\end{cases} \tag{14}$$

From equations (10) and (11), the total inventory in the ith cycle is

$$INV_i=-a\left[\begin{aligned}&\frac{T_{i-1}^2}{2}+\frac{\alpha\beta}{(\beta+1)(\beta+2)}T_{i-1}^{\beta+2}+\frac{t_i^2}{2}\\ &-\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_i^{\beta+2}-t_iT_{i-1}-\frac{\alpha t_i}{(\beta+1)}T_{i-1}^{\beta+1}\end{aligned}\right.$$

$$+\frac{\alpha T_{i-1}}{(\beta+1)}t_i^{\beta+1}\Bigg]-b\left[\begin{array}{l}\frac{T_{i-1}^3}{3}+\frac{\alpha\beta}{(\beta+1)(\beta+3)}T_{i-1}^{\beta+3}\\ +\frac{t_i^3}{6}-\frac{\alpha\beta}{2(\beta+2)(\beta+3)}t_i^{\beta+3}\\ -\frac{T_{i-1}^2 t_i}{2}-\frac{\alpha}{\beta+2}T_{i-1}^{\beta+2}t_i\end{array}\right.$$

$$+\frac{\alpha}{2(\beta+1)}T_{i-1}^2 t_i^{\beta+1}\Bigg]+P\left[\begin{array}{l}-\frac{T_{i-1}{}^2}{2}+\frac{t_i'^2}{2} \qquad T_{i-1}^{\beta+2}\\ +\frac{\alpha\beta}{(\beta+1)(\beta+2)}\end{array}\right.$$

$$-\frac{\alpha\beta}{(\beta+1)(\beta+2)}t_i'^{\beta+2}+t_i't_i-T_{i-1}t_i$$

$$\left.\begin{array}{l}-\frac{\alpha}{\beta+1}t_i't_i^{\beta+1}+\frac{\alpha}{\beta+1}T_{i-1}t_i^{\beta+1}\\ +\frac{\alpha}{\beta+1}t_i'^{\beta+1}t_i-\frac{\alpha}{\beta+1}T_{i-1}^{\beta+1}t_i\end{array}\right] \tag{15}$$

Now using (14),we get

$$INV_i=-a\left[\begin{array}{l}\frac{H^2(i-1)^2}{2n^2}+\frac{\alpha\beta H^{\beta+2}(i-1)^{\beta+2}}{(\beta+1)(\beta+2)n^{\beta+2}}\\ +\frac{H^2(r+i-1)^2}{2n^2}\end{array}\right.$$

$$-\frac{\alpha\beta H^{\beta+2}(r+i-1)^{\beta+2}}{(\beta+1)(\beta+2)n^{\beta+2}}$$

$$\left.\begin{array}{r}-\frac{H^2(r+i-1)(i-1)}{n^2}-\frac{\alpha H^{\beta+2}(i-1)^{\beta+1}(r+i-1)}{(\beta+1)n^{\beta+2}}\\ +\frac{\alpha H^{\beta+2}(i-1)(r+i-1)^{\beta+1}}{(\beta+1)n^{\beta+2}}\end{array}\right]$$

$$-b\left[\frac{H^3(i-1)^3}{3n^3}+\frac{\alpha\beta H^{\beta+3}(i-1)^{\beta+3}}{(\beta+1)(\beta+2)n^{\beta+3}}+\frac{H^3(r+i-1)^3}{6n^3}\right.$$

$$-\frac{\alpha\beta H^{\beta+3}(r+i-1)^{\beta+3}}{2(\beta+2)(\beta+3)n^{\beta+3}}-\frac{H^2(r+i-1)(i-1)^2}{2n^3}$$

$$\left.\begin{array}{l}-\frac{\alpha H^{\beta+3}(i-1)^{\beta+2}(r+i-1)}{(\beta+2)n^{\beta+3}}\\ +\frac{\alpha H^{\beta+3}(i-1)^2(r+i-1)^{\beta+1}}{2(\beta+1)n^{\beta+3}}\end{array}\right]$$

$$+P\left[-\frac{H^2(i-1)^2}{2n^2}\right.$$

Using the condition $I(t_i)=0$, in equation (6) we have

$$-a\left(t_i+\frac{\alpha}{\beta+1}t_i^{\beta+1}-\alpha t_i^{\beta+1}\right)$$

$$-b\left(\frac{t_i^2}{2}+\frac{\alpha}{\beta+2}t_i^{\beta+2}-\frac{\alpha}{2}t_i^{\beta+2}\right)+$$

$$a\left(T_{i-1}+\frac{\alpha}{\beta+1}-\alpha t_i^{\beta}T_{i-1}\right)$$

$$+b\left(\frac{T_{i-1}^2}{2}-\frac{\alpha\beta}{2(\beta+2)}T_{i-1}^{\beta+2}+\frac{\alpha}{2}T_{i-1}^{\beta+2}-\frac{\alpha}{2}t_i^{\beta}T_{i-1}^2\right)$$

$$+P\left[t_i'-T_{i-1}+\frac{\alpha}{\beta+1}\left(t_i'^{\beta+1}-T_{i-1}^{\beta+1}\right)-\alpha t_i^{\beta}t_i'+\alpha t_i^{\beta}T_{i-1}\right]=0$$

Now using (14), we get

$$-a\left[\frac{H}{n}(r+i-1)-\frac{\alpha\beta H^{\beta+1}(r+i-1)^{\beta+1}}{(\beta+1)n^{\beta+1}}\right]$$

$$-b\left[\frac{H^2(r+i-1)^2}{2n^2}-\frac{\alpha\beta H^{\beta+2}(r+i-1)^{\beta+2}}{(\beta+2)n^{\beta+2}}\right]$$

$$+a\left[\frac{H}{n}(i-1)+\frac{\alpha H^{\beta+1}(i-1)^{\beta+1}}{(\beta+1)n^{\beta+1}}-\frac{\alpha H^{\beta+1}(r+i-1)^{\beta}(i-1)}{n^{\beta+1}}\right]$$

$$+P\left[\begin{array}{l}\frac{H(rk_i+i-1)}{n}-\frac{H(i-1)}{n}\\ +\frac{\alpha H^{\beta+1}(rk_i+i-1)^{\beta+1}}{(\beta+1)n^{\beta+1}}\\ -\frac{\alpha H^{\beta+1}(i-1)^{\beta+1}}{(\beta+1)n^{\beta+1}}\end{array}\right]$$

$$+P\left(\begin{array}{l}\frac{\alpha H^{\beta+1}(r+i-1)^{\beta}(i-1)}{n^{\beta+1}}\\ -\frac{\alpha H^{\beta+1}(r+i-1)^{\beta}(rk_i+i-1)}{n^{\beta+1}}\end{array}\right)$$

$$\left.+b\left[\frac{H^2(i-1)^2}{2n^2}-\frac{\alpha\beta H^{\beta+2}(i-1)^{\beta+2}}{2(\beta+2)n^{\beta+2}}+\frac{\alpha H^{\beta+2}(i-1)^{\beta+2}}{4n^{\beta+2}}\right.\begin{array}{r}\\ -\frac{\alpha H^{\beta+2}(r+i-1)^{\beta}(i-1)^2}{2n^{\beta+2}}\end{array}\right]=0 \tag{17}$$

Here $\alpha, \beta, H, r, i, n$ are known, only k_i's are unknown, by tedious calculation we will get the values of k ,from equations (13) and (14), the total shortage in the i th cycle is

$$SHOR_i=\frac{H^2}{6n^2}\left\{2\frac{bH}{n}(r+i-1)^3+3a(1-r)^2\right.$$

$$-3P(1-d_i)^2+\frac{bH}{n}i^3-3\frac{bH}{n}i(r+i-1)^2\Bigg\} \qquad (18)$$

The deteriorated stock in the i^{th} cycle is

$$\theta * INV_i \qquad (19)$$

Therefore the average cost during the time horizon $(0, H)$ is

$$ATVC=\frac{1}{H}\left[(C_h+\theta C_p)\sum_{i=1}^{n}INV_i+C_s\sum_{i=1}^{n}SHOR_i+nA_s\right] \qquad (20)$$

NUMERICAL EXAMPLES

Example: 1

For increasing demand rate we consider parameter values

$C_s=4.5, C_p=6.0, C_h=0.3, \alpha=0.01,$

$A_s=75, a=100, b=5, P=175, H=12, \beta=1$

in appropriate units. Then the optimal solution is:

$n^*=2, r^*=0.7912,\ INV^*=2315.9046$

and $SHOR^*=38.4928$

which are shown in the table 1.

Table 1. Parameter values for increasing demand rate
Tabela 1. Wartości parametrów dla wzrastającego popytu

n	**k**$_{\mathbf{i}}$	***R***	**INV**$_{\mathbf{i}}$	**Total inventory**	**Shortage**	**Average cost**
1	k_1=0.3541	0,7425	V_i=2256,9731	2256.9731	82.1032	104.7469
2	k_1=0,2681 k_i=0,2701	0,7912	V_1=125,8431 V_2= 2190,0615	2315.9046	38.4828	96.4114
3	k_1=0,2800 k_2= 0,2900 k_3=0,3001	0,8028	V_1=75,205 V_2=1204,280 V_3= 1932,558	3212.0434	25.1706	152.6029
4	k_1=0,6090 k_2=0,6942 k_3=0,2873 k_4=0,2671	0,8092	V_1= 525,7442 V_2=2320,2453 V_3=1141,9260 V_4= 1062,0520	5049.9675	18.4624	183.4224
5	k_1=0,5604 k_2=0,6686 k_3=0,7799 k_4=0,8898 k_5=0,9877	0,8069	V_1= 284,3433 V_2= 1431,417480 V_3= 2815,054199 V_4=4343,145 V_5=5834,1079	14708.0677	15.5168	478.3108

Example: 2

We have consider $\beta=2$, and all other parameters same, we get the optimum solution is

$n^*=3, r^*=0.8028, INV^*=1814.473, SHOR^*=25.1706$

Which are shown in the table 2.

Table 2. Paramter values
Tabela 2. Wartości parametrów

n	*r*	Average cost	Inventory	Shortage cost
1	0.7425	423.4493	12880.3544	82.1035
2	0.7912	85.8170	1962.7414	38.4928
3	0.8028	82.6231	1814.473	25.1706
4	0.8092	136.8056	3498.0764	18.4624
5	0.8069	88.2302	1705.3808	15.5268

CONCLUSION

The present paper deals with an production inventory model for a weibull deteriorating item having a linear time dependent demand and a uniform production rate. The model permits inventory shortage in each cycle, which is completely backlogged with in the cycle itself. The uniform production rate is actually the CDPR of the manufacturing machine. Every cycle starts with zero stock and production. As production continues, the inventory begins to accumulate after meeting current demands and adjusting for deterioration. Production stops after some time to make room for machine maintenance. To maintain the CDPR, the machine needs regular maintenance, which is ensured by stopping production in every cycle for a certain interval of time. The excess inventory accumulated during the production period is used to account for demand and deterioration in the no-production period. Due to several practical reasons, production may not restart as soon as the accumulated inventory is fully exhausted. Such practical reasons may be like a delay in machine maintenances, for example, lake of raw materials, shortage of labour, breakdown of the power unity, lake of capital, etc. due to this time lag in restarting production, shortage starts accumulating gradually. As soon as the bottlenecks in the way of restarting production are removed, production restarts and accumulated shortage are fully cleared by the end of the cycle itself. Thus each production-inventory cycle in the proposed model is self-complete in the sense that no stock of shortage of one cycle is carried to the next one. There is no carry- over effect from one cycle to another. This is a distinct departure from the classical approach to backlogging the shortage of one cycle in the next one. The finite planning horizon is divided into a finite number of production cycles of equal duration.

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ZARZĄDZANIE POZIOMEM ZAPASÓW PRODUKCYJNYCH MODELU WEIBULLA DLA LINIOWEGO POPYTU ORAZ STRAT

STRESZCZENIE. **Wstęp:** Fizyczny proces psucia się oraz obniżania wartości zapasów jest ważnym czynnikiem w realnych systemach magazynowych.
Material i metody: w pracy poddano dyskusji model Weibulla zarządzania zapasami produkcyjnymi dla skończonego okresu planowania oraz liniowego zmiennego w czasie popytu i określonej partii produkcyjnej, zezwalający na braki, które podlegają uzupełnieniu.
Wyniki i wnioski: opracowano model Weibulla zarządzania zapasami produkcyjnymi dla skończonego okresu planowania oraz liniowego zmiennego w czasie popytu, określonej partii produkcyjnej i braków. Określono optymalną ilość cykli produkcyjnych, minimalizujących średnie koszty systemu.

Słowa kluczowe: produkcja, braki, psucie się, zapas, popyt.

MANAGEMENT VON PRODUKTIONSVORRÄTEN ANHAND DES WEIBULL-MODELLS FÜR DIE BESTIMMUNG DER LINEAREN NACHFRAGE UND VERLUSTE

ZUSAMMENFASSUNG. Einleitung: Physisches Verderben und Wertsenkung von Produktionsvorräten stellen einen wichtigen Einflussfaktor in real bestehenden Lagerungssystemen dar.
Material und Methoden: Im Rahmen der Arbeit wurde das Weibull-Modell für das Management von Produktionsvorräten bei einem finiten Planungshorizont, einer linearen, in der Zeit variablen Nachfrage und einer bestimmten Produktionsgröße innerhalb eines Systems, das Auftreten von Verlusten zulässt und Vervollständigung von Fehlmengen ermöglicht, untersucht.
Ergebnisse und Fazit: Es wurde ein Weibull-Modell für das Management von Produktionsvorräten für das Management von Produktionsvorräten bei einem finiten Planungshorizont, einer linearen, in der Zeit variablen Nachfrage und einer bestimmten Produktionsgröße und Verlust-Niveau ausgearbeitet. Desweiteren wurde die optimale Anzahl von Produktionszyklen, die durchschnittliche Kosten des Systems minimalisieren, festgestellt.

Codewörter: Produktion, Verluste, Verderben, Vorrat, Nachfrage

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