



## THEORETICAL ASPECTS OF SYNTHETIC MEASUREMENT OF THE DEVELOPMENT DYNAMICS IN THE CONTEXT OF CITY

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**ABSTRACT. Background:** The paper presents the theoretical basis for the proposal of modeling of the dynamics of the modern cities' development by the use of a properly constructed synthetic indicator. Additionally to the possibility of the quantification of the development of social and economic systems of cities, its implementation allows the identification of nonlinear processes as phase transitions, which occur e.g. under influence of technological and social innovations. The economic and physical approach to this allows to learn more about the nature of these processes and to set new instruments supporting the management of urban areas in conditions of an increasing competitiveness.

**Methods:** The mathematical modeling of social and economical processes and economical and physical approach to dynamics of systems of nonlinear development.

**Results and conclusions:** Based on conducted simulation researches, it can be concluded that the synthetic measure of the development of urban areas can be a good tool supporting the city management by local authorities. The economical and physical approach to the nonlinear dynamics of urban systems marks out new areas for further researches, the determination of minimum required conditions (the necessary level) for stimulation of the phase transition and the analysis of factors allowing to avoid the negative consequences of a phase transition, especially in smaller cities areas, seems to be the most important ones.

**Key words:** city management, nonlinear development of city, phase transition, synthetic measurement of development dynamics.

### INTRODUCTION

The urban areas, being the dynamic systems, like substances in physical processes, are subjected to transformation processes. One type of such transformation is a change of the state of matter - a phase transition. One of the most known and clear example of such phase transition is the transition of the state of matter of the water: from solid one (ice), through liquid one up to the gas one (steam). The transition between these states is conducted by the processes of freezing, melting, liquefaction, condensation, sublimation and resublimation. The urban agglomerations undergo the analogous processes of social and economical transformations, and as results of them, the changes can occur in their role, size or the range of their tasks on various levels (beginning from local one up to international one). Although most of occurrences of social and economical life of cities can be described by linear functions [Domański 2000], they can pass not exactly to results of observations, e.g. the increase of the effect is not proportional to the increase of one of the reasons. They can be not only of the continuous nature but also of incidental one, which causes additional complications to relations in social and economical systems. The identification of positive and negative nonlinearity can be useful in the recombination of factors, which have impact on the development of the infrastructure environment. It is very important, that thanks to technical and social innovations, the new possibilities

of the development of societies and communities and, at the same time, the economical development of cities is possible [Domański 2006]. The transformations, which occur due to them, are typical transformations of the nonlinear nature - described as phase transition in economical and social aspects. The new structure, which is created during each phase transition, has different properties from the previous one [Domanski 2000].

While analysing the analogy between states of matter of physical bodies and the processes of the development of cities, it must be stated, that the thermodynamics reactions can be described by the use of physical equations, which explain the factors and their influences on the transition processes (e.g. pressure, temperature or others) but there is a lack of such quantitative description of the analysis of phase transitions of cities. In other words, there is no synthetic indicator, like the temperature in the thermodynamics, which allows describing, determining and explaining whether the community of a given city was, is or will be in the phase transition. It is also of a great value to attempt to identify such factors, responsible for these transitions, interpreted as a quantity jump in the development process of an information society.

Due to the shortage of such form of the description of processes of the urban transformation (indispensable as one of tools of the city management), the aim of this paper is to propose the solution for the synthetic measurement of the development dynamics, which would enable the quantitative analysis of processes of phase transitions of cities.

## **MICRO- AND MACRO-VARIABLES AND INDICATORS OF THE DYNAMICS**

There are many indicators which describe various social and economic aspects of cities' dynamics. For example, the dynamics of the economic growth can be described by the use of the gross domestic product indicator (GDP), as well as by the unemployment rate or the production of the electricity. It was stated that the GDP indicator is quite well correlated with both the employment rate and the production of the electricity, and therefore the analysis only of the GDP indicator should be enough to reflect the economic situation of the city. Taking into consideration additionally the employment rate or the production of the electricity could complicate unnecessarily the quantitative analysis and weaken the statistical analysis. In this case, the GDP indicator is the macrovariable, which reflects the dynamics of microvariables such as the employment rate or the production of the electricity. However, the proper definition of macrovariables is a difficult task, taking into consideration the complexity of social and economical processes occurring within the cities as well as the accessibility of statistical data.

Based on statistical data, the following procedure for the determination of macrovariables is proposed:

1. To create the correlation matrix (of Pearson correlation coefficients) using both all available variables and their functions: logarithmic and quadratic ones.
2. To identify potential macrovariables, which will have the high correlation ratios with other variables (microvariables).
3. To apply the linear regression analysis for potential macrovariables: a macrovariable as a variable being explained and microvariables as explanatory ones. Therefore it will be possible to check the quality of the relationship between variables and their potential causality. It can be expressed, that the macrovariable  $x$  will be in relation with microvariables  $a_1, a_2, \dots, a_n$  according to the equation:

$$x = \beta + \alpha_1 a_1 + \alpha_2 a_2 + \dots + \alpha_n a_n + \varepsilon$$

where:

- x – macrovariable (explained)
- $a_1, a_2, a_3, \dots, a_n$  – microvariables (explanatory)
- $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n$  – regression coefficients
- $\beta$  – constant coefficient
- $\varepsilon$  – value of statistical error

In this case, if the above mentioned relation is fulfilled, i.e. it is statistically significant and could be explained in a logical way, the coefficient x could be considered to be the macrovariable. For example:

Assuming, the correlation between the values of GDP indicators and the indicators of employment rate and the production of the electricity is statistically significant and can be expressed by the following equation:

$$GDP = \beta + \alpha_1 EMPLOYM + \alpha_2 PROD.ELECTR + \varepsilon$$

It means, that both the analysis of the employment dynamics as well as the analysis of the production of the electricity are taken into consideration during the analysis of the GDP dynamics. Therefore the GDP is the macrovariable and the indicators of the employment rate and the production of the electricity are microvariables.

4. Microvariables, recognized to be significant and not correlated with other available indicators due to the lack of data, can be taken as macrovariables.

## SYNTHETIC INDICATOR OF THE DYNAMICS

The synthetic indicator of the development dynamics of cities  $\Omega$  in year  $t$  can be defined as the weighted mean of the growth rate of variables, which describes the social and economical processes of cities and can be presented by the following equation:

$$\Omega_t = \sum_{i=1}^N w_i z_i$$

where:

- $\Omega$  – value of synthetic indicator of city dynamics
- w – arithmetical weight of variable  $i$  in year  $t$
- z – growth rate of variable  $i$  in year  $t$
- N – number of variables describing city dynamics.

As a result of that, the given indicator includes information about the dynamics of individual variables and can be a new (synthetic) variable describing the progress of city transformation processes, i.e. having the similar role as the temperature in thermodynamic processes.

It can be noticed, that this indicator is dependent on the value of two factors: the weight of the variable  $w$  and the value of growth rate of the variable  $z$ .

Many various social and economical factors have the influence on the city development. It can be very often observed, that not only its own dynamics but also the interdependence with other factors leads to synergistic effects and creates very complex situation of the development of urban areas. The cognition of these processes (at least partially) is a very challenging task. Due to the fact, that various

factors affect differently the development of urban areas, it is necessary to assign the proper weight to each of them, which will describe the function of its dynamics during the process of the city development. The arithmetical weight of a given variable indicates the significance of factors represented by this value in the process of the city development and their potential to induce phase transitions.

The Multinomial Logit Model (MLM), known in scientific literature, was applied to determine these weights. It can be presented in the following generalized form [Ben-Akiva, Lerman 1985]:

$$P(i) = \frac{e^{V_i}}{\sum_{j=1}^N e^{V_j}}$$

where:

$P(i)$  – probability of the occurrence of a given even (result)  $i$

$N$  – number of possible events (results)

$V_j$  – function describing the nature of a given event  $j$ , which has influence on its chances to its existence, the higher the value of this function, the higher the probability of the occurrence,

$V_i$  – the value of the function  $V$  for an event  $i$ .

The Multinomial Logit Model can be applied in economical and mathematical analyses, e.g. the discrete decision analysis or the stochastic analysis of inhabitants' flows in urban areas [Pawlak 2008]. Its application for the explanation of the arithmetical weight in case of variables describing the dynamics of the development of the urban centre is based on the nature of such form of a function:

1. MLM takes into account the fact, that the sums of probabilities of possible results of a given experiment as well as the arithmetical weights for each mean, have to have a value equal to 1, i.e.:

$$\sum_{i=1}^N P(i) = 1$$

2. MLM allows to describe arithmetical weights of any number of events or (in the context of conducted researches) macrovariables.
3. For each event, if the value of the function  $V_i$  increases, then the value of obtained arithmetical weight will lead asymptotically to value 1. It means, there are no limits for the value of the function  $V_i$ , because the obtained weight will always be within the range from 0 to 1. It allows quite a big flexibility in the choice of a type and values of this function for individual macrovariables.
4. Transparent and computational convenience of the estimation of parameters of this function.

Taking into account the above mentioned facts, the following version of mathematical presentation of arithmetical weights of individual variables is proposed:

$$w_{it} = \frac{e^{\beta_{it}}}{\sum_{j=1}^N e^{\beta_{jt}}}$$

where:

$w_{it}$  – weight of a variable  $i$  in year  $t$

$\beta_{it}$  – function describing the meaning of the dynamics for any variable  $j$  in year  $t$

$\beta_{it}$  – the value of the function describing the meaning of the dynamics for any variable  $i$  in year  $t$

$N$  – number of variables

$t$  – the reference year

Having the above presented form, it should be considered how to estimate the value of the function  $\beta$  for each variable in year  $t$ . Practically it means that it must be estimated, which factors have the higher potential to stimulate the development. The higher this potential in a given period of a variable  $i$ , the higher value of the function  $\beta$ . The time dependence is also significant, due to the fact that the importance and the role of individual factors stimulating the development can be changed over a period of time. For example, the high share of heavy and mining industries could be stimulating factors for the development of a given city in a given period. However such heritage could be a big problem considering the progressive technological progress or the economical transformation in the direction of the economy based on modern electrical technologies. American Detroit, British Manchester or Polish Katowice are examples of cities, which suffered from such process.

The weight of a variable can be also of a synthetic nature, what is often of a great significance when the general competitiveness of cities is analyzed. Individual components can have the following shares [Watson, Sudhir 2012]: economical power (30%), institutional efficiency (15%), human resources (15%), financial maturity (10%), global importance of the city (10%), physical capital (10%), environmental and natural threats (5%) and the social and cultural character of the city (5%).

Beside the arithmetical weight of a variable, its dynamics influences also the way and the nature of the development of the city. Having the variable  $x_i$ , it is possible to determine the value of its growth rate  $z_i$  in year  $t$ :

$$z_{it} = \frac{d \ln(x_i)}{dt} = \frac{dx_i}{x_i dt} \approx \frac{\Delta x_i}{x_i \Delta t}$$

Assuming that period  $t$  is equal to one year, the equation can be simplified to the following form:

$$z_i \approx \frac{x_i(t) - x_i(t-1)}{x_i(t-1)}$$

Therefore it is the gain of the value of the variable between year  $t-1$  and year  $t$  (given in a form of a fraction or percentage). Such a form allows using available statistical data in an optimal way for the determination of the general dynamics of various factors, which have impact on the development.

## GENERALIZED FORM OF THE SYNTHETIC INDICATOR OF THE DYNAMICS OF THE CITY DEVELOPMENT

Based on the above contemplations, it is possible to create the synthetic indicator of the dynamics of city development for a given year, which can be defined as a mathematical function dependent on analyzed variables:

$$\Omega_i(t) = \sum_{i=1}^N w_i z_i = \sum_{i=1}^N \left( \frac{e^{\beta_i t}}{\sum_{j=1}^N e^{\beta_j t}} * \frac{\partial \ln[(x)_i(t)]}{\partial t} \right) \approx \sum_{i=1}^N \left( \frac{e^{\beta_i t}}{\sum_{j=1}^N e^{\beta_j t}} * \frac{x_i(t) - x_i(t-1)}{x_i(t-1)} \right)$$

The indicator is the weighted mean of the growth rate of variables. The weights of them are described by the polynomial logit function dependent on time and parameters of the function  $\beta$ , which explains the meaning of a given variable in the process of the city development. Practically, this

indicator is the synthetic expression of available variables, describing the development of an urban centre.

Taking into consideration the fact, that the above mentioned function describes the development rate of a city in a general way, it can be assumed, that the function  $D(t)$  fulfilling the following dependence:

$$D(t) = \int_{t=0}^T \Omega dt = \int_{t=0}^T \left[ \sum_{i=1}^N \left( \frac{e^{\beta_i t}}{\sum_{j=1}^N e^{\beta_j t}} * \frac{\partial \ln[(x)_i]}{\partial t} \right) \right] dt$$

will be the description of the level of the city development. In other words, assuming that the initial level of the city development is equal to 0, the function describes the inconstancy of the development in the synthetic way.

Due to the fact, that social and economical factors subject also to the short-term fluctuation, e.g. one-year one, it is necessary to reduce the effects of short-term fluctuations on the dynamics, i.e. to suppress the influence of such fluctuations by the use of proper statistical methods. For example - the *k-means* method can be implemented here, which enables to minimize the effect of short-term fluctuations on the analysis of the proper trend of the city development. Therefore, the proper analysis process must be conducted on the set of values of the synthetic indicator of the dynamics of the city development to display the trend of the development rate of the city.

## TESTING THE HYPOTHESIS OF THE NONLINEAR DEVELOPMENT OF A CITY

Based on the estimated trend of the synthetic indicator of the development dynamics, the linear regression method should be used once again. Due to the fact, that phase transitions are the nonlinear phenomena, their detection consists in the localisation of the nonlinear function, describing the development of the city. It can be done in two equivalent ways:

1. By the use of the sythetical indicator of the development dynamics  $\Omega$ : if the development of the city is of the linear nature, then the function  $\Omega$ , as a function describing the development rate, should be a constant function. In such case, the line of the simple linear regression can be fit in the values of the function  $\Omega$ . This line, like any linear function, will have its own directional and constant coefficients. In case of a constant function, the directional coefficient should be equal to zero. Then the test for *zero* hypothesis should be conducted for the directional coefficient not equal to zero for such regression line and it should be based on t-Student test. In case of the rejection of such hypothesis, it can be concluded, that the development of this city is not of linear nature.
2. By the use of function  $D$ , which describes the normalized level of the city development. If the development is of a linear nature, then the function  $D(t)$  should be a linear function. Therefore, based on the values of the function estimated for each year, the simple linear regression analysis can be done and especially the analysis of statistical significance of the estimated directional coefficient and the determination coefficient. Based on that, it can be concluded whether the phenomena, typical for nonlinear dynamics of the system, took place.

In practice, the first method is the preferred one, because it does not require the transformation of the discrete function into a constant one - the process, which will be necessary to perform the further integration. Additionally the further proceeding (test for *zero* hypothesis) will be quite clear one as well as enabling to obtain three variants of results (indispensable for the confirmation of the reliability

of the synthetic indicator of the dynamics of the city development, used as a tool for the management of urban areas):

1. The city development was of nonlinear nature and there was a present of phase transitions,
2. The city development was of linear nature, but non-monotonic one (there were both increasing as well as decreasing periods),
3. there was a statistical error and the *zero* hypothesis was rejected despite of its truthfulness.

## CONCLUSIONS

The theoretical basis of the proposal of modelling of the development dynamics of modern cities, by the use of properly constructed synthetic indicator, was presented. Beside the possibility of the quantification of the development of social and economical systems of cities, its use allows to identify the non-linearity of these processes as phase transitions - which occur e.g. under the influence of technological and social innovations. It should be an irreversible process and lead to the movement of the whole city system and, at the same time, of their inhabitants to the new quality, which is e.g. the city of a greater efficiency and more friendly to its inhabitants. The physic-economical approach allows learning a nature of these processes in a better way and shows new areas for further researches. The most important ones seem to be the determination of the minimum of indispensable requirements (required level) stimulating the phase transition as well as the analysis of factors allowing to avoid the negative phase transition - especially in smaller urban centres.

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## TEORETYCZNE ASPEKTY SYNTETYCZNEGO POMIARU DYNAMIKI ROZWOJU JAKO INSTRUMENTU ZARZĄDZANIA MIASTEM

**STRESZCZENIE. Wstęp:** Artykuł zawiera teoretyczne podstawy propozycji modelowania dynamiki rozwoju współczesnych miast, przy pomocy odpowiednio skonstruowanego syntetycznego wskaźnika. Jego zastosowanie oprócz możliwości kwantyfikacji rozwoju systemów społeczno-gospodarczych miast, pozwala przede wszystkim na zidentyfikowanie nieliniowości tych procesów jako przejść fazowych - występujących przykładowo pod wpływem innowacji technologicznych i społecznych. Ujęcie ekonofizyczne pozwala niewątpliwie lepiej poznać naturę tych procesów oraz wyznacza nowe instrumentarium wspomagające zarządzanie obszarami zurbanizowanymi, w warunkach rosnącej konkurencyjności.

**Metody:** Modelowanie matematyczne procesów społeczno-gospodarczych, ekonofizyczne ujęcie dynamiki systemów rozwoju nieliniowego.

**Wyniki i wnioski:** Z przeprowadzonych badań o charakterze symulacyjnym wynika, że syntetyczny pomiar dynamiki rozwoju obszarów zurbanizowanych może być dobrym instrumentem wspomagającym władze samorządowe w procesach

zarządzania miastami. Ujęcie ekonofizyczne nieliniowej dynamiki systemów miejskich wyznacza nowe obszary poznawcze dla dalszych badań, wśród których za najistotniejsze wydaje się określenie minimum warunków koniecznych (niezbędnego poziomu) stymulującego przejście fazowe oraz analiza czynników pozwalających na uniknięcie społeczeństwom negatywnych przejść fazowych - zwłaszcza w mniejszych ośrodkach miejskich.

**Słowa kluczowe:** zarządzanie miastem, rozwój nieliniowy miast, przejścia fazowe, pomiar syntetyczny dynamiki rozwoju.

## **THEORETISCHE ASPEKTE DER SYNTHETISCHEN BEMESSUNG VON ENTWICKLUNGSDYNAMIK ALS BRAUCHBARES INSTRUMENT FÜR EFFIZIENTES STADT-MANAGEMENT**

**ZUSAMMENFASSUNG. Einleitung:** Der Artikel beinhaltet theoretische Grundlagen für Modellierung der Entwicklungsdynamik von gegenwärtigen Städten mittels einer entsprechend generierten, synthetischen Kennziffer. Deren Anwendung erlaubt vor allem, außer der Möglichkeit einer Quantifikation der Entwicklung von städtischen, sozial-wirtschaftlichen Systemen, die Nichtlinearität der Prozesse als Phasen-Übergänge, die unter dem Einfluss technologisch und sozial bedingter Innovationen vorkommen, zu ermitteln. Solch ein ökonomisch-physisches Herangehen an den Themenkomplex ermöglicht, die Natur der betreffenden Prozesse zweifelsohne besser kennen zu lernen, und die neue Methodik kennzeichnet im wachsenden Wettbewerbskampf ein neues Instrumentarium, das das Management von städtischen Ballungsgebieten effektiv unterstützen kann.

**Methoden:** Mathematische Modellierung von sozial-wirtschaftlichen Prozessen, das ökonomisch-physische Herangehen an die Systeme von nichtlinearer Entwicklungsdynamik.

**Ergebnisse und Fazit:** Aus den durchgeführten Forschungen von simulationsmäßigem Charakter geht eindeutig hervor, dass die synthetische Bemessung der Entwicklungsdynamik von städtischen Ballungsgebieten ein für die jeweilige Stadtverwaltung brauchbares Instrument für Unterstützung der städtischen Management-Prozesse werden kann. Das ökonomisch-physische Herangehen an die nichtlineare Entwicklungsdynamik von städtischen Systemen kennzeichnet neue Erkundungsgebiete für die weitere Erforschung, wobei die Ermittlung eines unentbehrlichen Minimums (Mindest-Niveau) von den die Phasen-Übergänge stimulierenden Bedingungen sowie die Analyse der Faktoren, die gegenwärtige Gesellschaften negative Phasen-Übergänge meiden lassen, am wichtigsten zu sein scheinen - insbesondere innerhalb der Städte kleineren Typs.

**Codewörter:** Stadt-Management, nichtlineare Stadtentwicklung, Phasen-Übergänge, synthetische Bemessung der Entwicklungsdynamik.

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