A PRODUCTION INVENTORY MODEL FOR AN ITEM WITH THREE PARAMETER WEIBULL DETERIORATION AND PRICE DISCOUNT

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ABSTRACT. Background: Deterioration is a natural process for most of the items as such it cannot be ignored in study of inventory control and management. In recent years great deal of study is devoted in developing inventory models for deteriorating items considering various practical situations. Price discount for partially deteriorated items is considerably a new concept introduced in developing various models.

Methods: This paper deals with the development of an inventory model for Weibull deteriorating items. Here production and demand rate are considered to be constant and the holding cost per unit is assumed to be constant with respect to time. Completely deteriorated units are discarded and partially deteriorated items are allowed to carry a discount. Shortages are not allowed.

Results and conclusions: A Production Inventory model for an item with three parameter Weibull deterioration with price discount for partially deteriorated item have been proposed in this paper. Here the optimal cycle time for the model has been derived and the result is illustrated with the help of numerical example. Sensitivity analysis has been carried out to analyze the changes in the optimal solution with respect to the change in other parameters.

Key words: Production quantity model, Weibull deterioration, Price discount.

INTRODUCTION

One of the important concerns of the inventory management is to decide as to when and how much is to be ordered or manufactured so that the total cost associated with the inventory system can be kept at minimum. When the inventory is subject to significant deterioration, it becomes more important as loss due to deterioration cannot be ignored in this case. Study in this direction have resulted in continuous modification of inventory modelling for deteriorating items by including more and more practical features. The impact of product deterioration should not be neglected in the decision process of production lot size. Researchers are engaged in analyzing inventory models for deteriorating items such as volatile liquids, medicines, electronic components, fashion goods, fruits, vegetables, etc. An order level inventory model with constant deterioration was developed by Aggarwal [1978]. Earlier some researchers like Ghare [1963] considered exponentially decaying inventory for a constant demand. Optimum production planning for a deteriorating item was developed by Hwang [1986]. A production inventory model for decaying raw materials and a decaying single finished product system was developed by Raffat [1985], [1991]. Optimal pricing and lot sizing under conditions of perishability and partial backordering was studied by Misra [1975] and Abad [1996]. Goyal and Giri [2001] have also given many deteriorating inventory models. Then Economic lot scheduling problem was studied by Gary et.al [2005] and Deng et.al [2007]. Yang and Wei [2003] developed an integrated multilot-size production inventory model. Then Sugapriya and Jeyaraman [2008] studied a common production cycle time for an EPQ model of non instantaneous deteriorating items allowing price discount using permissible delay in payments. Inventory management of time dependent deteriorating with salvage value was developed by Mishra [2008]. Optimal policy for a deteriorating

In the present paper a production inventory model has been developed considering three parameter Weibull deterioration with price discount for partially deteriorated items. The holding cost is assumed to be constant and shortages are not allowed for this model. In section 2 assumptions and notations required for the development of the model are given. The optimum cycle time, holding cost and total variable cost of the model is derived in the Section 3. An illustrative numerical example, a sensitivity table and conclusion are given in section 4, 5 and 6 respectively.

**BASIC ASSUMPTIONS AND NOTATIONS**

The following are the assumptions required for development of the model:

1. The demand rate for the product is known and finite.
2. Shortage is not allowed.
3. Planning horizon is infinite.
4. Once a unit of the product is produced, it is available to meet the demand.
5. Price discount is allowed for partially deteriorated items.
6. There is no replacement or repair for a deteriorated item.

The notations that are employed here:

- \( p \) : Production rate per unit time.
- \( d \) : Actual demand of the product per unit time
- \( A \) : Set up cost
- \( \theta \) : Weibull three parameter deterioration rate (unit/unit time), \( \theta = \alpha \beta (t - \gamma)^{\beta-1} \), where \( 0 < \alpha < 1 \), \( \beta > 1 \), \( 0 < \gamma < 1 \), where \( \alpha \) is called scale parameter and \( \beta \) is called shape parameter and \( \gamma \) is called the location parameter.
- \( h \) : Inventory carrying cost per unit per unit time which is constant.
- \( k \) : Production cost per unit.
- \( l \) : Price discount per unit cost.
- \( T \) : Optimal cycle time.
- \( T_1 \) : Production period.
- \( T_2 \) : Time during which there is no production. i.e., \( T_1 = T - T_2 \).
- \( I_1(t) \) : Inventory level for product during the production period, i.e. \( 0 \leq t \leq T_1 \).
- \( I_2(t) \) : Inventory level of the product during the period when there is no production i.e. \( T_1 \leq t \leq T_2 \).
- \( I(M) \) : Maximum inventory level of the product.
- \( TVC(T) \) : Total cost/unit time.
MATHEMATICAL MODEL

At \( t = 0 \), the inventory level is zero. The production and supply start simultaneously and the production stops when the maximum inventory \( I(M) \) is reached at time \( t = T_1 \). During this period of time inventory built up at a rate \( p - d \) and there is no deterioration. After time \( T_1 \), the produced units start deterioration and supply is continued at the discount rate. As the demand remains constant for the product the inventory level reduces to zero and then the production run begins. Thus the inventory level of the product at time \( t \) over the period \([0, T]\) can be represented by the following differential equations

\[
\frac{dI_1(t)}{dt} = p - d \quad 0 \leq t \leq T_1
\]

and

\[
\frac{dI_2(t)}{dt} + \theta I_2(t) = -d \quad 0 \leq t \leq T_2
\]

Where \( \theta = \alpha \beta (t - \gamma)^{\beta - 1} \), where \( 0 < \alpha < 1, \beta > 1, 0 < \gamma << 1 \)
\( \alpha \) is called scale parameter and \( \beta \) is called shape parameter and \( \gamma \) is called the location parameter.

Here the boundary conditions are \( I_1(0) = I_2(T_2) = 0 \)

Solving equation (1) and (2), we get

\[
I_1(t) = (p - d)t, \quad 0 \leq t \leq T_1
\]

\[
I_2(t) = d \left[ T_2 - t + \frac{\alpha}{\beta + 1} (T_2 - \gamma)^{\beta + 1} - \frac{\alpha}{\beta + 1} (t - \gamma)^{\beta + 1} - \alpha T_2 (t - \gamma)^{\beta} + \alpha t (t - \gamma)^{\beta} + \alpha (t - \gamma)^{2\beta + 1} \right], \quad 0 \leq t \leq T_2
\]

The production cost per unit time is

\[
PC = p k \frac{T_i}{T}
\]

The set up cost per unit time is

\[
SC = \frac{A}{T}
\]

The Holding Cost is

\[
HC = \frac{1}{T} \left[ \int_0^{T_i} h(t) I_1(t) dt + \int_0^{T_i} h(t) I_2(t) dt \right] = \frac{1}{T} \left[ \int_0^{T_i} h(p - d) t \right] +
\]

\[
h d \left[ T_2 - t + \frac{\alpha}{\beta + 1} (T_2 - \gamma)^{\beta + 1} - \frac{\alpha}{\beta + 1} (t - \gamma)^{\beta + 1} - \alpha T_2 (t - \gamma)^{\beta} + \alpha t (t - \gamma)^{\beta} + \alpha (t - \gamma)^{2\beta + 1} \right] dt
\]
Integrating the above we get

\[
HC = \frac{h(p - d)T_1^2}{2T} + \frac{hd}{T} \left[ T_2^2 - \frac{2\alpha(T_2 - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)} + \frac{\alpha(T_2 - \gamma)^{\beta+1}}{(\beta + 1)} - \frac{\alpha(T_2 - \gamma)^{2\beta+2}}{2(\beta + 1)} + \frac{2\alpha(-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)} \right] + \frac{hd}{T} \left[ \frac{\alpha T_2(-\gamma)^{\beta+1}}{(\beta + 1)} - \frac{\alpha(-\gamma)^{2\beta+2}}{2(\beta + 1)} \right]
\]

Let us express \( T_1 \) and \( T_2 \) in terms of \( T \)

We know \( I_1(T_1) = I_2(0) \)

\[
(p - d)T_1 = d \left[ T_2 + \frac{\alpha}{\beta + 1}(T_2 - \gamma)^{\beta+1} - \frac{\alpha}{\beta + 1}(-\gamma)^{\beta+1} - \alpha T_2(-\gamma)^{\beta} + \alpha(-\gamma)^{2\beta+1} \right]
\]

Neglecting the terms involving second and higher power of \( \gamma \) as \( 0 < \gamma << 1 \) and \( T_2 \) from the right hand side to get a suitable solution, we have

\[
(p - d)T_1 = d T_2
\]

\[
(p - d)(T - T_2) = d T_2
\]

\[
\frac{T_2}{p} = \frac{(p - d)T}{p} = xT, \quad \text{where, let } x = \frac{p - d}{p}
\]

\[
T_1 = \frac{dT}{p}
\]

Using these values of \( T_1 \) and \( T_2 \) in equation (7) we get

\[
HC = \frac{hd xT}{2} - \frac{2\alpha hd(xT - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} + \frac{hd \alpha x(xT - \gamma)^{\beta+1}}{(\beta + 1)} + \frac{hd \alpha (xT - \gamma)^{2\beta+2}}{2(\beta + 1)T} + \frac{2\alpha h d(-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} + \frac{hd \alpha (-\gamma)^{\beta+1}}{(\beta + 1)} - \frac{hd \alpha (-\gamma)^{2\beta+2}}{2(\beta + 1)T}
\]

**Deterioration cost**

The number of units that deteriorate in a cycle is the difference between the maximum inventory and the number of units used to meet the demand. Hence the deterioration cost per unit time is given as

\[
DC = \frac{k}{T} \left[ I_2(0) - \int_0^T d d t \right]
\]

\[
= \frac{kd}{T} \left[ \frac{\alpha(T_2 - \gamma)^{\beta+1}}{(\beta + 1)} - \frac{\alpha(-\gamma)^{\beta+1}}{(\beta + 1)} - \alpha T_2(-\gamma)^{\beta} + \alpha(-\gamma)^{2\beta+1} \right]
\]
Price discount

Price discount is offered as a fraction of production cost for the units in the Period \([0, T_2]\)

\[
PD = \frac{kl T_2}{T} \int_0^T dT
\]

\[
= \frac{kl dT_2}{T}
\]

(12)

Therefore the average total cost per unit time is given by

\[
TVC(T) = PC + SC + HC + PD + DC
\]

\[
= \frac{pkT_1}{T} + kT + \frac{h d xT}{2} - \frac{2\alpha h d (xT - \gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T} - \frac{\alpha h d (xT - \gamma)^{\beta+1}}{(\beta + 1)T} + \frac{\alpha h d x (xT - \gamma)^{\beta+1}}{2(\beta + 1)T}
\]

\[
+ \frac{2\alpha h d (\gamma^{\beta+2})}{(\beta + 1)(\beta + 2)T} + \frac{h d \alpha x(-\gamma)^{\beta+1}}{(\beta + 1)} - \frac{h d \alpha (\gamma)^{2\beta+2}}{2(\beta + 1)T} + \frac{k l dT_2}{T}
\]

\[
+ \frac{k d T}{T} \left[ \alpha (T_2 - \gamma)^{\beta+1} - \frac{\alpha (\gamma)^{2\beta+2}}{\beta + 1} - \alpha T_2 (-\gamma)^{\beta} + \alpha (\gamma)^{2\beta+1} \right]
\]

(13)

Putting the values of \(T_2\) and \(T_1\) in terms of \(T\) from equation (8) and (9) respectively, equation (13) becomes

\[
TVC(T) = k d + \frac{A}{T} + \frac{h d xT}{2} - \frac{2\alpha h d (xT - \gamma)^{\beta+2} - (\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T}
\]

\[
+ \frac{h d \alpha}{2(\beta + 1)T} ((xT - \gamma)^{2\beta+2} - (\gamma)^{2\beta+2}) + \frac{h d \alpha x (xT - \gamma)^{\beta+1}}{(\beta + 1)} + \frac{\alpha h d x (xT - \gamma)^{\beta+1}}{(\beta + 1)} - \frac{\alpha h d x (xT - \gamma)^{\beta+1}}{(\beta + 1)} + \frac{\alpha h d x (\gamma)^{2\beta+1}}{(\beta + 1)} + \frac{k l d x}{T}
\]

(14)

To find the minimum total cost, we calculate the value of \(T\) from

\[
\frac{d}{dT} (TVC(T)) = 0
\]

\[
\Rightarrow -\frac{A}{T^2} + \frac{h d x}{2} - \frac{2\alpha h d}{(\beta + 1)(\beta + 2)T^2} [T x (\beta + 2) (xT - \gamma)^{\beta+1} - (xT - \gamma)^{\beta+2} + (\gamma)^{\beta+2}]
\]

\[
+ \frac{\alpha h d}{2(\beta + 1)T^2} [T x (2\beta + 2) (xT - \gamma)^{2\beta+1} - (xT - \gamma)^{2\beta+2} + (\gamma)^{2\beta+2}]
\]

\[
+ \frac{\alpha k d}{(\beta + 1)T^2} [T x (\beta + 1) (xT - \gamma)^{\beta} - (xT - \gamma)^{\beta+1} + (\gamma)^{\beta+1}]
\]

\[
+ \alpha h d x^2 (xT - \gamma)^{\beta} - \frac{\alpha k d (\gamma)^{2\beta+1}}{T^2} = 0
\]

(15)
The value of $T$ calculated from (16) will minimize the $TVC$ if

$$\frac{d^2}{dT^2}(TVC(T)) > 0$$

$$\Rightarrow \frac{2A}{T^3} - \frac{2\alpha h d x}{(\beta + 1)T^2} [T_x (\beta + 1)(xT - \gamma)^\beta - (xT - \gamma)^{\beta+1}]$$

$$+ \frac{2\alpha h d}{(\beta + 1)(\beta + 2)T^4} [T^2 x (\beta + 2)(xT - \gamma)^{\beta+1} - 2(xT - \gamma)^{\beta+2} T] + \frac{4\alpha h d (-\gamma)^{\beta+2}}{(\beta + 1)(\beta + 2)T^3}$$

$$+ \frac{\alpha h d x}{T^2} [T_x (2\beta + 1)(xT - \gamma)^{2\beta} - (xT - \gamma)^{2\beta+1}]

- \frac{\alpha h d x}{2(\beta + 1)T^4} [T^2 x (2\beta + 2)(xT - \gamma)^{2\beta+1} - 2(xT - \gamma)^{2\beta+2} T] - \frac{\alpha h d (-\gamma)^{2\beta+2}}{(\beta + 1) T^3}

$$+ \frac{\alpha k d x}{T^2} [T_x \beta (xT - \gamma)^{\beta-1} - (xT - \gamma)^\beta]

- \frac{\alpha k d}{(\beta + 1)T^4} [T^2 x (\beta + 1)(xT - \gamma)^\beta - 2(xT - \gamma)^{\beta+1} T]

- \frac{2\alpha k d (-\gamma)^{\beta+1}}{(\beta + 1) T^3} + \alpha h d \beta x^3 (xT - \gamma)^{\beta-1} + \frac{2\alpha k d (-\gamma)^{2\beta+1}}{T^5} > 0 \quad (16)$$

**NUMERICAL EXAMPLE**

Let $A = Rs 2000 /set up, \quad p = 200 \text{ units/unit time}, \quad d = 50 \text{ unit/unit time}, \alpha = 0.6, \beta = 10, \gamma = 0.4, \quad k = Rs 60 /unit, \quad l = 0.05, \quad h = 2$. Using equation (15), (10), (14) and (16) we get the optimum values of $T^* = 1.83038, HC^* = 72.1144, \quad TVC^* = 4343.27, \quad \frac{d^2}{dT^2}(TVC(T)) = 4776.65 > 0$ respectively.

**SENSITIVITY ANALYSIS**

We now perform the sensitivity analysis of the optimal solution of the model for changes in $A, \alpha, \beta, \gamma, k, p$ and $d$ parameter values associated with the system. We change one parameter at a time keeping the other parameters unchanged for study of sensitivity analysis. The original values of all the parameters for sensitivity analysis are taken from the example given above. Sensitivity analysis is performed by changing the values of all the parameters from -50% to +50%, one by one in the model which are given in the following table 1.
Table 1. Sensitivity Analysis
Tabela 1. Analiza wrażliwości

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From the table 1 we can conclude the following:

i. \( T^* \) is directly proportional to \( A, d, \beta, \gamma \) but inversely proportional to \( p, \alpha, k \).

ii. \( HC^* \) is directly proportional to \( A, p, d, \gamma \) but inversely proportional to \( \alpha, \beta, k \).

iii. \( TVC^* \) is directly proportional to \( A, d, p, \alpha, k \) but inversely proportional to \( \beta, \gamma \).

CONCLUSION

Here, a Production inventory model has been developed for an item with three parameter Weibull deterioration where the holding cost is constant per unit, per unit time. We have assumed here that the production and demand rate are constant and shortages are not allowed. Completely deteriorated items are discarded and partially deteriorated items are offered for sale with a discount meeting the demand. The optimum production cycle time, holding cost and total variable cost has been derived for the developed model. Sensitivity analysis shows how the different parameters affect the production cycle time, holding cost and total variable cost. It is clearly seen from the table that to minimise the total cost, the set up cost, production rate, demand rate, scale parameter and the production cost per unit should be minimised whereas the value of the shape parameter and location parameter should be maximised.

REFERENCES


MODEL ZARZĄDZANIA PRODUKCJĄ Z TRÓJPARAMETROWYM WPŁYWEM PSUCIA SIĘ PRODUKTÓW I UPUSTEM CENOWYM

STRESZCZENIE. Wstęp: Psucie się jest naturalnym procesem większości produktów i nie może być ignorowane przez zarządzających produkcją. W ostatnich latach opublikowano wiele prac poświęconych różnym modelom zarządzania zapasem i produkcją, uwzględniających psucie się produktów w różnych warunkach praktycznych. Upust cenowy stosowany dla częściowo zepsutych produktów jest stosunkowo nową koncepcją, wprowadzoną w wielu rozwijanych obecnie modelach.

Metody: Praca ta porusza zagadnienia związane z opracowaniem modelu dla artykułów podlegających psuciu się według modelu Weibulla. Wielkość produkcji i popytu są wielkościami statycznymi, zakłada się, że koszt utrzymania jednostki towaru jest stały w czasie. Całkowicie zepsute artykuły są usuwane z zapasu, natomiast częściowo zepsute mogą być sprzedane z upustem cenowym. Nie dopuszcza się braków towarowych.


Słowa kluczowe: model zarządzania produkcją, proces psucia się Weibulla, upust cenowy.
MANAGEMENT-MODELL FÜR DIE DURCH DEN DREI-PARAMETER-VERDORBEN DER PRODUKTE UND DEN DADURCH BEDINGTEN PREISNACHLASS CHARAKTERISIERTE PRODUKTION


Codewörter: Modell für Produktionsmanagement, Verderbnisprozess gemäß dem Weibull-Modell, Preisnachlass.

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