



## OPTIMIZATION IN NET PLANNING

Vasilij Alekseevič Novikov, Dmitrij Sergeevič Charitonov

MITKO, Minsk, Belarus

**ABSTRACT.** The paper presents the new approach to optimization of net graphs by using the tools available in VBA Excel. The calculation of both pessimistic and optimistic variants of optimized cases is presented using the different kinds of restrains.

**Key words:** net planning, optimization of critical route, net graph of logistic chain.

## INTRODUCTION

Net planning is one of the most important responsibilities of any manager. Nowadays it seems to be more and more actual in situation of development of new methods of logistic optimization. In logistics this task is even more understood as optimization of logistic chain as a whole. The effectiveness of the whole company depends on correctly planned net flows. One can find a lot of help during such projects by using specialised programmes e.g. MS Project [Kudriawciew 2003]. Such programmes enable to prepare net plans and presentation of given data and information, including Gantt diagram.

However there are new problems connecting with development of logistic methods not included in classic methods of net planning. The classic approach is to take fix periods of time to fulfil given tasks. In many contemporary programmes it is possible to set up the minimum and maximum period of time needed for defined task. Based on that feature one can prepare the optimistic and pessimistic net plans of jobs. One more disadvantage of classic mathematical methods is, they do not allow to prepare algorithms for process of modification of net flows, e.g. to optimize logistic chain in accordance to problematic or restricted resources.

From practical point of view, especially valuable is the possibility to choose the most optimal solution among a few available variants. In such cases the data are presented as a matrix of times of each job for each worker. Such projects include both the process of appointing of jobs and of net planning. Generally one can assume to make two or more jobs by one worker. The processing of this problem using mathematical methods enables to see the parallel solutions of given tasks and to prepare net diagram in accordance with restricted resources. The last step of this process is made manually based on visualisation of Gantt diagram.

In presented example there are 9 jobs to be fulfilled and therefore the matrix looks as presented on Figure 1.

$$A = \begin{pmatrix} 5 & 4 & 43 & 12 & 32 & 22 & 21 & 6 & 90 \\ 4 & 12 & 34 & 34 & 123 & 52 & 46 & 66 & 76 \\ 2 & 8 & 32 & 67 & 221 & 32 & 12 & 19 & 99 \\ 11 & 7 & 12 & 78 & 21 & 94 & 135 & 5 & 7 \\ 100 & 3 & 6 & 21 & 56 & 31 & 19 & 55 & 6 \\ 12 & 65 & 22 & 41 & 7 & 83 & 22 & 17 & 88 \\ 43 & 44 & 2 & 72 & 87 & 25 & 55 & 34 & 175 \\ 21 & 5 & 54 & 53 & 95 & 35 & 110 & 43 & 43 \\ 1000 & 8 & 11 & 33 & 63 & 16 & 19 & 45 & 18 \end{pmatrix}$$

Fig. 1. Net matrix for 9 jobs  
 Rys. 1. Macierz sieci dla 9 zadań

Workers are given in the rows of this matrix and jobs are given in the columns. If the worker  $i$  cannot take the job  $j$  than the value  $A_{ij}$  is respectively higher.

For further calculations for net diagram for given 9 jobs the graph presented at Fig. 2 was chosen.

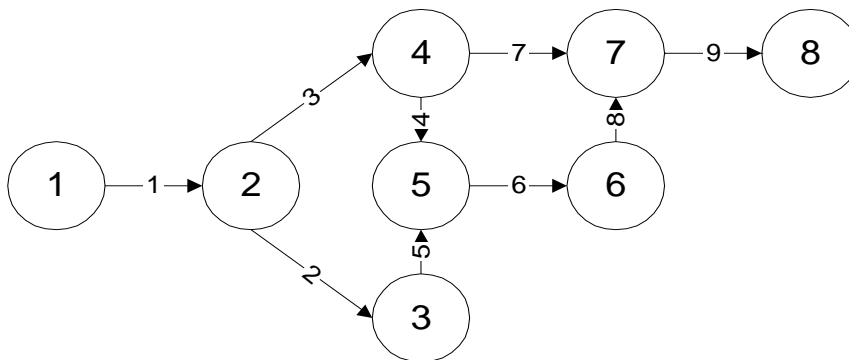


Fig. 2. Net graph  
 Rys. 2. Diagram sieci

Nodes of this graph are the points of beginning and ending of jobs and the lines are the processes of fulfillment of jobs. Every node has its own number. The time needed for every job is given on the lines respectively.

According to general requirements the nodes have to be numbered in very restricted way. This part of the process is called graph ranking. Number 1 is always placed at the beginning of net graph. The next nodes are numbered in accordance with the sequence how the jobs will succeed each other. Corresponding jobs should be marked in the same colour and in other unambiguous way. The process of preparing the ranking of parts of net has to be preceded manually and requires a lot of experience and attention especially when there are a lot of nodes. This part is very difficult to obtain the right results, because during the process of calculation and optimization the diagram could be modified at every step. The general algorithm of evaluation of parameters of net graph with any pattern of numbering of nodes is provided. Moreover the algorithm allows that some nodes could be not numbered and such situation could be the condition for optimization process. Although the presented algorithm looks for solution a little slower than previous ones but it includes in the process every data and condition. The realization of this algorithm by VBA is as follows:

```
Dim ii(), jj(), kk(), m(), rk(), pn(), pk(), kod()
Dim ie
Dim kpytj As Single
Public Sub pytj()
ie = Cells(1, 1)
ReDim ii(1 To ie), jj(1 To ie), kk(1 To ie), m(1 To ie)
ReDim rk(1 To ie), pn(1 To ie), pk(1 To ie), kod(1 To ie)
For i = 1 To ie
ii(i) = Cells(i + 1, 1): jj(i) = Cells(i + 1, 2): kk(i) = Cells(i + 1, 3)
Next i
f = xxxx(): Cells(1, 4) = "rn": Cells(1, 5) = "rk": Cells(1, 6) = "pn": Cells(1, 7) = "pk"
For i = 1 To ie
Cells(i + 1, 4) = m(i): Cells(i + 1, 5) = rk(i): Cells(i + 1, 6) = pn(i): Cells(i + 1, 7) = pk(i)
If rk(i) = pk(i) Then: Cells(i + 1, 8) = "*"
Else: Cells(i + 1, 8) = ""
End If: Next i
Cells(1, 8) = kpytj: Cells(1, 9) = "êď_ïóòü"
End Sub
Public Function xxxx(): kodd = 0
While kodd = 0
kodd = 1
For i = 1 To ie: If kod(i) = 0 Then: kodd = 0: ikod = 0
For j = 1 To ie: If kod(j) = 0 Then: If ii(i) = jj(j) Then
ikod = 1: GoTo abc
End If: End If: Next j
abc: If ikod = 0 Then: rkk = 0
For j = 1 To ie
If kod(j) = 1 Then: If ii(i) = jj(j) Then: If rkk < rk(j) Then
rkk = rk(j)
End If: End If: End If: Next j
m(i) = rkk: rk(i) = rkk + kk(i): kod(i) = 1
End If: End If: Next i: Wend
kpytj = 0
For i = 1 To ie: kod(i) = 0: If kpytj < rk(i) Then: kpytj = rk(i)
End If: Next i: kodd = 0
While kodd = 0
kodd = 1
For i = 1 To ie: If kod(i) = 0 Then
kodd = 0: ikod = 0
For j = 1 To ie: If kod(j) = 0 Then: If ii(j) = jj(i) Then
ikod = 1: GoTo abc1
End If: End If: Next j
abc1: If ikod = 0 Then
rkk = kpytj
```

```

For j = 1 To ie
If kod(j) = 1 Then: If ii(j) = jj(i) Then: If rkk > pn(j) Then
rkk = pn(j)
End If: End If: End If: Next j
pk(i) = rkk: pn(i) = rkk - kk(i): kod(i) = 1
End If: End If: Next i: Wend
xxxx = 0
End Function
    
```

The algorithm is calculated in Excel and results are presented at Figure 3. Two first columns present the beginning and ending of jobs. The third column – the period of time needed for a job. The obtained solution gives the rescheduling of point of start and finish of the jobs and the length of critical route (кр\_путь). Stars indicate the segments of critical route.

	A	B	C	D	E	F	G	H	I
1	9			rn	rk	pn	pk		31 кр_путь
2	1	2	1	0	1	0	1	*	
3	2	4	3	1	4	1	4	*	
4	2	3	2	1	3	1	3	*	
5	3	5	5	3	8	3	8	*	
6	4	5	4	4	8	4	8	*	
7	4	7	7	4	11	15	22		
8	5	6	6	8	14	8	14	*	
9	6	7	8	14	22	14	22	*	
10	7	8	9	22	31	22	31	*	

Fig. 3. Results of net planning  
 Rys. 3. Wyniki planowania sieciowego

To find the optimal solution for net graph presented at Figure 1, one can use the matrix X identical in size with matrix A. The symbol  $x_{ij}$  mean the job  $j$  is performed by worker  $i$ . In opposite situation  $x_{ij}=0$ .

The aim of the function is to minimize the critical route of net graph. If the period of time needed for job  $i$  is described by  $c_i$ , than for given (simple) example only three routes are possibly:

$$d_1 = c_1 + c_2 + c_5 + c_6 + c_8 + c_9$$

$$d_2 = c_1 + c_3 + c_7 + c_9$$

$$d_3 = c_1 + c_3 + c_4 + c_6 + c_8 + c_9$$

Such a simple example was chosen only to make easier to show the process of mathematical method for settlement of parallel branches of the net graph. The value  $K = \max(d)$  presents the critical route.

One job can be carried out only by one worker:

$$\sum_i x_{ij} = 1 \quad (1)$$

In connection with equation (1) the length of period of time needed for work  $c_i$  is equal to:

$$c_j = \sum a_{ij}x_{ij} \quad (2)$$

To prepare the optimist variant it is necessary to solve the equation to find the minimum value of K with limits (1) and (3).

$$Sx_j = \sum_j x_{ij} = 1 \quad (3)$$

The limitation (3) is very simple variant of this problem and allows only the condition that one worker can do one job.

This can be solved easily in Excel. The results are presented in Figure 4.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	
1			Матрица А										Матрица X								Sxi	C		d1	d2	d3
2	12	23	6	13	5	12	23	12	11		0	0	0	0	0	0	0	0	0	0	0	0		1	1	1
3	12	17	10	13	11	12	9	14	18		0	0	0	0	0	0	0	0	0	0	0	0		1	0	0
4	18	13	22	13	16	19	8	15	18		0	0	0	0	0	0	0	0	0	0	0	0		0	1	1
5	10	13	16	14	13	16	13	22	13		0	0	0	0	0	0	0	0	0	0	0	0		0	1	0
6	12	34	21	12	32	21	25	13	17		0	0	0	0	0	0	0	0	0	0	0	0		1	0	0
7	26	13	14	11	21	9	14	17	15		0	0	0	0	0	0	0	0	0	0	0	0		1	1	0
8	14	17	21	22	16	17	18	20	8		0	0	0	0	0	0	0	0	0	0	0	0		0	0	1
9	9	21	22	31	18	24	15	9	25		0	0	0	0	0	0	0	0	0	0	0	0		1	1	0
10	30	19	13	20	9	12	5	14	31		0	0	0	0	0	0	0	0	0	0	0	0		1	1	1
11																										
12										Sxj	0	0	0	0	0	0	0	0	0	0	0	0		0	0	0
13																										K

Fig. 4. Results from Excel (матрица А – Matrix A, матрица X – matric X)  
 Rys. 4. Wyniki otrzymane w Excelu (матрица А – macierz A, матрица X – macierz X)

To find the solution one can use the function for searching of results, available in standard Excel, presented at Figure 5.

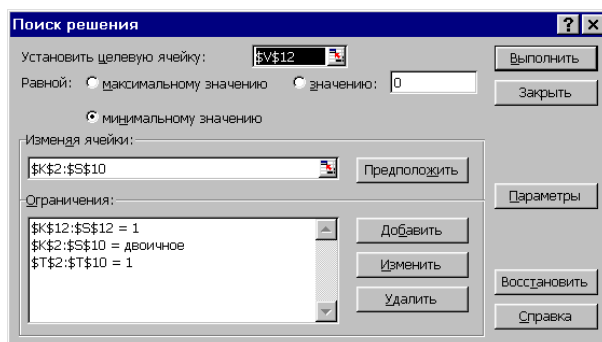


Fig. 5. Dialog window of Search function in Excel  
 Rys. 5. Okno dialogowe funkcji wyszukiującej w Excelu

The results of these calculations are presented in Figure 6.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	
1	Матрица А										Матрица X										Sxi	C		d1	d2	d3
2	12	23	6	13	5	12	23	12	11		1	0	0	0	0	0	0	0	0	1	12			1	1	1
3	12	17	10	13	11	12	9	14	18		0	0	0	0	0	0	0	1	0	1	13			1	0	0
4	18	13	22	13	16	19	8	15	18		0	0	0	1	0	0	0	0	0	1	13			0	1	1
5	10	13	16	14	13	16	13	22	13		0	1	0	0	0	0	0	0	0	1	13			0	1	0
6	12	34	21	12	32	21	25	13	17		0	0	0	0	0	0	1	0	0	1	18			1	0	0
7	26	13	14	11	21	9	14	17	15		0	0	0	0	0	1	0	0	0	1	9			1	1	0
8	14	17	21	22	16	17	18	20	8		0	-0	0	0	0	0	0	0	1	1	25			0	0	1
9	9	21	22	31	18	24	15	9	25		0	0	0	0	1	0	0	0	0	1	14			1	1	0
10	30	19	13	20	9	12	5	14	31		0	0	1	0	0	0	0	0	0	1	8			1	1	1
11																										
12										Sxj	1	1	1	1	1	1	1	1	1				74	74	69	58
13																							K			

Fig. 6. Results in case – one worker per one job (матрица А – matrix A, матрица X – matrix X)  
 Rys. 6. Wyniki przy założeniu – jeden pracownik jedna praca (матрица А – macierz A, матрица X – macierz X)

Values equal to 1 in matrix X presented at Figure 6 mean that the job  $j$  (columns) was performed by worker  $i$  (rows). Minimal critical route for this example is equal to 74.

To obtain the pessimistic variant of the case, it is necessary to exclude all  $x_{ij}$  for which the job  $j$  cannot be performed by worker  $i$ . In presented example these limits are valid for:

$$x_{51}=0; x_{91}=0; x_{62}=0; x_{63}=0; \quad (4)$$

The maximum of function K, calculated with defined limits (4), gives the pessimistic variant of the case.

In the presented example one worker can perform only one job as the presented net graph allows such situation. But there is a possibility, that with the smaller number of workers the net graph will be more optimized.

In such example it is necessary to assume that one worker can carry out more than one job. The key contribution to obtain this is to include parallelism into the matrix M. Each row  $i$  of that matrix represents the analyzed job and each column  $j$  represents the job connected with  $i$ . If the job  $i$  cannot be made at the same time as job  $j$ , then  $m_{ij}=1$ . In opposite situation  $m_{ij}=0$ . Diagonal  $m_{ij}$  is always equal to 1. The matrix of parallelism M can be constructed according to different finish times of jobs, which is presented below. In the given example the matrix M can be fixed in such a way as presented at Figure 7.

1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	1	0	0	0
0	1	1	0	1	0	0	0	0	0
0	1	0	1	1	0	1	0	0	0
0	0	1	1	1	0	1	0	0	0
0	0	0	0	0	1	1	0	0	0
0	1	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	1	1	0
0	0	0	0	0	0	0	0	0	1

Fig. 7. Matrix M  
 Rys. 7. Macierz M

The first job cannot be performed by one worker at the same time with other jobs. The second job cannot be performed at the same time as jobs 1, 5, 6, 8 and 9. Matrix M is always symmetric.

There is no use in the process of optimization to prepare only the matrix of parallelism without any formal limitations. The matrix N is prepared based on matrix M:

$$n_{mi} = \sum_j m_{nj} x_{ij} \quad (5)$$

The solution of the example by using the matrix N only gives different results if the constrain (3) will be changed to  $n_{mi} \leq 1$ .

The formula (5) of matrix N is calculated by the use of Macro function in VBA. The dialog input window of that function is presented at Figure 8.

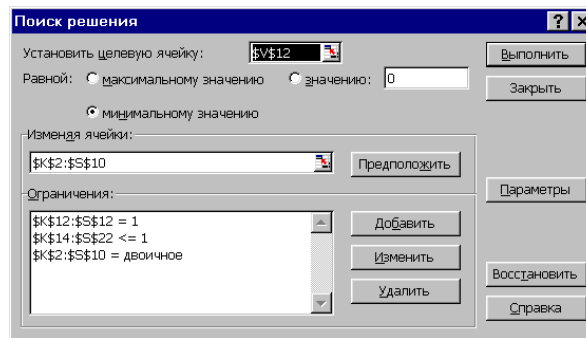


Fig. 8. Dialog window of Search function in Excel  
 Rys. 8. Okno dialogowe funkcji wyszukującej w Excelu

The solution for optimistic variant is presented at Figure 9. The critical route is equal to 64. Comparing to previous results, obtained without matrix M, the value was lowered from 74 to 64 and the number of needed workers was lowered from 9 to 7.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y			
1			Матрица А												Матрица X										C	d1	d2	d3
2	12	23	6	13	5	12	23	12	11		0	0	0	0	0	0	1	0	0		12		1	1	1			
3	12	17	10	13	11	12	9	14	18		0	0	1	0	0	1	0	0	0		13		1	0	0			
4	18	13	22	13	16	19	8	15	18		0	0	0	0	0	0	0	0	0		10		0	1	1			
5	10	13	16	14	13	16	13	22	13		0	1	0	0	0	0	0	0	0		11		0	1	0			
6	12	34	21	12	32	21	25	13	17		1	0	0	0	0	0	0	0	0		16		1	0	0			
7	26	13	14	11	21	9	14	17	15		0	0	0	1	0	0	0	0	0		12		1	1	0			
8	14	17	21	22	16	17	18	20	8		0	0	0	0	1	0	0	0	0		23		0	0	1			
9	9	21	22	31	18	24	15	9	2		0	0	0	0	0	0	0	1	1		9		1	1	0			
10	30	19	13	20	29	12	5	14	31		0	0	0	0	0	0	0	0	0		2		1	1	1			
11																												
12										Sxi	1	1	1	1	1	1	1	1	1		64	64	56	47				
13			Матрица М																				K					
14	1	0	0	0	0	0	0	0	0		0	0	0	0	1	0	0	0	0									
15	0	1	1	1	0	0	0	1	0		1	1	0	1	0	1	0	0	0									
16	0	1	1	0	1	0	0	0	0		0	1	0	1	0	0	1	0	0									
17	0	1	0	1	1	0	1	0	0		1	0	0	1	0	1	1	0	0									
18	0	0	1	1	1	0	1	0	0		1	1	0	0	0	1	1	0	0									
19	0	0	0	0	0	1	1	0	0		1	1	0	0	0	0	0	0	0									
20	0	1	0	1	1	1	1	1	0		1	1	0	1	0	1	1	1	0									
21	0	0	0	0	0	0	1	1	0		1	0	0	0	0	0	0	1	0									
22	0	0	0	0	0	0	0	0	1		0	0	0	0	0	0	0	1	0									
23											Матрица N																	

Fig. 9. Solution obtained by using the matrix of parallelism M (матрица А, X, М, N respect. matrix A, X, M, N)  
 Rys. 9. Rozwiązanie otrzymane przy użyciu matrycy M (матрица А, X, М, N odpowiednio macierz A, X, M, N)

To obtain the pessimistic variant of this case the function  $K$  is to be maximized with additional constrains (4).

The obtained results of optimistic and pessimistic variants optimize only the length of critical route. At the same time when using the matrix of parallelism it is necessary to optimize considering the jobs, not included in critical route. This complementary optimization can be processed by achieving the minimum  $\sum_j c_j$  with additional constrain of  $K \leq K_{omn}$ , where  $K_{omn}$  is the importance

for the process of optimization of critical route.

To obtain the final solution of full optimization of the process it is necessary to use the algorithm for calculation of critical route which includes simultaneously the earlier and later start and finish times of the jobs. Assuming:

PK – earlier start time,

ΠK – later start time,

PH – earlier finish time,

ΠH – later finish time,

The matrix of parallelism  $M$  can be automatically calculated by the use of following algorithm (in the algorithm the vectors  $KΠH$  and  $KPK$  are additionally included):

1.  $KΠH = 0$ ;  $KPK = 0$  for  $\forall j, m = 1$
2. Fixing the job  $i = 1, m_{i1} = 1$
3. if  $ΠK_i \leq PH_j$ , then  $m_{ij} = 0$  for  $\forall j \neq i$ , in the opposite situation p. 4
4. if  $PH_i \geq ΠK_j$ , then  $m_{ij} = 0$  for  $\forall j \neq i$ , in the opposite situation p. 5
5. if  $KΠH_j \neq 1$  and  $KPK_i \neq 1$  and  $PK_j \leq ΠH_i$ , then  $m_{ij} = 0$  for  $\forall j \neq i, KΠH_i = 1, KPK_j = 1$ , in the opposite situation p. 6
6. if  $KΠH_i \neq 1$  and  $KPK_j \neq 1$  and  $PK_i \leq ΠH_j$ , then  $m_{ij} = 0$  for  $\forall j \neq i, KΠH_j = 1, KPK_i = 1$ , in the opposite situation p. 6
7.  $i = i + 1$ , go to p. 3.

Presented methodology of using the matrix of parallelism gives the possibility to optimize the net graph according to numbers of parallel chains, because the quantity in the matrix is defined by the quantity in each row of the matrix. To lower the number of parallel chains it is necessary to transfer only one branch of the net graph with one value in matrix  $M$  into critical route. The choice of essential branch of the net graph can be made by analyzing all available variant and choosing the most optimal one. Such a work cannot be made only by using the tools available in Excel; therefore it is necessary to carry out a double cycle of the process of optimization. In the process of preparing the needed program the most important part of the task is to create the algorithm for optimization conducted by implementation the methodology of two cycles.

## SUMMARY

The presented methodology of optimization of logistic chains is based on key matrix of parallelism, which enables to fully formalize the process of optimization of any complex system, including the



possibility of modification of net graph due to minimization of restricted resources. The presented method can be used in any other situation to optimize the net graph.

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## OPTYMALIZACJA W PLANOWANIU SIECIOWYM

**STRESZCZENIE.** Praca przedstawia nowe podejście do zagadnienia optymalizacji w planowaniu sieciowym przy użyciu metod dostępnych w arkuszu kalkulacyjnym Excel. Przedstawiono wyliczanie wariantów optymistycznych i pesymistycznych danego zagadnienia przy zastosowaniu różnych ograniczeń.

**Słowa kluczowe:** planowanie sieciowe, optymalizacja ścieżki krytycznej, schemat graficzny łańcucha dostaw.

## OPTIMIERUNG IN DER NETZWERKPLANUNG

**ZUSAMMENFASSUNG.** Der Beitrag stellt einen neuen Ansatz zur Optimierung in der Netzwerkplanung unter Anwendung der in dem Tabellenkalkulationsprogramm Excel verfügbaren Methoden, dar. Es wird die Berechnung sowohl pessimistischer als auch optimistischer Varianten bei unterschiedlicher Einschränkungen dargestellt.

**Codewörter:** Netzwerkplanung, Optimierung des kritischen Pfades, grafische Darstellung der Lieferketten.

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Vasilij Alekseevič Novikov,  
Dmitrij Sergeevič Charitonov  
MITKO  
Minsk, Belarus  
e-mail: [vanovikov@tut.by](mailto:vanovikov@tut.by)