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ORIGINAL PAPER

IMPACT OF PUBLICITY EFFORT AND VARIABLE ORDERING COST IN MULTI-PRODUCT ORDER QUANTITY MODEL OF UNITS LOST SALES DUE TO DETERIORATION

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ABSTRACT. Background: This model investigates the instantaneous multi-product economic order quantity model by allocating the percentage of units lost due to deterioration in an on-hand inventory with promotional investment and functional major ordering cost. The objective is to maximize the net profit so as to determine the order quantity, publicity effort factor, the cycle length and number of units lost due to deterioration.

Methods: The mathematical model with algorithm is developed to find some important characteristics for the concavity of the net profit function. Numerical examples are provided to illustrate the results of proposed model which benefits the retailer for ordering the deteriorating items. Finally, sensitivity analysis of the net profit for the major inventory parameters is also carried out.

Results and conclusions: The proposed model is a general framework that considers wasting/ none wasting the percentage of on-hand multi-product inventory due to deterioration with publicity effort cost and functional major ordering cost simultaneously.

Key words: Multi-product, Publicity effort, Variable ordering cost, Deterioration, Profit maximization.

INTRODUCTION

Most of the literature on inventory control and production planning has dealt with the assumption that the demand for a product will continue infinitely in the future either in a deterministic or in a stochastic fashion. This assumption does not always hold true. Inventory management plays a crucial role in businesses since it can help retail companies reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth. The mathematical modelling of real-world inventory problems necessitates the simplification of assumptions to make the mathematics flexible. However, excessive simplification of assumptions results in mathematical models that do not represent the inventory situation to be analyzed.

Many models have been proposed to deal with a variety of inventory problems. The classical analysis of inventory control considers three costs for holding inventories. These costs are the procurement cost, carrying cost and shortage cost. The classical analysis builds a model of an inventory system and calculates the EOQ which minimize these three costs so that their sum is satisfying minimization criterion. One of the unrealistic assumptions is that items stocked preserve their physical characteristics during their stay in inventory. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness; vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks.

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The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries as well as retail industry also. In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver [1994] developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Mishra [2012] explored the inventory model for time dependent holding cost and deterioration with salvage value where shortages are allowed. Gupta and Gerchak [1995] examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modelled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable [Tabatabaei, Sadjadi, Makui, 2017]. Goyal and Gunasekaran [1995] considered the effect of different marketing policies, e.g. the price per unit product and the advertisement frequency on the demand of a perishable item. Bose, Goswami and Chaudhuri [1995] considered an economic order quantity (EOQ) inventory model for deteriorating goods developed with a linear, positive trend in demand allowing inventory shortages and backlogging [Sana, 2015]. Bose, Goswami and Chaudhuri [1995] and Hariga [1996] investigated the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga [1994] argued that the analysis of Bose, Goswami and Chaudhuri [1995] contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Padmanabhan and Vrat [1995] presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. A non-linear profit-maximization entropic order quantity model for deteriorating items with stock dependent demand rate is explained.

The most recent work found in the literature is that of Hariga [1995] who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Pattnaik [2011] assumes instant deterioration of perishable items with constant demand where discounts are allowed. Salameh, Jabar and Nouehed [1999] studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked.

Roy and Maiti [1997] presented fuzzy EOQ model with demand dependent unit cost under limited storage capacity. Tsao and Sheen [2008] explored dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. In the real world, procurement and inventory control are truly large scale problems, often involving more than hundreds of items. In a multi-item distribution channel, considerable savings can be realized during the replenishment by coordinating the ordering of several different items. Multi-item replenishment strategies are already widely applied in the real world. In these industries, a supplier normally produces different products for a single customer and ships to the customer simultaneously in a single truck.

In the grocery supply industry or a fast moving consumer goods industry different types of refrigerated goods (General Mills yogurt, Derived Milk products etc.) can be shipped in the same truck to the same supermarket or retail store Hammer [2001], Tsao and Sheen [2012] and others have developed models and algorithms for solving multi-item replenishment problems for different constraints. Karimi et al. [2015] introduced closed loop production systems to economic improvement, deliver goods to customers with the best quality, decrease in the return rate of expired material and decrease environmental pollution and energy usage. In this study, they solved a multi-product, multi-period closed loop supply chain network in Kalleh dairy company, considering the return rate under uncertainty. The objective of this study is to develop a supply chain model including raw material suppliers, manufacturers, distributors and a recycle center for returned products. Ghorabaee et al. [2017] developed an Integration of reverse logistics processes into supply chain network design which can help to achieve a network for multi-product, multi-period. The framework of the proposed approach includes green supplier evaluation and a mathematical model in an uncertain environment. Because multi-echelon coordination is frequently applied in current business practice, it is an essential component in inventory model for retailer's perspective. Hence the multi-product EOQ model is the focus of the present study.

The objective of this model is to determine optimal replenishment quantities in an instantaneous profit maximization multi-product model with publicity effort, functional major ordering cost and units lost due to deterioration.

The above mentioned inventory literatures with deterioration and percentage of on-hand inventory due to deterioration have the basic assumption that the retailer owns a storage room with optimal order quantity. In recent years, companies have started to recognize that a trade-off exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these retail companies have mainly relied on qualitative judgment. This multi-product model postulates that measuring the behaviour of the production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with wasting the percentage of on-hand inventory due to deterioration with functional major ordering cost. Then comparative analysis of the optimal results with none wasting percentage of on-hand inventory with publicity effort cost due to deterioration traditional model is incorporated. The major assumptions used in the above research articles are summarized in Table 1.

Table 1. Summary of the Related Researches

Author(s) and published Year	Structure of the model	Item	Demand	Demand patterns	Publicity Effort Cost	Deterio-ration	Ordering Cost	Planning	Units Lost due to Deterioration	Model
Hariga (1994)	Crisp (EOQ)	Single	Time	Non-stationary	No	Yes	Constant	Finite	No	Cost
Roy et al. (1997)	Fuzzy (EOQ)	Single	Constant (Deterministic)	Constant	No	No	Functional	Infinite	No	Cost
Salameh et al. (1999)	Crisp (EOQ)	Single	Constant (Deterministic)	Constant	No	Yes	Constant	Finite	Yes	Cost
Tsao et al. (2008)	Crisp (EOQ)	Single	Time and Price	Linear and Decreasing	No	Yes	Constant	Finite	No	Profit
Pattnaik (2011)	Crisp (EOQ)	Single	Constant (Deterministic)	Constant	No	Yes	Constant	Finite	No	Profit
Tsao et al. (2012)	Multi-Echelon Supply Chain	Multi	Price	Linear Decreasing	No	No	Constant	Finite	No	Profit
Present Model(2018)	Multi-Product (EOQ)	Multi	Constant (Deterministic)	Constant	Yes	Yes	Functional	Finite	Yes	Profit

The remainder of the model is organized as follows. In Section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in Section 3. Algorithm through steps is outlined in Section 4 to obtain the best solution for the multi-product model. The solution procedure is given in Section 5. In Section 6, numerical example is presented to illustrate the development of the model. The sensitivity analysis is carried out in Section 7 to observe the changes in the optimal solution. Finally Section 8 deals with the summary and the concluding remarks.

ASSUMPTIONS AND NOTATIONS

k Number of items considered,

r_i Consumption rate for item i,

t_{ci} Cycle length for item i,

t_c Cycle length, $t_c = \sum_{i=1}^{k} t_{ci}$

t_c^* Optimal cycle length, $t_c^* = \sum_{i=1}^{k} t_{ci}^*$

h_i Holding cost of item i per unit per unit of time,

HC (q_i, ρ_i) Holding cost per cycle,

ρ_i The publicity effort factor per cycle,

PEC (ρ_i) The publicity effort cost per cycle, $PE\,(\rho_i) = \tau_i(\rho_i - 1)^2 {r_i}^{\beta_i}$ where, $\tau_i > 0$ and β_i is a constant,

c_i Purchasing cost for item i,

p_i Selling Price for item i,

α_i Percentage of on-hand inventory of item i that is lost due to deterioration,

q_i Order quantity of item i, $q = \sum_{i=1}^{k} q_i$

$A \times ({q_i}^{\gamma_i - 1})$ Major ordering cost per cycle where, $0 < \gamma_i < 1$, A is positive constant

a_i Minor ordering cost for item i

q_i^* Traditional optimal ordering quantity for item ,$q^* = \sum_{i=1}^{k} q_i^*$

q_i^{**} Modified optimal ordering quantity for item i,$q^{**} = \sum_{i=1}^{k} q_i^{**}$

$\varphi(t_i)$ On-hand inventory level at time t_i of item i,

$\pi_1(q_i, \rho_i)$ Net profit per cycle

$\pi\,(q_i, \rho_i)$ Average profit per cycle, $\pi\,(q_i, \rho_i) = \frac{\pi_1(q_i)}{t_c}$

$\pi_1^*(q_i, \rho_i)$ Optimal net profit per cycle

$\pi^*(q_i, \rho_i)$ Optimal average profit per cycle

MATHEMATICAL MODEL

Denote $\varphi(t_i)$ as the on-hand inventory level at time t_i, i=1,2,...........k. During a change in time from point t_i to $t_i + \mathrm{d}t_i$, where $t_i + \mathrm{d}t_i > t_i$, the on-hand inventory drops from $\varphi(t_i)$ to $\varphi(t_i + \mathrm{d}t_i)$. Then $\varphi(t_i + \mathrm{d}t_i)$ is:

$$\varphi(t_i + \mathrm{d}t_i) = \varphi(t_i) - r_i\rho_i dt_i - \alpha_i\varphi(t_i)dt_i, i = 1,2, \dots\dots\dots\dots\dots k$$

$\varphi(t_i + \mathrm{d}t_i)$ can be re-written as: $\frac{\varphi(t_i+\mathrm{d}t_i)-\varphi(t_i)}{dt_i} = -r_i\rho_i - \alpha_i\varphi(t_i)$ and $dt_i \to 0$, equation $\frac{\varphi(t_i+\mathrm{d}t_i)-\varphi(t_i)}{dt_i}$ reduces to: $\frac{d\varphi(t_i)}{dt_i} + \alpha_i\varphi(t_i) + r_i\rho_i = 0$

It is a differential equation, solution is $\varphi(\mathrm{t_i}) = \frac{-\mathrm{r_i}\rho_i}{\alpha_i} + \left(\mathrm{q_i} + \frac{\mathrm{r_i}\rho_i}{\alpha_i}\right) \times \mathrm{e}^{-\alpha_i t_i}$

Where $\mathrm{q_i}$is the order quantity which is instantaneously replenished at the beginning of each cycle of length $\mathrm{t_{ci}}$ units of time. The stock is replenished by $\mathrm{q_i}$ units each time these units are totally depleted as a result of outside demand and deterioration. The cycle length of item i, $\mathrm{t_{ci}}$ is determined by first substituting $\mathrm{t_{ci}}$into equation $\varphi(t_i)$ and then setting it equal to zero to get the cycle length: $\mathrm{t_c} = \sum_{i=1}^{k} \mathrm{t_{ci}} = \sum_{i=1}^{k} \frac{1}{\alpha_i} \ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)$.

Here $\varphi(\mathrm{t_i})$ and $\mathrm{t_{ci}}$ are used to develop the mathematical model for item i. Then the total number of units lost per cycle, L, is given as: $L = \sum_{i=1}^{k} L_i = \sum_{i=1}^{k} \mathrm{r_i}\rho_i \left[\frac{\mathrm{q_i}}{\mathrm{r_i}\rho_i} - \frac{1}{\alpha_i} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]$. It is worthy to mention that as $\alpha_i$approaches to zero, t_{ci} approaches to $\frac{\mathrm{q_i}}{\mathrm{r_i}\rho_i}$ and the cycle length is $t_c = \sum_{i=1}^{k} t_{ci} = \sum_{i=1}^{k} \frac{\mathrm{q_i}}{\mathrm{r_i}\rho_i}$ and L, the number of units lost per cycle due to deterioration is zero.

The total cost per cycle is $TC(\mathrm{q_i}, \rho_i)$ which is the sum of the major ordering cost per order, minor ordering cost per order, the holding cost, purchasing cost and publicity effort cost per cycle respectively.

$\mathrm{HC}(\mathrm{q_i}, \rho_i)$ is obtained from equation $\varphi(\mathrm{t_i})$ as:

$$HC = \sum_{i=1}^{k} \int_0^{t_{ci}} h_i\varphi(t_i)dt_i = \sum_{i=1}^{k} h_i \int_0^{\frac{1}{\alpha_i} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)} \left[-\frac{r_i\rho_i}{\alpha_i} + \left(q_i + \frac{r_i\rho_i}{\alpha_i}\right) \times \mathrm{e}^{-\alpha_i t_i}\right] dt_i$$

$$\text{So, } HC = \sum_{i=1}^{k} \mathrm{h_i} \left[\frac{\mathrm{q_i}}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]$$

$TC(q_i,\rho_i) = Major\ Ordering\ Cost\ (MAOC) + Minor\ Ordering\ Cost\ (MOC) + Holding\ Cost\ (HC) + Purchasing\ Cost\ (PC) + Publicity\ Effort\ Cost\ (PEC)$

$$TC(q_i,\rho_i) = \sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + h_i\left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right] + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]$$

Where, $MAOC = \sum_{i=1}^{k} A\times q_i^{(\gamma_i-1)} A,\ MOC = \sum_{i=1}^{k} a_i,\ HC = \sum_{i=1}^{k} h_i\left[\frac{q_i}{\alpha_i} - \frac{r_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i}{r_i}\right)\right], PC = \sum_{i=1}^{k} c_i q_i$ and and $PEC = \sum_{i=1}^{k}\tau_i(\rho_i-1)^2 r_i^{\beta_i}$

The total cost per unit of time, $TCU(q_i,\rho_i)$, is given by dividing equation $TC(q_i,\rho_i)$ by t_c to give:

$$TCU(q_i,\rho_i) = TC(q_i,\rho_i)\times\sum_{i=1}^{k}\left[\frac{1}{\alpha_i}\ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]^{-1} = \left[\sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + h_i\left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right] + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]\right]\times\sum_{i=1}^{k}\left[\frac{1}{\alpha_i}\ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right]^{-1}$$

As α_i approaches zero in equation $TCU(q_i,\rho_i)$reduces to $TCU(q_i,\rho_i) = \frac{TC(q_i)}{t_c} = \frac{\sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]}{\sum_{i=1}^{k}\frac{q_i}{r_i\rho_i}}$, where, $TC(q_i,\rho_i) = \sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]$.

The average profit $\pi(q_i,\rho_i)$ per unit time is obtained by dividing t_c in $\pi_1(q_i,\rho_i)$. The total profit per cycle is $\pi_1(q_i,\rho_i)$.

$$\pi_1(q_i,\rho_i) = Sales\ Revenue\ (SR) - Total\ Cost\ (TC) = \sum_{i=1}^{k}[(q_i - L_i)\times p_i] - \sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right] = \sum_{i=1}^{k}[(q_i)\times p_i] - \sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + \frac{h_i q_i^2}{2r_i\rho_i} + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]$$

where, $L_i$is the number of units lost per cycle due to deterioration.

As α_i is not approaching to zero in equation $TCU(q_i,\rho_i)$ the average profit $\pi(q_i,\rho_i)$ per unit time is obtained by dividing t_c in $\pi_1(q_i,\rho_i)$. The total profit per cycle is $\pi_1(q_i,\rho_i)$.

$$\pi_1(q_i,\rho_i) = Sales\ Revenue\ (SR) - Total\ Cost\ (TC) = \sum_{i=1}^{k}[(q_i - L_i)\times p_i] - \sum_{i=1}^{k}\left[A\times q_i^{(\gamma_i-1)} + a_i + h_i\left[\frac{q_i}{\alpha_i} - \frac{r_i\rho_i}{\alpha_i^2} ln\left(\frac{\alpha_i q_i + r_i\rho_i}{r_i\rho_i}\right)\right] + c_i q_i + \tau_i(\rho_i-1)^2 r_i^{\beta_i}\right]$$

Hence the profit maximization problem is:

Maximize $\pi_1(q_i,\rho_i)$

$\forall\ q_i \geq 0\ and\ \rho_i \geq 0\ for\ i = 1,2,3\ \ldots..k$

ALGORITHM

Step 1: Set numerical values for the inventory parameters.

Step 2: Set $\frac{d\pi_1(q_i,\rho_i)}{dq_i} = 0\ and\ \frac{d\pi_1(q_i,\rho_i)}{d\rho_i} i = 1,2,3 \ldots \ldots k$ and solve by Lingo 13.0 for q_i and ρ_i.

Find out the appropriate scenario and for that obtain multi-product profit per cycle.

Step 3: Check sufficiency condition graphically.

OPTIMIZATION

The optimal ordering quantity q_i and publicity effort ρ_i per cycle can be determined by differentiating equation $\pi_1(q_i,\rho_i)$ with respect to q_i and ρ_i separately, setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is jointly concave in terms of ordering quantity q_i and promotional effort factor ρ_i. The second partial derivates of equation $\pi_1(q_i,\rho_i)$with respect to q_i and ρ_i are strictly negative and the determinant of Hessian matrix is positive. Considering the following propositions:

Proposition 1. The net profit $\pi_1(q_i,\rho_i)$ per cycle is concave in q_i.

Conditions for optimal q_i is:

$$\frac{\partial\pi_1(q_i,\rho_i)}{\partial q_i} = \frac{r_i\rho_i}{(\alpha_i q_i + r_i\rho_i)\alpha_i}(\alpha_i p_i + h_i) - \left(A(\gamma_i-1)q_i^{\tau_i-2} + c_i + \frac{h_i}{\alpha_i}\right) = 0$$

The second order partial derivative of the net profit per cycle with respect to q_i can be expressed as:

$\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i^2} = -\frac{r_i\rho_i}{(\alpha_i q_i + r_i\rho_i)^2}(p_i\alpha_i + h_i) - \left(A(\gamma_i - 1)(\gamma_i - 2){q_i}^{\gamma_i - 3}\right) = -\left[\frac{r_i\rho_i}{(\alpha_i q_i + r_i\rho_i)^2}(p_i\alpha_i + h_i) + \left(A(\gamma_i - 1)(\gamma_i - 2){q_i}^{\gamma_i - 3}\right)\right] < 0$

Since $r_i\rho_i > 0$, $(\gamma_i - 1)(\gamma_i - 2) > 0$ and $(p_i\alpha_i + h_i\alpha_i) > 0$ so, $\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i^2}$ is strictly negative.

Proposition 2. The net profit $\pi_1(q_i,\rho_i)$ per cycle is concave in ρ_i.

Conditions for optimal ρ_i is:

$\frac{\partial \pi_1(q_i,\rho_i)}{\partial \rho_i} = \left(\frac{1}{\alpha_i} ln\left(\frac{\alpha_i q_i}{r_i\rho_i} + 1\right) - \left(\frac{q_i}{(\alpha_i q_i + r_i\rho_i)}\right)\right)(\frac{r_i}{\alpha_i} \times (\alpha_i p_i + h_i)) - 2\tau_i(\rho_i - 1){r_i}^{\beta_i} = 0$

The second order partial derivative of the net profit per cycle with respect to ρ_i is

$\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial \rho_i^2} = -\frac{r_i{q_i}^2}{(r_i\rho_i + \alpha_i q_i)^2}(\alpha_i p_i + h_i) - 2\tau_i{r_i}^{\beta_i}$

Since $(p_i\alpha_i + h_i\alpha_i)>0$, $\tau_i > 0$, $r_i>0$, it is found that $\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial \rho_i^2}$ is strictly negative.

Propositions 1 and 2 show that the second partial derivatives of equation π_1 $(q_i,\rho_i)$with respect to q_i and ρ_i separately are strictly negative. The next step is to check that the determinant of the Hessian matrix is positive, i.e.

$\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i^2} \times \frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial \rho_i^2} - \left(\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i \partial \rho_i}\right)^2 > 0$

$\left(\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i^2}\right), \left(\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial \rho_i^2}\right)$ shown in $\frac{\partial \pi_1(q_i,\rho_i)}{\partial q_i}$ and $\frac{\partial \pi_1(q_i,\rho_i)}{\partial \rho_i}$

and $\frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial q_i \partial \rho_i} = \frac{\partial^2 \pi_1(q_i,\rho_i)}{\partial \rho_i \partial q_i} = \frac{r_i q_i}{(\alpha_i q_i + r_i\rho_i)^2}(\alpha_i p_i + h_i)$

The net profit per unit time we have the following maximization problem.

Maximize $\pi_1(q_i,\rho_i)$

Subject to

$\left[\left(\frac{r_i(\alpha_i p_i + h_i)^2}{(r_i\rho_i + \alpha_i q_i)^2}\right)\left[2\tau_i{r_i}^{\beta_i}\rho_i + A{q_i}^{\gamma_i - 1}(\gamma_i - 1)(\gamma_i - 2) + \frac{\rho_i r_i {q_i}^2}{(r_i\rho_i + \alpha_i q_i)^2} - r_i{q_i}^2\right] + 2\tau_i{r_i}^{\alpha_1}A(\gamma_i - 1)(\gamma_i - 2){q_i}^{(\gamma_i - 3)}\right] > 0$

$\forall q_i, \rho_i \geq 0$

The objective is to determine the optimal values of q_i and ρ_i to maximize the net profit function. It is very difficult to derive the optimal values of q_i and ρ_i, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here Lingo 13.0 software is used to derive the optimal values of the decision variables.

NUMERICAL EXAMPLE

We consider ten different items that need to be replenished jointly, namely items 1-10. The model is illustrated through the numerical example where the numerical data is given in Table 2.

Table 2. Numerical Data for the Example

$p_1 = Rs.125, p_2 = Rs.126, p_3 = Rs.127, p_4 = Rs.128, p_5 = Rs.129, p_6 = Rs.130, p_7 = Rs.131, p_8 = Rs.132, p_9 = Rs.133, p_{10} = Rs.134$ *per unit*
$c_1 = Rs.100, c_2 = Rs.102, c_3 = Rs.104, c_4 = Rs.106, c_5 = Rs.108, c_6 = Rs.109, c_7 = Rs.110, c_8 = Rs.112, c_9 = Rs.115, c_{10} = Rs.116$ *per unit*
$h_1 = Rs.5, h_2 = Rs.5.5, h_3 = Rs.6, h_4 = Rs.6.5, h_5 = Rs.7, h_6 = Rs.7.1, h_7 = Rs.7.2, h_8 = Rs.7.3, h_9 = Rs.7.4, h_{10} = Rs.7.5$ *per unit per unit of time*
$r_1 = 1000, r_2 = 1050, r_3 = 1100, r_4 = 1150, r_5 = 1200, r_6 = 1210, r_7 = 1220, r_8 = 1225, r_9 = 1230, r_{10} = 1235$ *units per unit of time*
$\alpha_1 = 0.01, \alpha_2 = 0.02, \alpha_3 = 0.03, \alpha_4 = 0.04, \alpha_5 = 0.05, \alpha_6 = 0.06, \alpha_7 = 0.07, \alpha_8 = 0.08, \alpha_9 = 0.09, \alpha_{10} = 0.1$
$\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = \beta_9 = \beta_{10} = 2$
$a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = a_9 = a_{10} = Rs.1$ *per item*
$\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \gamma_8 = \gamma_9 = \gamma_{10} = 0.5$
$\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6 = \tau_7 = \tau_8 = \tau_9 = \tau_{10} = 2$
$A = Rs.200$ *per order*

Fig. 1 represents the relationship between the order quantity q_i and fuctional major ordering cost MAOC. Fig. 2 shows the relationship between the order quantity q_i and units lost per cycle due to deterioration L_i, Fig. 3 to Fig. 12 represent the three dimensional mesh plot of order quantity q_i, publicity effort factor ρ_i and net profit per cycle π_1. These figures show concavity of the net profit function per cycle. The optimal solution that maximizes $\pi_1(q_i, \rho_i)$, t_c^*,q_i^{**}, ρ_i^*,L^*, MAOC and PEC are determined by using Lingo 13.0 version software and the results are tabulated in Table 3.

Table 3. Optimal Values of the Proposed Model

Model	Iteration	q_i^{**}	t_c^*	L^*	MAOC	ρ_i^*	PEC	π_1^*	π^*
Multi-product	5378	4220.248,3372.022,2792.053,2367.018,2039.698,1869.063,1724.64,1510.5711,1249.742,1165.991	8.2756688	12968.66	44.94955	1.012844,1.008908,1.006432,1.004770,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	795.1205	240644.8	29078.59
Multi-product	865	4220.182,3371.945,2791.964,2366.916,2039.583,1868.942,1724.514,1510.429,1249.570,1165.813	8.275271	12967.95	-	1.012844,1.008907,1.006431,1.004770,1.003604,1.003242,1.002937,1.002432,1.001801,1.001665	795.0252	240489.8	29061.25675
% Change	-	-	0.0048	.0055	-	-	0.012	0.0644	0.0596
Multi-product	58	4166.667,3342.175,2774.123,2355.68,2032.258,1862.903,1719.463,1506.765,1247.324,1163.874	8.275271	12887.7	-	-	-	239694.7	28965.1783
% Change	-	-	0.0481	0.6243	-	-	-	0.3948	0.39

Table 4. Comparative Analysis of a Single-Product Model and Multi-Product Model

Model	Item	Iteration	t_c^*	q_i^*	L^*	MAOC	ρ_i^*	PEC	π_1^*	π^*
Single-product	1	96	4.08226	4220.248	85.55468	3.078655	1.012844	329.9389	51701.5	12664.92
	2	353	3.08588	3372.022	102.9863	3.444172	1.008908	174.9594	39452.64	12784.89
	3	117	2.431157	2792.053	100.5812	3.785020	1.006432	100.1039	31223.28	12842.97
	4	89	1.968897	2367.018	91.98498	4.110827	1.00477	60.18685	25288.46	12843.97
	5	90	1.625741	2032.373	81.48381	4.436376	1	0	20756.24	12767.25
	6	103	1.472667	1869.063	81.35932	4.6026132	1.003242	30.77834	19010.69	12909.03
	7	85	1.344225	1724.641	79.86835	4.815936	1.002938	25.69237	17509.16	13025.47
	8	66	1.173299	1506.907	69.61584	5.15213	1	0	14591.24	12436.09
	9	98	1.001802	1249.742	53.78885	5.657437	1.001802	9.820012	10903.33	11233.93
	10	98	0.9007397	1165.991	51.7245	5.857096	1.001666	8.465561	10163.10	11283.06
Total	-	-	19.08667	22300.06	798.9478	44.94026	10.0426	739.9453	240599.6	124791.6
Muti-product	1-10	5378	8.2756688	4220.248,3372.022,2792.053,2367.018,2039.698,1869.063,1510.5711,1249.742,1165.991	12968.66	44.94955	1.012844,1.008908,1.006432,1.004770,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	795.1205	240644.8	29078.59
% Change	-	-	130.6359	-	93.8394	0.0207	-	6.9392	0.0188	329.1529

Comparative analysis of a multi-product model with and without publicity effort cost and for fixed major ordering cost with the present multi-product model is shown in Table 3. It is observed that the multi-product net profit per cycle of the present model is 6.44% and 1.88% more than that of the multi-product model with publicity and fixed major ordering cost and the other multi-product model without publicity policy and fixed major ordering cost respectively. So, considerable savings can be realized during the replenishment by the ordering of several different multi-items with implication of publicity policy with functional ordering cost. So, multi-product retailers' publicity and ordering multi-product strategies for deteriorating items are widely used in the real world for retailers' perspective. It indicates the present model incorporated with publicity effort cost, functional major ordering cost and units lost due to deterioration may draw the better decisions in managerial uncertain space with retailer's perspective.

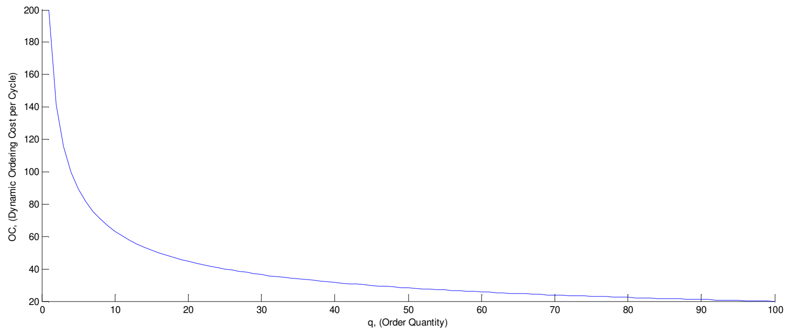

Fig. 1. Two dimensional plot of Order Quantity, q_i and Functional Major Ordering Cost

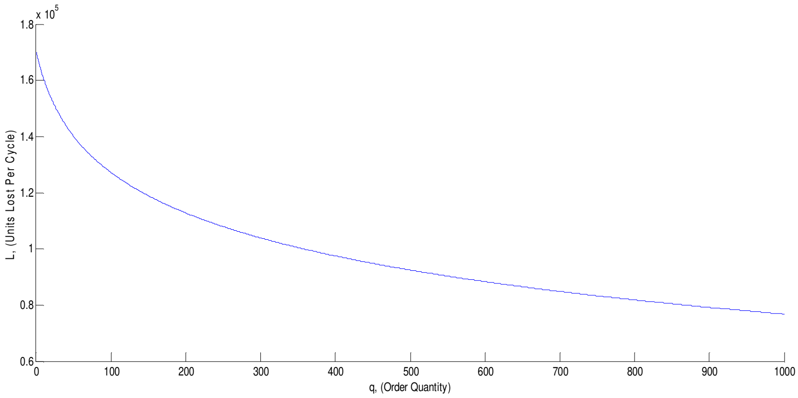

Fig. 2. Two Dimensional Plot of Order Quantity q_i and Units Lost per Cycle L_i

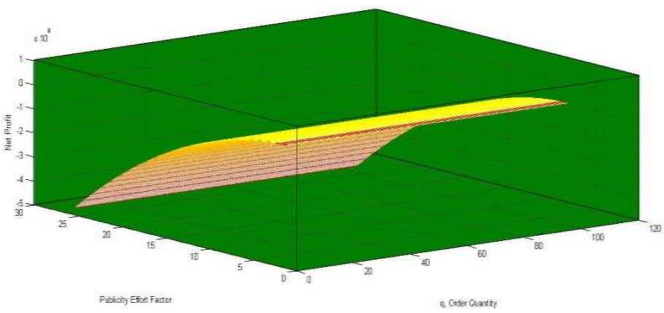

Fig. 3. Two Three Dimensional Mesh Plot of Order Quantity q_1, Publicity Effort Factor ρ_1 and Net Profit per Cycle π_1 (q_1,ρ_1)

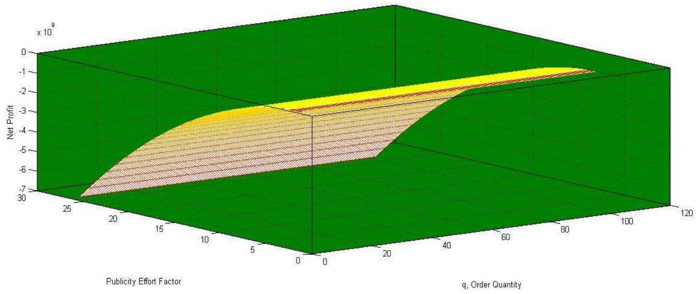

Fig. 4. Two Three Dimensional Mesh Plot of Order Quantity q_2, Publicity Effort Factor ρ_2 and Net Profit per Cycle π_1 (q_2,ρ_2)

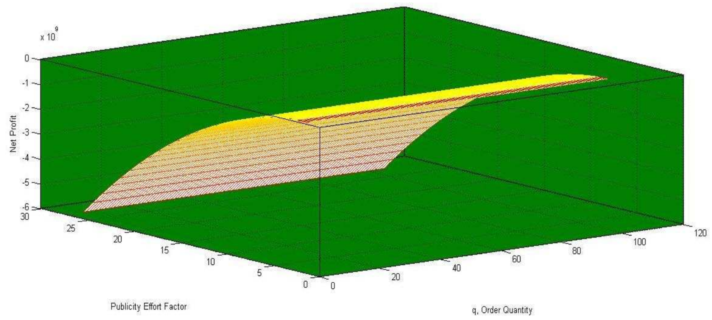

Fig. 5. Two Three Dimensional Mesh Plot of Order Quantity q_3, Publicity Effort Factor ρ_3 and Net Profit per Cycle $\pi_1(q_3,\rho_3)$

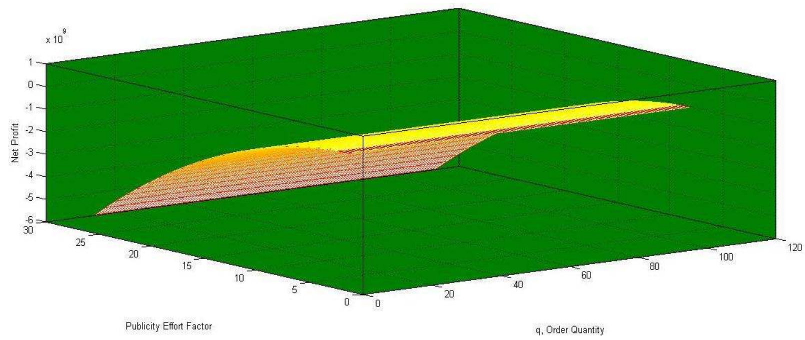

Fig. 6. Two Three Dimensional Mesh Plot of Order Quantity q_4, Publicity Effort Factor ρ_4 and Net Profit per Cycle $\pi_1(q_4,\rho_4)$

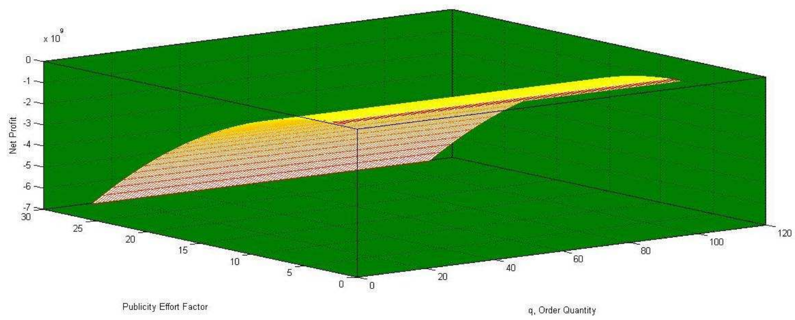

Fig. 7. Two Three Dimensional Mesh Plot of Order Quantity q_5, Publicity Effort Factor ρ_5 and Net Profit per Cycle $\pi_1(q_5,\rho_5)$

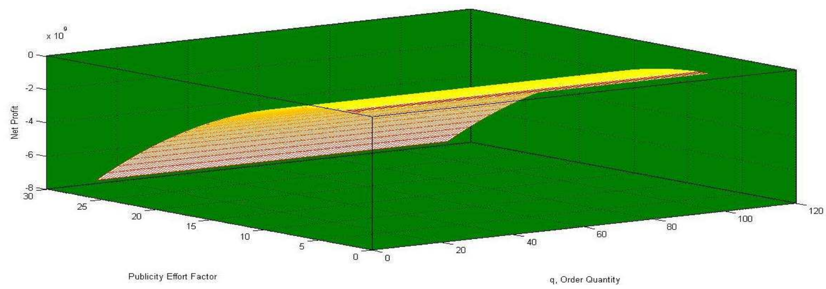

Fig. 8. Two Three Dimensional Mesh Plot of Order Quantity q_6, Publicity Effort Factor ρ_6 and Net Profit per Cycle $\pi_1(q_6,\rho_6)$

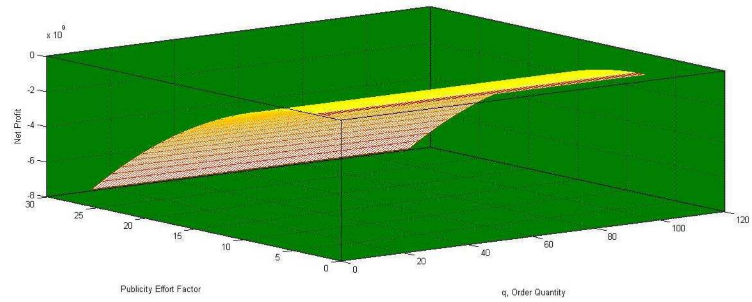

Fig. 9. Two Three Dimensional Mesh Plot of Order Quantity q_7, Publicity Effort Factor ρ_7 and Net Profit per Cycle $\pi_1 (q_7,\rho_7)$

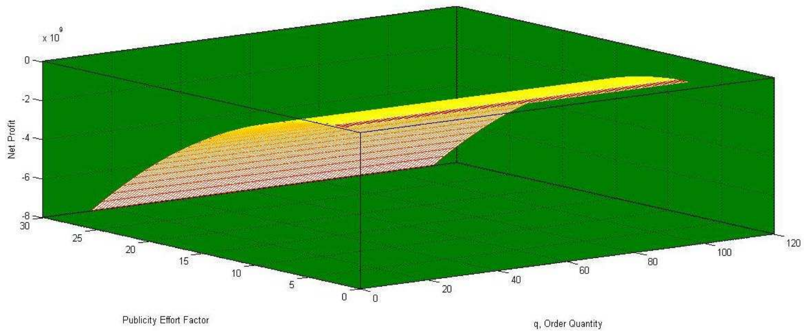

Fig. 10. Two Three Dimensional Mesh Plot of Order Quantity q_8, Publicity Effort Factor ρ_8 and Net Profit per Cycle $\pi_1 (q_8,\rho_8)$

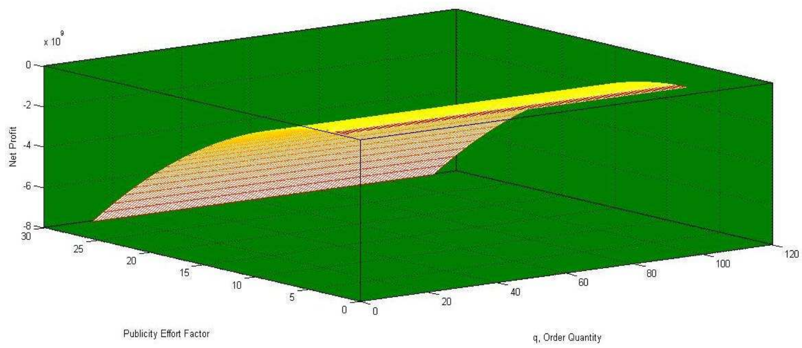

Fig. 11. Two Three Dimensional Mesh Plot of Order Quantity q_9, Publicity Effort Factor ρ_9 and Net Profit per Cycle $\pi_1 (q_9,\rho_9)$

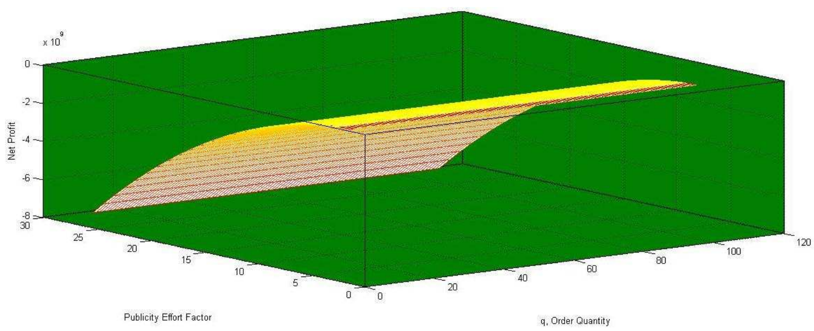

Fig. 12. Two Three Dimensional Mesh Plot of Order Quantity q_{10}, Publicity Effort Factor ρ_{10} and Net Profit per Cycle $\pi_1 (q_{10},\rho_{10})$

SENSITIVITY ANALYSIS

It is interesting to investigate the influence of the major inventory parameters, $p_i, r_i, h_i, c_i, a_i, A, \gamma_i, \tau_i, \beta_i and\ \alpha_i$ on retailers' perspective multi-product order quantity model. The computational results shown in Table 5 indicate the following managerial phenomena:

- $q_i, i = 1,2, \ldots 10$ order quantities, t_c the cycle length, L total units lost due to deterioration and ρ_i the publicity effort factor are highly sensitive, MAOC functional major ordering cost is sensitive, PEC publicity effort cost per cycle is highly sensitive, π_1 the net profit per cycle and π the average profit per cycle are highly sensitive to the parameter p_i selling price for item i.
- $q_i, i = 1,2, \ldots 10$ order quantities are sensitive, t_c the cycle length is insensitive L total units lost due to deterioration is sensitive, ρ_i the publicity effort factor are insensitive, MAOC functional major ordering cost is moderately sensitive, PEC publicity effort cost per cycle is insensitive, π_1 the net profit per cycle and π the average profit per cycle are sensitive to the parameter $r_i$the consumption rate for item i.
- $q_i, i = 1,2, \ldots 10$ order quantities and t_c the cycle length, L total units lost due to deterioration is sensitive, ρ_i the publicity effort factor are insensitive, MAOC functional major ordering cost, PEC publicity effort cost per cycle and π_1 the net profit per cycle are sensitive and π the average profit per cycle is moderately sensitive to the parameter $h_i$holding cost of item i per unit per unit of time.
- $q_i, i = 1,2, \ldots 10$ order quantities, t_c the cycle length, L total units lost due to deterioration, ρ_i the publicity effort factor, MAOC functional major ordering cost, PEC publicity effort cost per cycle, π_1 the net profit per cycle and π the average profit per cycle are sensitive to the parameter c_i purchasing cost for item i.
- $q_i, i = 1,2, \ldots 10$ order quantities are insensitive, t_c the cycle length, L total units lost due to deterioration, ρ_i the publicity effort factor, MAOC functional major ordering cost, PEC publicity effort cost per cycle are insensitive, π_1 the net profit per cycle and π the average profit per cycle is moderately sensitive to the parameter a_i minor ordering cost of item i.
- $q_i, i = 1,2, \ldots 10$ order quantities, t_c the cycle length, L total units lost due to deterioration, ρ_i the publicity effort factor are insensitive, MAOC functional major ordering cost is highly sensitive, PEC publicity effort cost per cycle is insensitive, π_1 the net profit per cycleand π the average profit per cycle are moderately sensitive to the parameter A major ordering cost per order.
- $q_i, i = 1,2, \ldots 10$ order quantities and t_c the cycle length are insensitive, L total units lost due to deterioration is moderately sensitive, ρ_i the publicity effort factor is insensitive, MAOC functional major ordering cost is highly sensitive, PEC publicity effort cost per cycle is insensitive, π_1 the net profit per cycle and π the average profit per cycle is moderately sensitive to the parameter γ_i of the publicity effort cost per cycle.
- $q_i, i = 1,2, \ldots 10$ order quantities are sensitive, t_c the cycle length, L total units lost due to deterioration, ρ_i the publicity effort factor are insensitive, MAOC functional major ordering cost is moderately sensitive, PEC publicity effort cost per cycle is highly sensitive, π_1 the net profit per cycle and π the average profit per cycle are moderately sensitive to the parameter τ_i of the publicity effort cost per cycle.
- $q_i, i = 1,2, \ldots 10$ order quantities are highly sensitive, t_c the cycle length is insensitive, L total units lost due to deterioration is highly sensitive, ρ_i the publicity effort factor, MAOC functional major ordering cost, PEC publicity effort cost per cycle, π_1 the net profit per cycle and π the average profit per cycle are highly sensitive to the parameter β_i of the publicity effort cost per cycle.
- $q_i, i = 1,2, \ldots 10$ order quantities are sensitive, t_c the cycle length, L total units

lost due to deterioration are sensitive, ρ_i the publicity effort factor are insensitive, MAOC functional major ordering cost and PEC publicity effort cost per cycle are sensitive, π_1 the net profit per cycle is highly sensitive and π the average profit per cycle is moderately sensitive to the parameter α_i of the percentage of units lost due to deterioration.

Fig. 13 is about net profit per cycle variations with respect to inventory parameters. The profit increases slightly with increase in per unit selling price, consumption rate and one parameter of publicity effort cost and then decreasing and the profit increases slightly with increase in consumption rate. The profit decreases with increase in holding cost per unit per unit time, purchasing cost per unit for item i and the profit decreases slightly with increase in one parameter of publicity effort cost, minor ordering cost, parameter of functional major ordering cost, MAOC and percentage of units lost due to deterioration respectively. This suggests that the retailer should work on the holding cost per unit per unit of time, purchasing cost per unit, minor ordering cost, parameters of publicity cost function, functional major ordering cost and percentage of units lost due to deterioration for item i. The retailer should put large order with implementing publicity strategy and implementation of appropriate preservation technology to save in ordering cost and wastage cost as a result profit of retailers can be increased significantly.

Table 4. Sensitivity Analyses of the Significant Parameters

Parameter	Value	Iteration	t_c^*	L^*	q_i^*	ρ_i^*	MAOC	PEC	π_1^*	π^*	% Change
p_i	126,127,128,129,130,131,132,133,134,135	6323	11.83294	19218.46	5979.433,4818.722,4032.471,3460.682,3023.335,2769.234,2554.343,2272.408,1948.699,1817.747	1.025039,1.013727,1.013103,1.009975,1.007756,1.006969,1.006309,1.005389,1.004291,1.003966	36.82914	3250.313	503265	42530.85	109.1319
	127, 128,129,130,131,132,133,134,135,136	301	15.28144	25725.07	7808.809,6307.79,5300.66,4573.65,4020.962,3680.894,3393.627,3042.034,2653.794,2474.798	1.041171,1.029443,1.022024,1.016989,1.013402,1.012029,1.010882,1.003546,1.00201,1.000231	31.89842	9140.703	860131.6	56286.03	257.4279
	128,129,130,131,132,133,134,135,136,137	1206	18.60528	32514.9	9727.495,7850697,6604.34,5711.394,5036.643,4607.349,4245.226,3821.727,3366.939,3138.779	1.061179,1.043987,1.033127,1.025753,1.02049,1.018372,1.016607,1.014541,1.012235,1.011299	28.47849	20657.67	1310622	7443.55	444.6293
r_i	1100,1150,1200,1250,1300,1310,1320,1325,1330,1335	3031	8.293054	14116.24	4636.911,3690.321,3044.242,2571.855,2209.049,2023.018,1865.575,1633.567,1351.144,1260.225	1.002995,1.002715,1.002249,1.001666,1.001541,1.011676,1.008133,1.005896,1.004389,1.003327	43.15352	795.1090	262181.9	31614.64	8.949747
	1200,1250,1300,1350,1400,1410,1420,1425,1430,1435	4715	8.275588	15263.37	5053.575,4008.621,3296.432,2776.693,2378.399,2176.972,2006.51,1756.563,1452.547,1354.46	1.010703,1.007482,1.005442,1.004063,1.003090,1.002782,1.002524,1.002091,1.001549,1.001434	41.55751	795.0998	283718.7	34283.81	17.89937
	1300,1350,1400,1450,1500,1510,1520,1525,1530,1535	7646	8.275555	16410.73	5470.239,4326.92,3548.622,2981.532,2547.75,2330.927,2147.446,1879.56,1553.95,1448.695	1.00988,1.006928,1.005053,1.003783,1.002884,1.002598,1.002358,1.001954,1.001448,1.00134	40.12685	795.0921	305255.3	36886.38	26.84891

h_i	6,6.5,7,7.5,8,8.1,8.2,8.3,8.4,8.5	2139	7.506565	11731.4	3610.892,2974.163,2514.617,2164.556,1887.001,1741.033,1615.895,1422.87,1182.992,1107.905	1.011031,1.007884,1.00581,1.004374,1.003342,1.003027,1.002758,1.002295,1.001708,1.001585	46.78103	626.6265	217878.5	29025.06	-9.46054
	7,7.5,8,8.5,9,9.1,9.2,9.3,9.4,9.5	9285	6.881033	10729.44	3155.279,2660.271,2287.33,1993.999,1755.575,1629.419,1520.051,1344.795,1123.012,1055.333	1.009666,1.007071,1.005298,1.004038,1.003116,1.002838,1.0026,1.002173,1.001624,1.001512	48.53508	508.9365	199416.5	28980.61	-17.1324
	8,8.5,9,9.5,10,10.1,10.2,10.3,10.4,10.5	4293	6.359818	9897.398	2801.749,2406.306,2097.722,1848.357,1641.264,1531.254,1434.639,1274.843,1068.822,1007.526	1.008602,1.006410,1.004869,1.003750,1.002918,1.002672,1.002458,1.002063,1.001548,1.001446	50.221701	422.9443	184073.6	28943.22	-23.5082
c_i	101,103,105,107,109,111,113,116,117	5583	8.14151278	12792.4	4040.63,3220.613,2660.433,2250.081,1934.12,1771.732,1634.392,1427.732,1634.392,1427.687,1174.149,1095.302	1.011824,1.008167,1.005872,1.004336,1.003261,1.002932,1.002657,1.002188,1.001602,1.001481,	46.18363	665.4669	218885.5	26885.11	-9.04208
	102,104,106,108,110,112,114,117,118	4425	4.803439	14681.21	3862.113,3070.311,2529.875,2134.149,1829.485,1675.333,1545.052,1345.667,1099.358,1025.384	1.010847,1.007459,1.005339,1.003923,1.002935,1.002639,1.002390, 1.002188,1.001414,1.001307	47.51344	552.6636	198223.4	41266.98	-17.6282
	103,105,107,109,111,113,115,118,119	6077	7.13019	11039.96	3684.672,2921.093,2433.359,2019.202,1725.781,1579.849,1456.608,1264.497,1025.355,956.2227	1.009914,1.006784,1.004832,1.003532,1.002628,1.002362,1.002139,1.001741,1.001239,1.001145	48.95266	455.1292	178649	25055.29	-25.7624
a_i	2,2,2,2,2,2,2,2,2,2	3146	8.275669	12968.66	4220.248,3372.022,2792.053,2367.018,2039.698,1869.063,1724.641,1510.571,1249.742,1165.991	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	44.94955	795.1205	240634.8	29077.38	-0.00416
	5,5,5,5,5,5,5,5,5,5	3637	8.275669	12968.66	4220.247,3372.022,2792.056,2367.017,2039.701,1869.063,1724.64,1510.568,1249.742,1165.99	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	44.94955	795.1168	240604.8	29073.76	-0.01662
	50,50,50,50,50,50,50,50,50,50	6167	8.275669	12968.66	4220.248,3372.022,2792.053,2367.017,2039.698,1869.063,1724.64,1510.571,1249.743,1165.991	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,29076.291.002432,1.001802,1.001666	44.94955	795.1204	240154.8	29019.38	-0.20362
A	300	7547	8.27555	12968.42	4220.281,3372.061,2792.098,2367.069,2039.756,1869.124,1724.704,1510.642571,1249.828,1165.08	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	67.42302	795.1681	240622.3	29076.29	-0.00935
	500	190	8.275951	12969.13	4220.346,3372.139,2792.188,2367.171,2039.871,1869.245,1724.83,1510.784,1250,1165.259	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	112.3673	795.2633	240577.4	29069.46	-0.02801
	800	335	8.277115	12971.25	4220.445,3372.255,2792.322,2367.324,2040.043,1869.426,1725.02,1510.997,1250.997,1166.526	1.012844,1.008908,1.006432,1.00477,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	179.7773	795.4062	240510	29057.23	-0.05602

γ_i	.6,.6,.6,.6,.6,.6,.6,.6,.6,.6	2062	8.275925	12969.16	4220.303,3372.085,2792.122,2367.093,2039.78,1869.148,1724.727,1510.665,1249.851,1166.102	1.012844,1.008928,1.006432,1.004771,1.003605,1.003242,1.002938,1.002432,1.001802,1.001666	95.73294	795.1918	240594.0	29071.55	-0.02111
	.7,.7,.7,.7,.7,.7,.7,.7,.7,.7	207	8.276355	12969.88	4220.391,3372.181,2792.226,2367.205,2039.9,1869.27,1724.851,1510.797,1250,1166.252	1.012845,1.008908,1.006432,1.004771,1.003605,1.003243,1.002939,1.002433,1.001802,1.001667	204.1958	795.2987	240485.6	29056.95	-0.06616
	.8,.8,.8,.8,.8,.8,.8,.8,.8,.8	286	8.276811	12970.68	4220.504,3372.3,2792.351,2367.335,2040.030,1869.406,1724.988,1510.939,1250.155,1166.407	1.012846,1.008909,1.006433,1.004771,1.003606,1.003243,1.002939,1.002433,1.001803,1.001667	436.2109	795.425	240253.6	29027.31	-0.16256
τ_i	3,3,3,3,3,3,3,3,3,3	441	8.275874	12941.92	4202.409,3362.099,2786.106,2363.272,2037.256,1867.05,1722.957,1509.35,1248.994,1165.345	1.008563,1.005938,1.004288,1.00318,1.002403,1.002161,1.001595,1.001621,1.001201,1.00111	44.98136	530.0805	240379.8	29045.85	-0.11012
	4,4,4,4,4,4,4,4,4,4	674	8.27567	12928.53	4193.489,3357.137,2783.132,2361.399,2036.035,1866.043,1722.115,1508.739,1248.619,1165.022	1.006422,1.004454,1.003216,1.002385,1.001802,1.001621,1.001469,1.001216,1.000901,1.000833	44.99729	397.5605	240247.3	29030.56	-0.16518
	5,5,5,5,5,5,5,5,5,5	10859	8.275671	12920.51	4188.138,3354.16,2781.348,2360.276,2035.303,1865.439,1721.61,1508.372,1248.394,1164.828	1.005138,1.003563,1.002573,1.001908,1.001442,1.001297,1.001175,1.000973,1.000721,1.000666	45.00687	318.0484	240167.8	29020.95	-0.19822
β_i	1,1,1,1,1,1,1,1,1,1	185	8.27531	99319.27	57681.98,34600.44,22399.22,15277.40,10822.13,9170.142,7881.59,5995.372,4010.715,3557.94	13.84367,10.35266,8.074332,6.48533,5.32518,4.922471,4.583716,3.978922,3.215379,3.056892	20.22025	850650.2	1090517	131779.6	353.1646
	2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1,2.1	9152	8.275671	12928.34	4193.553,3357.1,2783.069,2361.335,2035.978,1865.993,1722.071,1508.701,1248.599,1165.004	1.006437,1.004443,1.003193,1.002358,1.001774,1.001594,1.001443,1.001194,1.000884,1.000817	44.99772	395.9313	240245.6	29030.35	-0.16589
	2.3,2.3,2.3,2.3,2.3,2.3,2.3,2.3,2.3,2.3	13870	8.275682	12898.31	4173.469,3345.98,2776.369,2357.155,2033.258,1863.72,1720.214,1507.337,1247.744,1164.28	1.001617,1.001105,1.000787,1.000576,1.000429,1.000385,1.000348,1.000288,1.000213,1.000197	45.03358	98.19572	239947.9	28994.34	-0.2896
α_i	..02,.03,.04,.05,.06,.07,.08,.09,.1,.2	250	7.273427	11545.03	3610.445,2966.957,2504.561,2153.451,1875.738,1730.344,1605.753,1413.016,1173.536,725.08	1.010904,1.007795,1.005746,1.004326,1.003307,1.002995,1.00273,1.002272,1.00169,1.001029	48.30947	608.5429	212206.9	29175.64	-11.8174
	.03,.04,.05,.06,.07,.08,.09,.1,.2,.3	3518	6.404144	10256.46	3154.686,2648.8,2270.76,1975.24,1736.181,1610.797,1502.203,1327.299,729.2793,526.1964	1.009474,1.00693,1.005193,1.003958,1.003054,1.002783,1.002550,1.002131,1.001043,1.000745	52.70174	479.6698	187653.2	29301.84	-22.0207
	.04,.05,.06,.07,.08,.09,.1,.2,.3,.4	2828	5.625639	9064.817	2801.129,2392.284,2076.891,1824.277,1615.957,1506.704,1411.202,826.1996,529.0723,412.9809	1.008376,1.006238,1.004737,1.003648,1.002838,1.002599,10..2393,1.001317,1.000755,1.000584	57.8854	383.6185	165374.9	29396.64	-31.2784

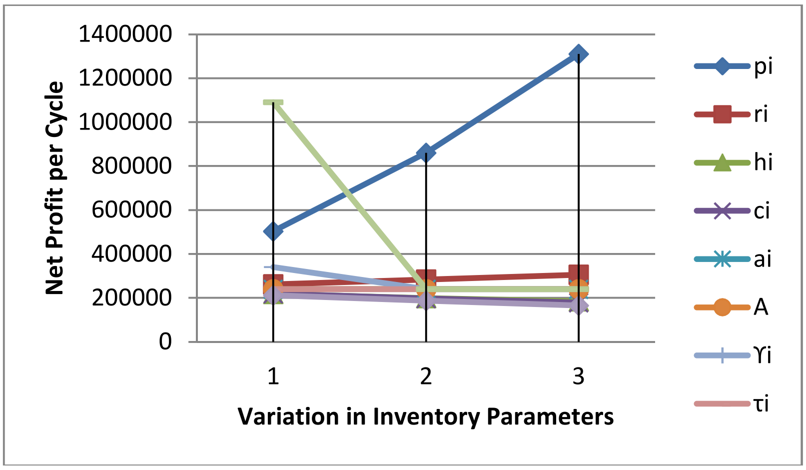

Fig. 13. Changes in Net Profit per Cycle with Variations in Inventory Parameters

CONCLUSION

Recently, research on sales promotions has shed much light on the effects of price promotions. Publicity effort factor plays a significant role in framing the publicity effort cost. In this model, it is analyzed that the effect of publicity effort cost for a modified multi-product EOQ model with a percentage of the on-hand inventory lost due to deterioration and functional major ordering cost as characteristic features and the inventory conditions govern the item stocked. This model provides a useful property for finding the optimal profit and ordering quantity with deteriorated units of lost sales. A new mathematical model with algorithm is developed and compared to the traditional EOQ model numerically. Finally, wasting the percentage of on-hand inventory due to deterioration effect was demonstrated numerically to have an adverse effect on the average profit per unit per cycle. Hence the utilization of units lost due to deterioration and publicity effort cost makes the scope of the applications broader. Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analyses with respect to the major inventory parameters. The model in this study is a general framework that considers wasting/ none wasting the percentage of on-hand multi-product inventory due to deterioration with publicity effort cost and functional major ordering cost simultaneously. To the best of its knowledge, this is the model that investigates the impact of publicity, units lost due to deterioration and functional major ordering cost simultaneously with retailer's perspective.

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REFERENCES

Bose S., Goswami A., Chaudhuri K.S., 1995. An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting, Journal of Operational Research Society, vol. 46, 775-782. www.tandfonline.com/doi/abs/10.1057/jors.1995.107.

Ghorabaee M.K., Amiri M., Olfat L., Firouzabadi S.M.A.L., 2017. Designing a multi-product multi-period supply chain network with reverse logistics and multiple objectives under uncertainty, Technological and Economic Development of Economy, 23(3), 520-548.

http://dx.doi.org/10.3846/20294913.2017.1312630.

Goyal S.K., Gunasekaran A., 1995. An integrated production-inventory-marketing model for deteriorating items, Computers and Industrial Engineering, 28, 755-762. http://dx.doi.org/10.1016/0360-8352(95)00016-T

Gupta D., Gerchak Y., 1995. Joint product durability and lot sizing models, European Journal of Operational Research, 84, 371-384. http://dx.doi.org/10.1016/0377-2217(93)E0273-Z

Hammer M., 2001. The superefficient company, Harvard Business Review, vol. 79, 82-91. https://hbr.org/2001/09/the-superefficient-company.

Hariga M., 1994. Economic analysis of dynamic inventory models with non-stationary costs and demand, International Journal of Production Economics, 36, 255-266. https://doi.org/10.1016/0925-5273(94)00039-5

Hariga M., 1995. An EOQ model for deteriorating items with shortages and time-varying demand, Journal of Operational Research Society, 46, 398-404. http://dx.doi.org/10.1057/jors.1995.54

Hariga M., 1996, Optimal EOQ models for deteriorating items with time-varying demand, Journal of Operational Research Society, 47, 1228-1246. http://dx.doi.org/10.1016/0925-5273(94)00039-5

Jain K., Silver E., 1994. A lot sizing for a product subject to obsolescence or perishability, European Journal of Operational Research, 75, 287-295. http://dx.doi.org/10.1016/0377-2217(94)90075-2

Karimi R., Ghezavati V.R., Damghani K.K., 2015. Optimization of multi-product, multi-period closed loop supply chain under uncertainty in product return rate: case study in Kalleh dairy company, Journal of Industrial and Systems Engineering, 8(3), 95-114. http://www.jise.ir/article_10151.html.

Mishra V.K., 2012. Inventory model for time dependent holding cost and deterioration with salvage value and shortages, The Journal of Mathematics and Computer Science, 4(1), 37-47. www.isr-publications.com/.../download-inventory-model-for-time-dependent-holding.

Padmanabhan G., Vrat P., 1995. EOQ models for perishable items under stock dependent selling rate, European Journal of Operational Research, 86, 281-292. http://dx.doi.org/10.1016/0377-2217(94)00103-J

Pattnaik M., 2011. A note on non linear optimal inventory policy involving instant deterioration of perishable items with price discounts, The Journal of Mathematics and Computer Science, 3(2), 145-155. www.isr-publications.com/.../download-a-note-on-non-linear-optimal-inventory-polic.. .

Pattnaik M., 2012. Models of Inventory Control, Lambart Academic Publishing Company, Germany.

Pattnaik M., 2012. An EOQ model for perishable items with constant demand and instant Deterioration, Decision, vol. 39(1), 55-61. www.sciepub.com/reference/77772.

Roy T.K., Maiti M., 1997. A Fuzzy EOQ model with demand dependent unit cost under limited storage capacity, European Journal of Operational Research, 99, 425 – 432. http://dx.doi.org/10.1016/S0377-2217(96)00163-4

Salameh M.K., Jaber M.Y., Noueihed, N., 1993. Effect of deteriorating items on the instantaneous replenishment model, Production Planning and Control, 10(2), 175-180. http://dx.doi.org/10.1080/095372899233325

Sana S.S., 2015. An EOQ model for stochastic demand for limited capacity of own warehouse. Ann Oper Res, 233, 1, 383–399. http://dx.doi.org/10.1007/s10479-013-1510-5

Tabatabaei M.S.R., Sadjadi S.J., Makui A. 2017. Optimal pricing and marketing planning for deteriorating items. PLoS ONE 12(3): e0172758. http://dx.doi.org/10.1371/journal.pone.0172758

Tsao Y.C., Sheen G.J., 2008. Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment, Computers and Operations Research, 35, 3562-3580. www.sciepub.com/reference/77782.

Tsao Y.C., Sheen G.J., 2012, A multi-item supply chain with credit periods and weight freight cost discounts, International Journal of Production Economics, 135, 106-115. http://dx.doi.org/10.1016/j.ijpe.2010.11.013

WPŁYW WYDATKÓW I ZMIENNYCH KOSZTÓW W MODELU ZAMAWIANIA WIELO-ASORTYMENTOWYM NA STRATY SPRZEDAŻY W WYNIKU NISZCZENIA

STRESZCZENIE. **Wstęp:** W pracy poddano analizie model ekonomicznej wielkości partii dla zamówień wielo-asortymentowych poprzez alokację procentu jednostek utraconych w wyniku zniszczenia oraz poprzez inwentaryzację przy uwzględnieniu inwestycji w promocję oraz koszty zamówień. Celem pracy było maksymalizacji zysku netto poprzez odpowiednie kształtowanie wielkości zamówienia, długości cyklu odtworzeniowego oraz ilości jednostek, ulegających zniszczeniu.
Metody: Opracowany matematyczny algorytm w celu znalezienia ważnych charakterystyk wklęsłości funkcji zysku netto. Zaprezentowany przykłady w celu zilustrowania wyników uzyskanych przy zastosowaniu opracowanego modelu oraz jego zalet. Na końcu przeprowadzono analizę wrażliwości zysku netto dla głównych parametrów inwentaryzacyjnych.
Wyniki i wnioski: Proponowany model stanowi ogólny schemat uwzględniający utratę procentową zapasów w wyniku zniszczenia przy uwzględnieniu zmiennych kosztów związanych z zamawianiem towarów.

Słowa kluczowe: wieloasortymentowość, zmienny koszt zamówienia, zniszczenie, maksymalizacja zysków.

EINFLUSS VON AUSGABEN UND VARIABLEN KOSTEN IM MEHRSORTIMENT-BESTELLUNGSMODELL AUF VERLUSTE BEI VERKAUF INFOLGE EINES VERDERBS

ZUSAMMENFASSUNG. Einleitung: Im Rahmen der vorliegenden Arbeit wurde das Modell einer wirtschaftlichen Losgröße für die Mehrsortiment-Bestellung anhand einer Allokation des Prozentsatzes von verlorengegangenen Einheiten infolge eines Verderbs und mithilfe einer Inventarisierung bei Berücksichtigung von Investitionen in die Promotion und Bestellungskosten analysiert. Das Ziel der Arbeit war es, den Netto-Gewinn durch eine entsprechende Gestaltung von Bestellungsgrößen, ferner von der Dauer des Wiederbeschaffungszyklus und der Anzahl von den einem Verderb unterliegenden Einheiten zu maximieren.
Methoden: Als die brauchbare Methode dafür gilt der ausgearbeitete mathematische Algorithmus zwecks der Ermittlung von relevanten Charakteristika der Höhlung der Funktion vom Netto-Gewinn. Ferner das dargestellte Beispiel für die Projizierung der unter Anwendung des ausgearbeiteten Modells gewonnenen Ergebnissen und dessen Vorteile. Zum Ausgang der Forschung wurde eine Analyse der Empfindlichkeit des Netto-Gewinns für die grundlegenden Inventarisierungsparameter durchgeführt.

Ergebnisse und Fazit: Das unterbreitete Modell gilt als ein allgemeines Schema, das einen prozentuellen Verlust von Vorräten infolge eines Verderbs bei der Berücksichtigung von den variablen, mit Bestellung von Waren verbundenen Kosten mit berücksichtigt.

Codewörter: Mehrsortiment-Bestellung, variable Bestellungskosten, Verderb, Maximierung von Gewinnen

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