



## CLASS-BASED STORAGE WAREHOUSE DESIGN WITH DIAGONAL CROSS-AISLE

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**ABSTRACT. Background:** Unit-load (UL) warehouses are among the most diffuse solutions to store items stocked in pallets, while class-based storage (CBS) assignment is an effective strategy balancing the storage space need and the UL traceability. This paper proposes a design strategy to reduce the access time for UL-CBS systems based on the inclusion of one additional diagonal aisle crossing the racks and the parallel aisles accessing the storage bays.

**Methods:** The introduced design strategy is based on the analytic models to compute the access time for one diagonal-cross aisle warehouses with 2- and 3-CBS system. The minimization of the closed form expressions getting the average time to reach the storage bays allow reducing the travelled paths to store and retrieve (S&R) the ULs.

**Results:** Comparison of the average time to S&R the ULs between traditional warehouses and the proposed configuration highlights the positive impact of the diagonal cross-aisle inclusion. In addition, a case example for a company operating within the food sector shows performance increase of about 33% respect to the correspondent traditional warehouse configuration.

**Conclusion:** The interest in non-traditional warehousing systems raises because of the savings in the time to S&R the ULs. Analytic and even more general models to compute such savings are of help to best design the storage area increasing the inbound handling performances.

**Key words:** warehouse design, diagonal cross-aisle, class-based storage, access time model, optimization.

### INTRODUCTION AND BACKGROUND

Research on warehouse design and operation planning is essential to define the configuration and to drive the best management of storage systems avoiding delays, bottlenecks and the supply chain cost increase [Gu et al. 2007, Baker and Canessa 2009, Gu et al. 2010]. Wide literature exists on this topic, from multiple sources, e.g. management, industrial engineering, OR, mathematics, etc., while extensive reviews collect and categorize the existing approaches, models, methods and best-practices to tackle the warehouse configuration and management issues [Cormier and Gunn 1992, Van den Berg

and Zijm 1999, Rouwenhorst et al. 2000, Staudt et al. 2015].

Almost all the Authors stress the importance of a panel of design drivers heavily affecting the overall system performance.

- Warehouse plant layout: rack dimensions, pickup and delivery (P&D) point position and access options;
- Stored items: unit-load (UL) vs. picking systems, additional features and requirements;
- Level of automation of the storage and retrieval (S&R) activities;
- Item assignment criterion: random, full-turnover based storage, class-based storage (CBS), dedicated storage, etc.;

- Access policy to the storage bays: single command cycles, dual command cycles, picking tours, etc.

A specific combination of such drivers univocally defines the warehouse configuration allowing the analytic modelling of the operative environment leading to the mathematical formulation of the design problem. Recently, Thomas and Meller [2014] exemplify and generalize such a diffuse research process.

Finally, concerning the assessment metrics to evaluate the warehouse performances, the mostly adopted key performance indicator (KPI) is the average access distance to S&R a UL or an item from the generic storage bay. Such a value directly affects the system handling capacity, i.e. throughput, productivity and cost [Roodenbergen and Vis 2006, Zaerpour et al. 2013].

This paper aims at contributing to the introduced research stream focusing on the analytic optimization of UL manual warehouses organized according to the CBS assignment strategy and presenting non-traditional aisles to shorten the S&R paths.

The advantages coming from the CBS assignment strategy, with two, three or more classes, are widely discussed by the literature together with the definition of a mix of design models. Past and recent examples are in Eyan and Rosenblatt [1994], Van den Berg [1996], Muppani and Adil [2008], Bortolini et al. [2015] and Zhang et al. [2017]. CBS effectively combines space saving to a prioritization of the storage bays assigning the most easily accessible to the frequently asked items.

In addition, a rising research area is about warehouses with non-traditional aisles (called diagonal cross-aisles in the following), i.e. non-parallel aisles that are not orthogonal to the system walls. The inclusion of such aisles significantly increases the rack available configurations and access modes allowing potential savings in the S&R activities. Gue, Meller and, more recently, Öztürkoğlu jointly propose new warehouse configurations, called

flying-V and fishbone, and analyze the performances within random and full-turnover based UL systems even with multiple P&D points [Gue and Meller 2009, Pohl et al. 2011, Gue et al. 2012, Öztürkoğlu et al. 2012, Clark and Meller 2013, Gue and Meller 2014, Cardona et al. 2015]. Close to such configurations, the Authors, in their previous study [Bortolini et al. 2015], discuss about a non-traditional warehouse configuration with straight diagonal cross-aisles crossing the parallel racks (see Fig. 1) assessing its performances and suitability for an easy re-layout of existing traditional warehouses.

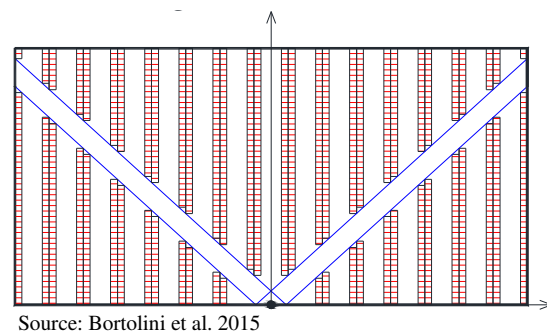


Fig. 1. Reference non-traditional warehouse with straight diagonal cross-aisle

This paper starts and refers to the previous one, addressing the storage system optimal design if the 2- and 3-CBS assignment strategy is adopted. Focusing on one diagonal cross-aisle configuration the model to get the optimal class shape is presented and applied comparing the average access distance toward the correspondent traditional configuration value.

According to proposed topic and goals, the remainder of this paper is organized as in the following. The next Section 2 introduces the model assumptions and the reference traditional configuration. Section 3 presents the model to optimize the 2- and 3-CBS system with one diagonal cross-aisle. Section 4 proposes a performance analysis based on data from a company operating within the food sector, while Section 5 concludes this paper outlining the next research steps.

## MODEL ASSUMPTIONS AND BASE CASE

According to the standard literature on the topic, the warehouse design model is based on the following assumptions [Öztürkoğlu et al. 2012, Clark and Meller 2013, Gue and Meller 2014, Bortolini et al. 2015, Cardona et al. 2015]:

1. The P&D point is located in the lower centre of the aisle front, representing the origin of axes;
2. The problem is symmetric so that the sole right half of the storage system is investigated and all notations and dimensions refer to such a part of the warehouse;
3. UL warehouse with single command cycles;
4. The items have the same dimensions and they are continuously allocated into the racks;
5. S&R locations are independent, while ULs are stored according to the 2- and 3-CBS assignment strategy. The ULs are split into the classes according to their demand level. The well-known demand curve  $G(i) = i^S$  is adopted [Hausman et al. 1976] where:
  - $i$  is the normalized cumulative storage area for all products, ranking them in decreasing order of the ratio between the demand and the storage area,  $i \in [0, 1]$ ;
  - $S$  is the so-called *skewness factor*,  $S \in (0, 1]$ ;
  - $G(i)$  is the normalized cumulative demand until the  $i$ -th percentile of the storage area. It represents the probability of access to a generic UL within the storage area  $i$ ,  $G(i) \in [0, 1]$ .
6. The warehouse shape factor is equal to 1, so that the normalized dimensionless rack measure is 1 [Hausman et al. 1976].

In particular, assumption 6. sets a squared area for each half of the storage system. Such assumption can be relaxed easily in future research and it is introduced here to simplify the model expressions (1) and because of the previous works confirm that a squared configuration generally leads to lower average access distance values (2).

## Base case

The base case refers to traditional warehouses with no diagonal cross-aisles. The following Fig. 2 depicts the normalized dimensionless system configuration, including three classes, i.e. ABC, and the key notations.

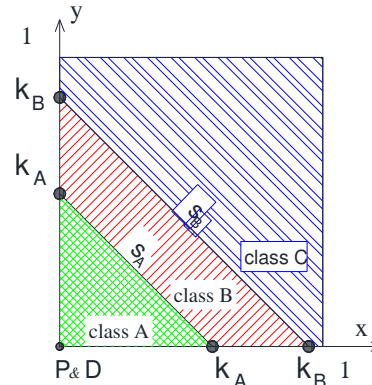


Fig. 2. Base case configuration with 3-CBS

The class shape is triangular and the correspondent boundary limits are:

$$S_A: y = -x + k_A \quad (1)$$

$$S_B: y = -x + k_B \quad (2)$$

The analytic expression of the distance function,  $f(x, y)$ , between the P&D point and the generic  $P(x, y)$  location is  $f(x, y) = x + y$  because of the orthogonal movements within the racks.

Assuming  $k_A < k_B \leq 1$  (the other cases are developed by the Authors and omitted here for brevity) the analytic expressions to get the average access distance for each class are from the application of the *Integral Mean Value Theorem* as in the following:

$$D_{03}^A = \frac{1}{A_A} \int_0^{k_A} \int_0^{S_A} f(x, y) dy dx \quad (3)$$

$$D_{03}^B = \frac{1}{A_B} \left( \int_0^{k_A} \int_{S_A}^{S_B} f(x, y) dy dx + \int_{k_A}^{k_B} \int_0^{S_B} f(x, y) dy dx \right) \quad (4)$$

$$D_{03}^C = \frac{1}{A_C} \left( \int_0^{k_B} \int_{S_B}^1 f(x, y) dy dx + \int_{k_B}^1 \int_0^1 f(x, y) dy dx \right) \quad (5)$$

where  $D_{\alpha\beta}^{\gamma}$  is the average access distance to class  $\gamma$  in a warehouse with  $\alpha$  diagonal cross-aisles and  $\beta$  classes.  $A_A, A_B$  and  $A_C$  are the class areas univocally defined given  $k_A$  and  $k_B$ . Given the average access distances for each class the overall average access distance comes weighting such values by the probability to access the class, obtained from the demand curve (see assumption 5). Analytically:

$$D_{03}^{ABC} = D_{03}^A \cdot A_A^S + D_{03}^B \cdot ((A_B + A_A)^S - A_A^S) + D_{03}^C \cdot (1 - (A_A + A_B)^S) \quad (6)$$

where the unknowns to optimize are  $k_A$  and  $k_B$ , given the skewness factor,  $S$ . Neglecting the boring math details and adopting the *Sequential Quadratic Programming* (SQP) numerical optimization algorithm [Bonnans et al. 2006] within a dedicated Maple working environment, Equation (6) is minimized for eight relevant values of  $S$ . Results are in Table 1.

Table 1. Base case, results for 3-CBS warehouse

Demand curve	$s$	$k_A$	$k_B$	$D_{03}^{ABC}$
Random	1.000	-	-	1.0000
20%-30%	0.748	0.4778	0.9741	0.9087
20%-40%	0.569	0.3968	0.9046	0.8207
20%-50%	0.431	0.3228	0.8355	0.7308
20%-60%	0.317	0.2517	0.7622	0.6332
20%-70%	0.222	0.1837	0.6825	0.5253
20%-80%	0.139	0.1165	0.5888	0.3978
20%-90%	0.065	0.0514	0.4661	0.2357

Table 2. Base case, results for 2-CBS warehouse

Demand curve	$s$	$k_A$	$D_{02}^{AB}$
Random	1.000	-	1.0000
20%-30%	0.748	0.7301	0.9262
20%-40%	0.569	0.6524	0.8552
20%-50%	0.431	0.5744	0.7820
20%-60%	0.317	0.4911	0.7011
20%-70%	0.222	0.4005	0.6088
20%-80%	0.139	0.2950	0.4933
20%-90%	0.065	0.1647	0.3299

The 2-CBS base case is a simplification of the previous 3-CBS by omitting  $s_B$  and  $k_B$ , i.e. class A and B, only, so that the unknown is  $k_A$ . Table 2 presents the results for this scenario.

Results highlight the convenience of having a low skewness factor because of the higher frequency of access to the bays closer to the P&D point. Starting from this benchmark, the next section introduces the diagonal cross-aisle and measures its positive impact on the travelled distance to access the bays.

### DIAGONAL CROSS-AISLE MODEL WITH CBS

The inclusion of the diagonal cross-aisle changes the storage system configuration and enlarges the path options to reach the bays.

In the following (see Fig. 3 about the 2-CBS configuration), the diagonal cross-aisle is referred as  $r_1: y = \tan(a) \cdot x$ . The angle  $a$  is to optimize.  $r_2$  is called *equidistance line* and represents the locus of points whose distance from the P&D point is the same adopting the standard orthogonal path or moving through the diagonal cross-aisle. This line is defined by the angle  $z$ , function of the angle  $a$  (see Bortolini et al. [2015] for analytic details).

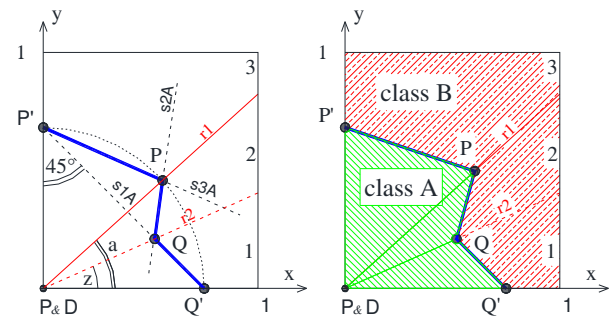


Fig. 3. Warehouse configuration with 2-CBS and one diagonal cross-aisle

$r_1$  and  $r_2$  divide the storage area into four zones accessed through the paths of Fig. 4, having the following equations:

$$f(x, y) = \begin{cases} f_1(x, y) = x + y \\ f_2(x, y) = \frac{x}{\cos(a)} - y + x \cdot \tan(a) \\ f_3(x, y) = \frac{x}{\cos(a)} + y + x \cdot \tan(a) \\ f_4(x, y) = \frac{1}{\sin(a)} + x - \frac{1}{\tan(a)} + 1 - y \end{cases} \quad (7)$$

Zone 4 collapses if  $a \leq 45^\circ$ .

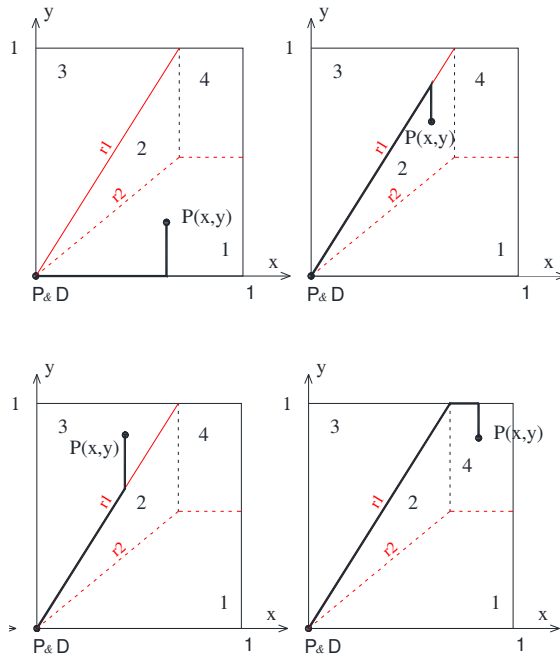


Fig. 4. Warehouse configuration with 2-CBS and one diagonal cross-aisle

### Class boundary shape

Previous Fig. 3 further shows the shape of the class boundary, i.e. the blue broken line. Due to the presence of the diagonal cross-aisle, the class boundary shape is different among the warehouse zones. Zone 1 is accessed through orthogonal path as in traditional warehouses. Consequently, the line is the same as before and called  $s_A^1$ . In Fig. 3,  $Q(Q_x, Q_y)$  is the common point between  $s_A^1$  and  $r_2$ , while  $P(P_x, P_y)$  belongs to  $r_1$  and its distance from the P&D point is  $k_A$ .  $P$  and  $Q$  univocally define  $s_A^2$ , while  $P$  and  $P'(0, k_A)$  univocally define  $s_A^3$ . Formally:

$$s_A: \begin{cases} s_A^1: y = -x + k_A \\ s_A^2: y = \left(\frac{1}{\cos(a)} + \tan(a)\right) \cdot x - k_A \\ s_A^3: y = -\left(\frac{1}{\cos(a)} + \tan(a)\right) \cdot x + k_A \end{cases} \quad (8)$$

In 3-CBS configuration,  $s_B$  is defined similarly, including  $k_B$  instead of  $k_A$ , with  $k_A < k_B$ . Furthermore, if  $k_A$  and/or  $k_B$  are higher than 1, the class boundary shapes are out of the warehouse area (partially or totally).

Despite the Authors fully develop and compare all the possible configurations, in the following, for brevity and because of the optima fall in this case, the  $k_A < k_B \leq 1$  configuration is discussed, only. In addition, for the same reasons, the  $a \leq 45^\circ$  scenario is presented, only.

### Analytic model to compute the average access distance

The analytic model to compute the average access distance in the case of one diagonal cross-aisle warehouse, 2-,  $D_{12}^{AB}$ , and 3-CBS system,  $D_{13}^{ABC}$ , follows the same approach of the base case. At first,  $D_{12}^A$ ,  $D_{12}^B$ ,  $D_{13}^A$ ,  $D_{13}^B$  and  $D_{13}^C$  are computed applying the *Integral Mean Value Theorem*. Integrals are extended over the class area according to the boundary shapes outlined in Section 3.1 and Equation (7) is used to get the equation of the travelled distance. Given the average access distances for each class, the warehouse average access distance comes from the application of Equation (6), as for the base case.

### 2-CBS and one diagonal cross-aisle model

Following the introduced notations, Equations (9) and (10) show how to express  $D_{12}^A$  and  $D_{12}^B$  as function of  $a$  and  $k_A$ . Fig. 3 supports the reader in understanding the integral extensions and the travelled distance equation to use (see Equation 7).

$$\begin{aligned}
 D_{12}^A = & \frac{1}{A_A} \left( \int_0^{Q_x} \int_0^{r_2} f_1(x, y) dy dx + \int_{Q_x}^{k_A} \int_0^{s_1^A} f_1(x, y) dy dx + \right. \\
 & \int_0^{Q_x} \int_{r_2}^{r_1} f_2(x, y) dy dx + \int_{Q_x}^{P_x} \int_{s_2^A}^{r_1} f_2(x, y) dy dx + \quad (9) \\
 & \left. \int_0^{P_x} \int_{r_1}^{s_3^A} f_3(x, y) dy dx \right) \\
 D_{12}^B = & \frac{1}{A_B} \left( \int_{Q_x}^{k_A} \int_{s_1^A}^{r_2} f_1(x, y) dy dx + \int_{k_A}^1 \int_0^{r_2} f_1(x, y) dy dx + \right. \\
 & \int_{Q_x}^{P_x} \int_{r_2}^{s_2^A} f_2(x, y) dy dx + \int_{P_x}^1 \int_{r_2}^{r_1} f_2(x, y) dy dx + \quad (10) \\
 & \left. \int_0^{P_x} \int_{s_3^A}^1 f_3(x, y) dy dx + \int_{P_x}^1 \int_{r_1}^1 f_3(x, y) dy dx \right)
 \end{aligned}$$

Adopting the *Sequential Quadratic Programming* (SQP) numerical optimization algorithm [Bonnans et al. 2006] within a dedicated Maple working environment,  $D_{12}^{AB}$  is minimized for the same eight relevant values of  $S$  shown for the base case. Results are in Table 3.

Table 3. One diagonal cross-aisle, results for 2-CBS warehouse

Demand curve	$s$	$a$	$k_A$	$D_{12}^{AB}$
Random	1.000	33.97°	-	0.8529
20%-30%	0.748	33.93°	0.6259	0.7923
20%-40%	0.569	33.90°	0.5596	0.7331
20%-50%	0.431	33.88°	0.4929	0.6715
20%-60%	0.317	33.86°	0.4214	0.6028
20%-70%	0.222	33.84°	0.3434	0.5240
20%-80%	0.139	33.83°	0.2527	0.4249
20%-90%	0.065	33.83°	0.1408	0.2841

Results confirm the trend discussed for the base case and 2-CBS showing a low sensitivity of the diagonal cross-aisle position on the skewness factor, i.e.  $a \approx 34^\circ$  in all cases.

### 3-CBS and one diagonal cross-aisle model

Extending the analysis by adding the third class, the equations and the optimization results become the followings (without any conceptual change, within equations, points  $P(P_x, P_y)$  and  $Q(Q_x, Q_y)$  are replaced with  $P^A(P_x^A, P_y^A)$ ,  $P^B(P_x^B, P_y^B)$ ,  $Q^A(Q_x^A, Q_y^A)$  and

$Q^B(Q_x^B, Q_y^B)$  because of the two class boundary shapes, i.e. between class A and B and class B and C).

$$\begin{aligned}
 D_{13}^A = & \frac{1}{A_A} \left( \int_0^{Q_x^A} \int_0^{r_2} f_1(x, y) dy dx + \right. \\
 & \int_{Q_x^A}^{k_A} \int_0^{s_1^A} f_1(x, y) dy dx + \int_0^{Q_x^A} \int_{r_2}^{r_1} f_2(x, y) dy dx + \\
 & \int_{Q_x^A}^{P_x^A} \int_{s_2^A}^{r_1} f_2(x, y) dy dx + \quad (11) \\
 & \left. \int_0^{P_x^A} \int_{r_1}^{s_3^A} f_3(x, y) dy dx \right)
 \end{aligned}$$

$$\begin{aligned}
 D_{13}^B = & \frac{1}{A_B} \left( \int_{Q_x^A}^{k_A} \int_{s_1^A}^{r_2} f_1(x, y) dy dx + \int_{k_A}^{Q_x^B} \int_0^{r_2} f_1(x, y) dy dx + \right. \\
 & \int_{Q_x^B}^{k_B} \int_0^{s_1^B} f_1(x, y) dy dx + \int_{Q_x^A}^{P_x^A} \int_{r_2}^{s_2^A} f_2(x, y) dy dx + \\
 & \int_{P_x^A}^{Q_x^B} \int_{r_2}^{r_1} f_2(x, y) dy dx + \\
 & \int_{Q_x^B}^{P_x^B} \int_{s_2^B}^{r_1} f_2(x, y) dy dx + \quad (12) \\
 & \left. \int_0^{P_x^A} \int_{s_3^A}^{s_3^B} f_3(x, y) dy dx + \int_{P_x^A}^{P_x^B} \int_{r_1}^{s_3^B} f_3(x, y) dy dx \right)
 \end{aligned}$$

$$\begin{aligned}
 D_{13}^C = & \frac{1}{A_C} \left( \int_{Q_x^B}^{k_B} \int_{s_1^B}^{r_2} f_1(x, y) dy dx + \int_{k_B}^1 \int_0^{r_2} f_1(x, y) dy dx + \right. \\
 & \int_{Q_x^B}^{P_x^B} \int_{r_2}^{s_2^B} f_2(x, y) dy dx + \\
 & \int_{P_x^B}^1 \int_{r_2}^{r_1} f_2(x, y) dy dx + \quad (13) \\
 & \left. \int_0^{P_x^B} \int_{s_3^B}^1 f_3(x, y) dy dx + \int_{P_x^B}^1 \int_{r_1}^1 f_3(x, y) dy dx \right)
 \end{aligned}$$

Results confirm the trend discussed for the base case and 3-CBS showing a low sensitivity of the diagonal cross-aisle position on the skewness factor and the class number, i.e.  $a \approx 34^\circ$  in all cases.

Table 4. One diagonal cross-aisle, results for 3-CBS warehouse

Demand curve	$s$	$a$	$k_A$	$k_B$	$D_{13}^{ABC}$
Random	1.000	33.97°	-	-	0.8529
20%-30%	0.748	33.92°	0.4052	0.8261	0.7781
20%-40%	0.569	33.88°	0.3375	0.7695	0.7048
20%-50%	0.431	33.84°	0.2752	0.7124	0.6291
20%-60%	0.317	33.81°	0.2150	0.6509	0.5460
20%-70%	0.222	33.79°	0.1571	0.5837	0.4536
20%-80%	0.139	33.77°	0.0998	0.5040	0.3439
20%-90%	0.065	33.76°	0.0440	0.3992	0.2039

### Space loss and result comparison

Despite the positive effect of the diagonal cross-aisle inclusion is evident, roughly, by directly comparing  $D_{12}^{AB}$  and  $D_{13}^{ABC}$  to  $D_{02}^{AB}$  and  $D_{03}^{ABC}$  respectively, to properly conclude on the effect of the diagonal cross-aisle on the average access distance, the storage space losses are to consider. In fact, the diagonal cross-aisle requires some space that is not available to store any UL. This means that and additional extra-space, equal to the space loss, is necessary to guarantee the same global storage capacity. Such an extra-space enlarges the warehouse further increasing the travelled distances respect to the base case.

The Authors, in Bortolini et al. [2015], detail the analytic approach to face this aspect and propose a method to correct the average access distance allowing the aforementioned result comparison. This approach requires, as input data, the diagonal cross-aisle and the rack width together with the warehouse (non-normalized) dimensions. The next Section 4 makes this comparison while presenting a model application for a company operating within the food sector.

### INDUSTRIAL APPLICATION AND PERFORMANCE ANALYSIS

The proposed approach is applied to re-design the UL storage system for a leading company producing and distributing long-life food products, e.g. in oil produces, pickled vegetables, sauces, etc. The storage system is

located in Tuscany, Italy, and manages over 30 million tins per year, with more than 800 items.

At first, the ABC analysis conducted over a reference year of S&R activities allows determining the demand curve skewness factor, equal to  $S=0.4472$  (fitting accuracy:  $R^2=99.5\%$ ). In addition, Fig. 5 presents the current rack layout with relevant dimensions.

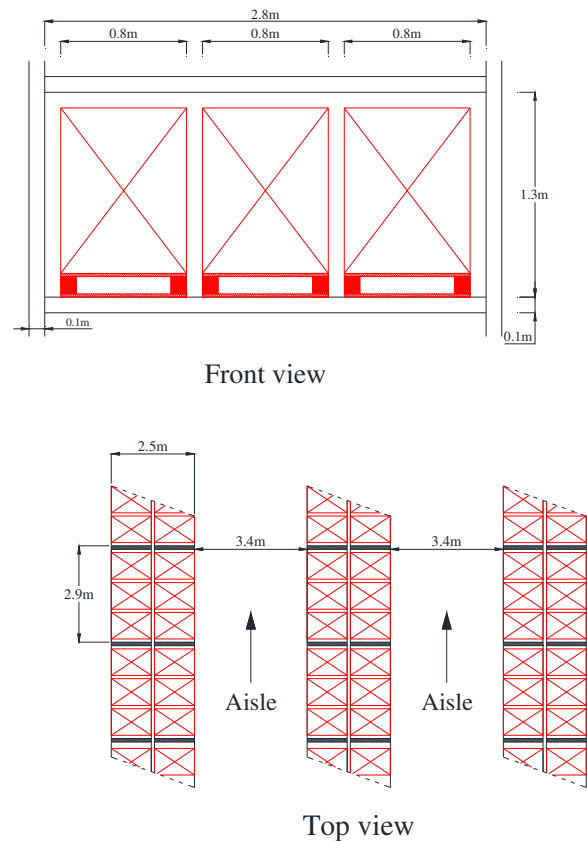


Fig. 5. Industrial application, rack layout

Actually, the warehouse plant area is  $106.1 \times 52.1$  m, with 18 aisles, 6 vertical levels, 18 spans per aisle and 3 bays per span and rack hosting ISO1 Eur-Epal ULs. Actually, no diagonal cross-aisle is present and the company adopts the random storage assignment policy.

## MODEL APPLICATION AND POTENTIAL SAVINGS

Starting from the current As-Is scenario, the proposed warehouse design model is applied developing the 2- and 3-CBS scenarios with no and one diagonal cross-aisle. The obtained results are in the next Table 5 together with the average access distance percentage gross savings (neglecting the space loss due to the diagonal cross-aisle).

Table 5. Industrial application, results and gross savings

Scenario	$a$	$k_A$	$k_B$	$D_{\alpha\beta}^Y$	Gross saving
As-Is (random)	-	-	-	1.0000	-
2-CBS 0 cross-aisle	-	0.5847 31.02m	-	0.7918	20.8%
3-CBS 0 cross-aisle	-	0.3322 17.62m	0.8446 44.81m	0.7427	25.7%
2-CBS 1 cross-aisle	33.88°	0.5017 26.62m	-	0.6798	32.0%
3-CBS 1 cross-aisle	33.85°	0.2831 15.02m	0.7199 38.19m	0.6391	36.1%

Following the analytic approach proposed by the Authors in Bortolini et al. [2015], the space loss is estimated and used to get the final

net savings, including the space loss, driving the warehouse re-design actions. The key outcomes of this phase are in Table 6.

Table 6. Industrial application, space loss and savings

Scenario	Space loss	Gross saving	Net saving
As-Is (random)	-	-	-
2-CBS 0 cross-aisle	-	20.8%	20.8%
3-CBS 0 cross-aisle	-	25.7%	25.7%
2-CBS 1 cross-aisle	9.45%	522.38m <sup>2</sup>	32.0%
3-CBS 1 cross-aisle	9.45%	522.38m <sup>2</sup>	36.1%

Despite the presence of the diagonal cross-aisle reduces the available storage area, i.e. the company has no possibility to enlarge the overall system dimensions because of the already existing structure, the advantage coming from the inclusion of the additional cross-aisle is significant not only if compared to the As-Is scenario but also toward the correspondent CBS base scenario.

After an internal discussion, the company decides to adopt the 3-CBS scenario with one diagonal cross-aisle layout so that the new schematic of the storage system is in accordance with Fig. 6.

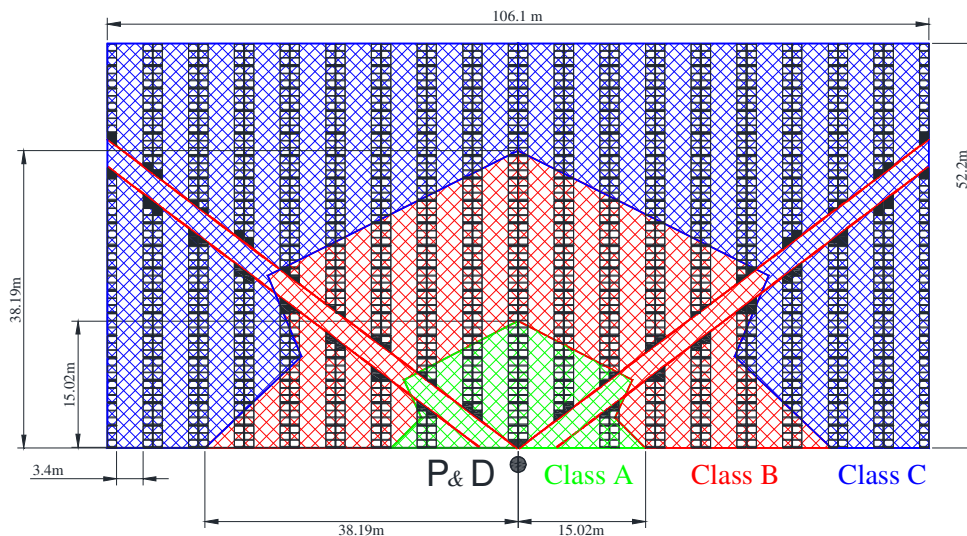


Fig. 6. Industrial application, warehouse re-layout



## CONCLUSIONS AND NEXT STEPS

Warehouses are relevant nodes of almost all supply chains smoothing the product flow against fluctuant production, distribution and consumption rates. At the same time, warehouses do not add value to the products, directly, and risk to become a cost and a source of inefficiency for the manufacturing companies and the distributors. The best design and re-design of such systems, together with their effective management through optimized operating strategies, are the viable paths to make warehouses sustainable in the long-term.

This paper aims at contributing to the applied research in the warehouse design presenting and applying an analytic model to optimize unit-load (UL) warehouses storing items according to the class-based storage (CBS) assignment strategy. Particularly, this paper proposes the model to compute the average travel distance to access the storage bays in the case of non-traditional systems adopting one diagonal cross-aisle to shorten the travel path of the handling systems. Starting from the discussion and performance assessment of 2- and 3-CBS traditional systems with parallel aisles and orthogonal paths, i.e. the base case, the correspondent diagonal cross-aisle configurations are presented. Deep discussion is on the class boundary shapes and about the optimization of the diagonal cross-aisle position and of the class dimensions varying the so-called skewness factor, function of the market demand curve.

Finally, through the discussion of an industrial case taken from the long-life food sector, the model is applied to a real context comparing the handling performances and further considering the space loss due to the presence of the additional aisle and the subsequent potential warehouse dimension increase. Results highlight the relevant distance and time saving in accessing the bays in presence of the diagonal cross-aisle toward the As-Is configuration and the CBS base case. Global savings range from 28.6% to 32.9% for

2- and 3-CBS systems with a global space loss of about 9.45%.

Starting from this work, the next research steps are in the direction of enlarging the model boundaries to assess the impact on both the average access distance and the space loss of more than one diagonal cross-aisle, concluding about the optimal number of aisles to adopt. In addition, model extensions to include rectangular warehouses, multiple pickup and delivery (P&D) points, the vertical handling movements and dual command storage and retrieval (S&R) operations are possible and expected by researchers and practitioners. Lastly, multi-scenario analyses over a range of real contexts may be of interest to define operative boundary convenience conditions and constraints for the inclusion of the proposed non-traditional warehouse configuration.

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## PROJEKT MAGAZYNU Z MIEJSCAMI SKŁADOWANIA ORAZ UKOŚNYMI OSIAMI JAKO DROGAMI DOJAZDU

**STRESZCZENIE. Wstęp:** magazyny z miejscami składowania są najczęściej stosowanym obecnie rozwiązaniem dla przechowywania dóbr ułożonych na paletach, zaś przypisanie miejsc składowania (CBS) jest efektywną strategią optymalizującą ograniczenia powierzchni i identyfikalność jednostek składowania. W pracy zaproponowano strategię planowania rozkładu magazynu umożliwiającą redukcję czasu dostępu do tej poszczególnych jednostek składowania poprzez wprowadzenie dodatkowej ukośnej osi w poprzek regałów oraz osi równoległych umożliwiających dostęp do miejsc magazynowania.

**Metody:** Wprowadzona strategia rozplanowania oparta jest na modelach analitycznych obliczających czas dostępu w magazynie z jedną osią ukośną w systemie 2-CBS oraz 3-CBS. Minimalizacja wyrażenie określającego średni czas dostępu do miejsca składowania pozwala na zmniejszenie długości ścieżki dostępu (S&R) do jednostki składowania.

**Wyniki:** Porównanie średniego czasu dostępu do jednostki składowania pomiędzy tradycyjnym magazynem a proponowaną konfiguracją pokazuje pozytywny wpływ zastosowania osi ukośnej. Dodatkowo, zastosowanie tego rozwiązania w przykładowym magazynie firmy działającej w branży spożywczej pokazało 33% wzrost efektywności w stosunku do konfiguracji w tradycyjnym magazynie.

**Wnioski:** Zainteresowanie w rozwiązaniach odbiegających od tradycyjnych magazynów jest coraz większe, ze względu na oferowane przez nie możliwości oszczędności czasu dostępu do jednostek składowania. Modele analityczne i ogólne są tutaj szczególnie przydatne do obliczania możliwych oszczędności i dzięki temu zaprojektowania optymalnych rozwiązań obejmujących również operacje manipulacyjne w obrębie magazynu.

**Słowa kluczowe:** projekt magazynu, ukośna oś, magazynowe oparte o jednostki składowania, model czasu dostępu, optymalizacja.

## PROJEKT EINES LAGERS MIT LAGERPLÄTZEN UND SCHRÄG VERLAUFENDEN ACHSEN ALS ZUFAHRTSGASSEN

**ZUSAMMENFASSUNG. Einleitung:** Die Lagereinrichtungen mit Lagerplätzen stellen heutzutage die meist angewendete Lösung für die Lagerung von palettierten Waren, und die eindeutige Zuweisung der Lagerplätze (CBS) eine die Lagerfläche und Identifizierbarkeit von Lagereinheiten optimierende Strategie dar. In der vorliegenden Arbeit wurde eine Strategie für die Planung der Aufteilung des Lagers vorgeschlagen. Sie ermöglicht eine Reduktion der Zeit eines Zugriffs zu den einzelnen Lagereinheiten mittels der Einführung der zusätzlichen, schräg und quer durch die Regalanlagen verlaufenden Achse, sowie der parallelen Achsen, die den Zugriff zu den Lagerplätzen sichern.

**Methoden:** Die eingeführte Verplanungsstrategie stützt sich auf die analytischen Modelle, die die Zugriffszeit im Lager mit einer schrägen Achse im 2-CBS-System und 3-CBS-System berechnen. Die Minimalisierung der durchschnittlichen Zugriffszeit zu den Lagerplätzen ermöglicht die Reduzierung der Länge des Zugriffspfades (S&R) zur Lagereinheit.

**Ergebnisse:** Der Vergleich der durchschnittlichen Zeit des Zugriffs zur Lagereinheit im Falle eines traditionellen Lagers und der vorgeschlagenen Lagerkonfiguration zeigt einen positiven Einfluss der eingesetzten Schrägachse auf. Die Anwendung dieser Lösung in einem Lager für Nahrungsmittel zeigte beispielweise einen 33%-gen Zuwachs der Lagereffizienz im Verhältnis zu der im traditionellen Lager bestehenden Infrastruktur auf.

**Fazit:** Das Interesse an den Lösungen, die über die traditionellen Lagerausrüstungen hinausgehen, wird immer größer, und dies angesichts der möglichen Erzielung von Zeitersparungen beim Zugriff zu den Lagereinheiten. Die allgemeinen und analytischen Modelle sind dabei besonders brauchbar für die Berechnung der möglichen Ersparungen und dank dessen für die Projektierung von optimalen Lösungen, die auch operative Handhabungen im Lager umfassen.

**Codewörter:** Lagerprojekt, Schrägachse, Lagerung von Lagereinheiten, Modell der Zugriffszeit, Optimierung

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