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ORIGINAL PAPER

# INFLUENCE OF DELIVERY QUANTITY ON SERVICE LEVEL IN THE STOCK REPLENISHMENT SYSTEM BASED ON REORDER LEVEL FOR IRREGULAR DISTRIBUTIONS OF DEMAND

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**ABSTRACT**. **Background.** Due to random changes in demand, inventory management is still – despite the development of alternative goods flow management concepts – an important issue both in terms of costs of maintenance and replenishment, as well as the level of service measured by inventory availability levels. One of the classical replenishment systems is Reorder Point replenishment, in which orders of fixed quantity are placed whenever effective stock reaches the reorder level B. There is a well known relationship between level B and the service level, most often defined as the probability of filling in all orders within a replenishment cycle, although in practice – as demonstrated in the author's earlier papers – some extensions of the commonly used formulas are necessary. The task becomes even more complicated in the case of unusual, peculiar patterns of demand distribution, significantly different from the norm, which is characteristic for slow moving goods. In this case a considerable influence of order/delivery quantities on actual stock levels triggering the replenishment cycle (lower than B) is observed. This phenomenon is reflected in the service level, which is lower than expected. The scientific aim of the research presented in the paper was to explain (based on results of simulation experiments) the observed phenomenon and to elaborate recommendations for establishing order quantities, which would have the slightest impact on service level.

**Method.** The subject of the research was to identify the scale of the phenomenon based on an example of an atypical demand distribution. To determine the influence of delivery quantity Q on difference Δ between reorder level B and the actual stock level at the moment when the order is placed, a simulation was conducted using a dedicated tool (a simulator in EXCEL). Since the relatively low scale of variability of difference Δ as a function of order quantity Q does not explain the scale of variability observed in the service level, a hypothesis was advanced and demonstrated (via the simulation) that the probability of a stock-out situation in two consecutive replenishment cycles depends on the nature and scale of changes of difference as a function of the difference between S and B levels steering the parameters of a BS replenishment system (being one of the MIN-MAX type replenishment systems)

**Results.** The results of the research on the influence of order/delivery quantity Q on service level (<SL) are presented. It has been shown that when determining order quantity Q, one has to take into account the difference Δ and its dependence on the difference between S and B levels which are the steering parameters of a BS replenishment system. A simple procedure is proposed to avoid the increased risk of a stock-out situation in two consecutive replenishment cycles, which leads to a decrease in service levels.

**Conclusion.** The results of the research presented here, as well as the proposed procedure, will support determination order quantity in a reorder point replenishment system (BQ), especially in case of peculiar, atypical demand distribution types. That makes it possible to avoid order quantities carrying a risk of decreased service levels. Elaboration of a complex model determining expected service level as a function of order/delivery quantity will require further investigation.

**Key words:** stock management, stock replenishment based on BQ system, point of reorder, service level, simulation.



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## INTRODUCTION

Despite the development of alternative concepts of managing flows of goods, stock management under conditions of random changes of demand remains an important issue, from the perspective of both stock maintenance and replenishment costs, and service level measured with the level of stock availability. One of the two classical stock replenishment systems is a system based on reorder point (level) in which an order with fixed quantity (Q) is placed when the available (i.e. effective) stock reaches level B (reorder level, reorder point). There is a simple dependence between the value of reorder level B and service level, most frequently defined as the probability to serve demand (□SL) in a specific stock replenishment cycle. In practice, however, as previous studies of the author [Krzyżaniak 2013] had shown, it is necessary to somewhat extend the commonly applied formulae. The issue gets even more complicated when the distribution of demand distribution is irregular and considerably deviates from normal distribution, which is characteristic particularly for slow-moving goods. What becomes evident here, is the significant influence of order quantity on the actual service level. The paper presents results of a study analysing this phenomenon.

## CHARACTERISTICS OF THE STOCK REPLENISHMENT SYSTEM BASED ON REORDER LEVEL

Among stock replenishment systems in which safety stock plays a crucial role in ensuring required level of availability of goods, the system based on reorder level is one of the two classical ones (apart from the system based on periodical review). In the terminology applied by the European Logistics Association, the system is referred to as BQ [ELA, 2009], where B designates reorder point/level, and Q is a set order/delivery quantity.

General rules governing the execution of the BQ system include:

1) Defining data which determine the controlling parametres:
   - calculating parametres of demand distribution in an adopted time unit (including random changes only),
   - defining replenishment lead time (LT) and its distribution,
   - setting the required service level (the type of definition and the target quantitative measure).

2) Setting the parameters which control stock replenishment, including:

   a) setting the reorder level B.

   In a classic approach, reorder level B is set on the basis of the following dependency:

$$B = D \cdot LT + SS \qquad (1)$$

where:

D – mean demand in an adopted time unit (e.g. mean daily/weekly demand)
LT – replenishment cycle lead time – between order and delivery, expressed in the same time units as the ones for which demand (D) has been defined.

SS - safety stock expressed in the following way:

$$SS = \omega \cdot \sigma_{DLT} \qquad (2)$$

where:

ω – safety coefficient, dependent on adopted service level and type of demand distribution,
$\sigma_{DLT}$ - standard deviation of demand in replenishment cycle time.

In general cases (random variability of demand and replenishment cycle time), the following formula applies:

$$\sigma_{DLT} = \sqrt{\sigma_D^2 \cdot LT + \sigma_{LT}^2 \cdot D^2} \qquad (3)$$

where:

$\sigma_D$ – standard deviation of demand in an adopted time unit (the same as for D), or a mean standard error of a forecast.

$\sigma_{LT}$ – standard deviation of replenishment lead time.

b) Defining order quantity (Q).

Literature describes a number of models (e.g. [Fertch 2003]). One of them is the economic order quantity model.

3) the stock replenishment procedure compliant with set parametres, which is characterised by the fact that after each transaction (release, receipt or booking):

– current status of effective stock $S_e$ is specified

$$S_e = S_W + S_o + S_{er} - S_b \qquad (4)$$

where:

$S_w$ - stock physically available in the warehouse (on-hand),

$S_o$ - orders placed, but not yet implemented,

$S_{er}$ -stock en route,

$S_b$ - stock already booked.

– Comparison of calculated effective stock level $S_e$ with reorder level B.

– Placement of an order with an adopted, fixed quantity Q, if $S_e \leq B$.

Figure 1 illustrates rules governing the implementation of the BQ system.

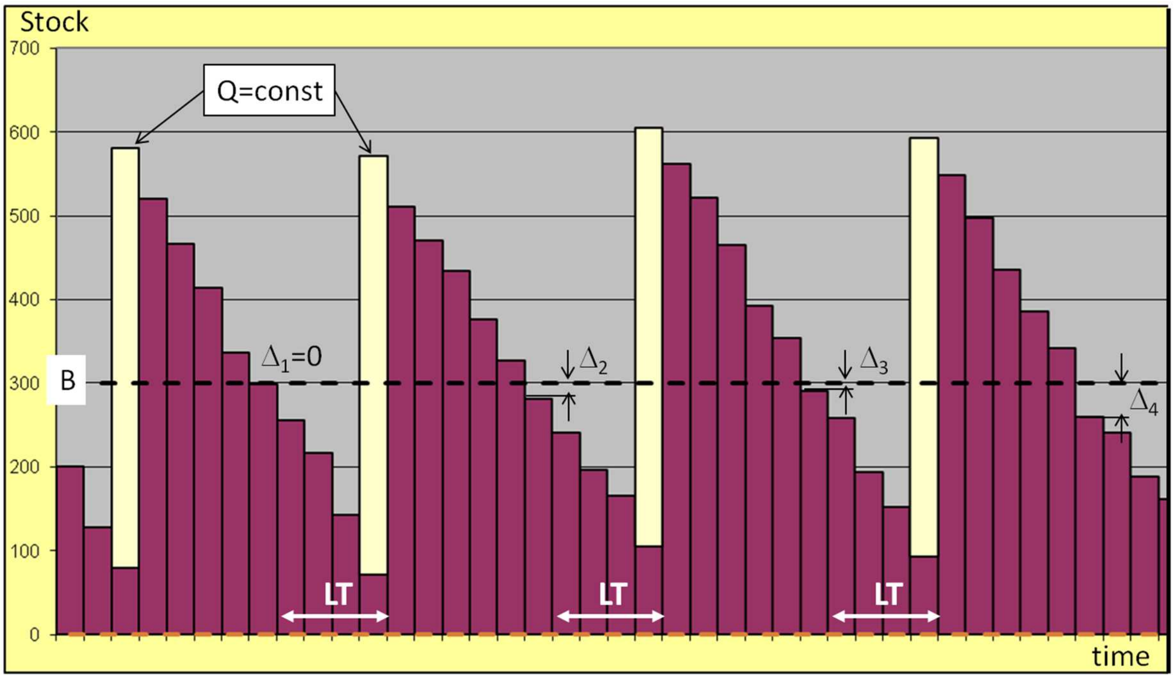


Fig. 1. Illustration of a rule for performing stock replenishment in the BQ system
Rys. 1. Ilustracja zasady realizacji odnawiania zapasu w systemie BQ

The figure 1 also shows volumes of the difference Δ between the assumed reorder level B and the actual effective stock level, which triggers replenishment cycle with the lead time LT. The essence of the difference Δ has been presented further in the article.

The discussed system is frequently used in practice, it has to be borne in mind, however, that in its classic form presented above, it has a number of restrictions. It forces the introduction of different kinds of modifications resulting, for example, from irregular demand whose distribution significantly differs from a normal one. One of them is the BS system, MIN-MAX class [e.g. Teunter R., Sani B., 2009, Babai M.Z., Jemai Z. Dallery Y., 2011]. The difference between the BS system and the BQ system, discussed in the present study, is that in a situation where the condition of $S_e \leq B$ for order placement is met, the order is not placed in fixed quantity Q (as in the case of the BQ system), but it is calculated as a difference between adopted level S and current effective stock level $S_e$ ($Q = S - S_e$). The BS system is also subject to continuous studies covering both issues related to proper determination of reorder point, particularly under conditions of discontinuous demand (e.g. for spare parts) [e.g. Porras E., Dekker R., 2008, Gamberini R. et al., 2014, Hahn G.J., Leucht A., 2015], and issues concerning the determination of optimum order quantity [e.g. Samal N.K., D.K. Pratihar D.K., 2014, Wan C.H., 2010].

## STUDYING THE INFLUENCE OF DELIVERY QUANTITY ON ACTUAL EFFECTIVE STOCK LEVEL UPON STARTING THE REPLENISHMENT CYCLE

One of the reasons resulting in the obligation to modify the value of reorder point B calculated from the classic formula (1) is the fact that usually when the replenishment cycle starts, stock $S_e$ is lower (often significantly lower) than level B. The phenomenon has been presented in Figure 1. In reality, it is hardly probable that the replenishment cycle will start with $S_e$ equal to B. It would result in actual service level being lower than assumed and expected. The problem seems to be neglected.

The author had observed the phenomenon earlier, during his study of the BS system [Krzyżaniak S. 2015, Krzyżaniak 2016]. There, the difference between reorder level B and actual effective stock level upon the start of replenishment cycle $S_e$ was defined as Δ and it will be used that way further in the present study:

$$\Delta = B - S_e \qquad (5)$$

It might be said that Δ defines the scale of transferring reorder level from assumed level B to actual level $S_e$.

For further considerations, the following abbreviations have been adopted:

**D** – arithmetic mean from all demand quantities in adopted periods τ,

$\sigma_D$ – standard deviation from all demand quantities,

**D*** – mean from non-zero demand quantities (the analysis and the calculation of mean quantity and of standard deviation does not cover periods without demand: $D_i = 0$),

$\sigma_{D^*}$ – standard deviation calculated with non-zero demand quantities.

All of the author's above-mentioned studies on the BS system present the model of calculating transfer Δ as a demand distribution function of D* and the difference between levels B and S (**r** = S – B). At the same time, the studies proved the influence of the difference **r** on actual service level calculated in accordance with a classic probabilistic definition describing service level as probability of non-occurrence of stock-out situation (stock deficit) in a period covering replenishment cycle LT: Service Level α (αSL) [Tempelmeier H. , 2000].

The factor that inspired the studies whose results have been presented in the present article was a question whether a similar phenomenon might occur in a system based on reorder level BQ. It can be proven [Krzyżaniak 2013] that in the case of fast-moving goods where demand distribution may be considered normal and a $D^* = D$, transfer Δ is practically constant (apart from the case of very small delivery quantities Q) and equal to a half of mean demand recorded in a period between the moments of possible orders (or a half of mean quantity of a single release), and does not show any dependency on delivery quantity Q. Thus, delivery quantity seems not to influence service level measured with the probability of non-occurrence of stock deficit αSL. It must be emphasized, however, that in the case of a service level indicator based on the fill rate degree βSL, the influence is meaningful.

A decision was taken to verify if such absence of dependences will occur also with reference to demand whose distribution is different from the normal one. To enable comparison of the studies with the studies previously carried out for the BS system, demand distribution illustrated in Figure 2 was adopted.

Simulation tests were performed with the use of a formerly developed by the author application in an EXCEL spreadsheet. Quantities adopted in the experiment have been presented in Table1.

At the first stage, the course of service level αSL changes was determined for selected controlling parametres, depending on delivery quantity Q. When defining reorder point B, attention should be drawn to a difficulty in determining safety coefficient ω (formula 2). For obvious reasons, coefficients resulting from normal distribution may not be applied here. In practice, each type of demand change and resulting demand distribution nature would require separate determination of the coefficient.

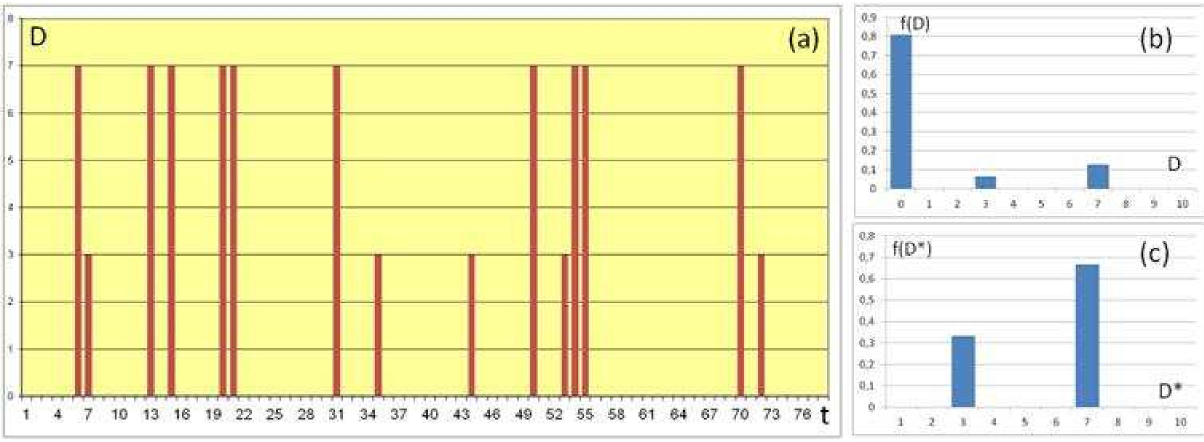


Fig. 2. Distribution of demand adopted for the purposes of simulation studies: (a) – Example time series resulting from the assumed demand distribution (b). Fig. (c) presents a demand distribution which excludes periods with no releases

Rys. 2. Przyjęty do badań symulacyjnych rozkład popytu: (a) – przykładowy szereg czasowy wynikający z założonego rozkładu częstości występowania popytu (b). Rysunek (c) przedstawia rozkład popytu nieuwzględniający okresy z brakiem wydań

Table 1. Quantities adopted in the experiment
Tabela 1. Wielkości przyjęte w eksperymencie

| Quantity | Value |
|---|---|
| Demand distribution (probability density distribution) | p(D=0) = 0.808<br>p(D=3) = 0.064<br>p(D=7) = 0.128 |
| Mean demand in the period | **D** = 1.09 |
| Standard deviation of periodic demand | $\sigma_D$ = 2.17 |
| Demand distribution (probability density distribution) excluding periods without demand | p(D=3/D>0) = 0.333<br>p(D=7/D>0) = 0.667 |
| Mean demand in the period (excluding periods without demand) | $\mathbf{D}^*$ = 5.67 |
| Standard deviation of periodic demand (excluding periods without demand) | $\sigma_{D*}$ = 1.885 |
| Replenishment cycle lead time | LT = 4 periods |
| Standard deviation of replenishment lead time. | $\sigma_{LT}$ = 0 |
| Reorder level | B = 12** |
| Delivery quantity - variable | Q ∈ ⟨13; 37⟩ |

** The quantity has been adopted by experiment. It allows the achievement of service level of αSL≈86% on average. This relatively low quantity was adopted on the basis of trial tests, which showed that the lower the expected service level, the more distinct the expected dependence αSL=f(Q)

The fundamental simulation test consisted in studying the influence of delivery quantity Q on service level αSL. The results of the study have been presented in Figure 3.

At the same time, changes in the difference (transfer) Δ in function Q was measured. Their course has been presented in Fig. 4.

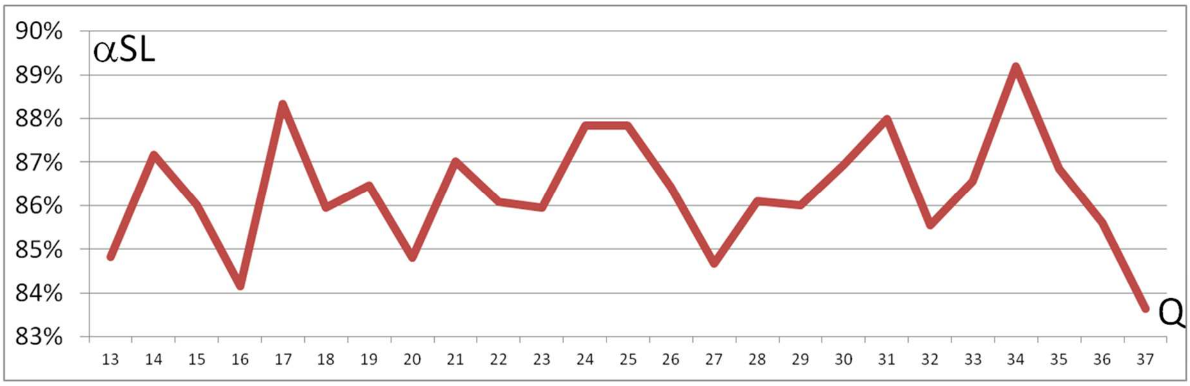


Fig. 3. The course of service level dependence (measured with the probability to serve demand αSL) on delivery quantity (for data provided in Table 1)
Rys. 3. Otrzymany drogą symulacji przebieg zależności poziomu obsługi (mierzonego prawdopodobieństwem obsłużenia popytu αSL) od wielkości dostawy (dla danych określonych w tablicy 1)

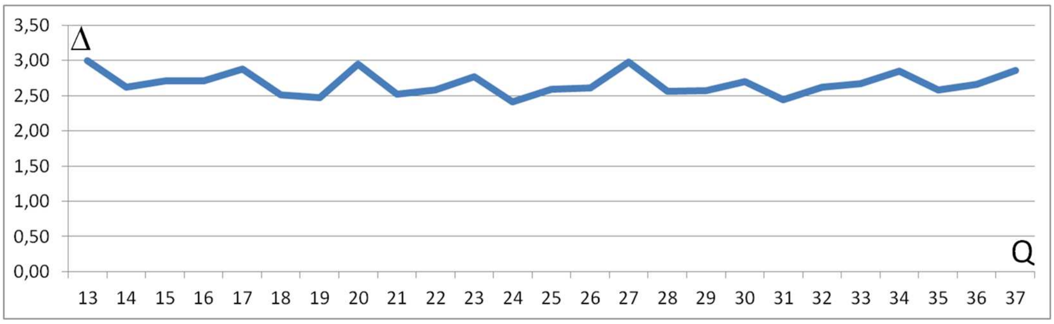


Fig. 4. The course of changes in difference (transfer) Δ between reorder level B and actual effective stock level upon starting the stock ($S_e$) replenishment cycle, obtained as a result of simulation. (for data specified in Table 1)
Rys. 4. Otrzymany drogą symulacji przebieg zmian różnicy (przesunięcia) Δ pomiędzy poziomem informacyjnym B, a rzeczywistym poziomem zapasu efektywnego w chwili rozpoczęcia cyklu uzupełnienia zapasu $S_e$. (dla danych określonych w tablicy 1)

Although there is a certain co-dependence between changes in difference Δ and changes in service level αSL (Figures 3 and 4), they cannot be used to explain the scale of fluctuations of the service level indcator αSL. While the influence of changes in difference Δ on service level in the case of the BS level is obvious and possible to explain [Krzyżaniak 2015], in the case of the BQ system these changes are impossible to explain only on the basis on observed difference changes Δ. It has been illustrated in Figure 5. It presents three courses: αSL = f(Q) for the BQ system, Δ = f(Q) for the BQ system and Δ = f(r) = f(S – B) for the BS system.

A model allowing the prediction of difference volume Δ as a function of delivery quantity Q, on the basis of the previously presented [e.g. Krzyżaniak 2015] model of determining difference Δ as the function of difference r=(S – B) for the BS system, has been developed. It is presented in Table 2.

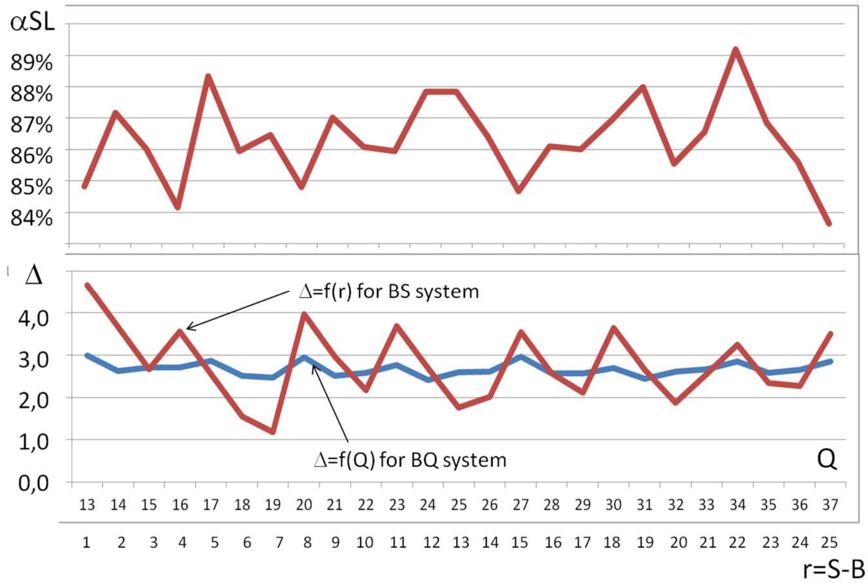


Fig. 5. Courses of dependence: αSL = f(Q) for the BQ system, Δ = f(Q) for the BQ system obtained by way of simulation and Δ = f(r) = f(S – B) for the BS system obtained by formula (6).
Rys. 5. Przebiegi zależności: αSL = f(Q) dla systemu BQ, Δ = f(Q) dla systemu BQ otrzymane drogą symulacji oraz Δ = f(r) = f(S – B) dla systemu BS otrzymany w oparciu o formułę (6).

Table 2. Rules of calculating difference Δ as the function of difference r=(S – B) for the BS system
Tabela 2. Zasady obliczania różnicy Δ jako funkcji różnicy r=(S – B) dla systemu BS

| | $D^*_i$ | $D^*_1=1$ | $D^*_2=2$ | $D^*_3=3$ | $D^*_4=4$ | . | $D^*_{max-1}$ | $D^*_{max}$ |
|---|---|---|---|---|---|---|---|---|
| r | $\Sigma f(D^*_i)=1$ | $f(D^*_1)$ | $f(D^*_2)$ | $f(D^*_3)$ | $f(D^*_4)$ | . | $f(D^*_{max-1})$ | $f(D^*_{max})$ |
| $r_1=1$ | **$\Delta(r_1)$** | **0** | **1** | **2** | **3** | . | **$D^*_{max-2}$** | **$D^*_{max-1}$** |
| $r_2=2$ | **$\Delta(r_2)$** | $\Delta(r_1)$ | **0** | **1** | **2** | . | **$D^*_{max-3}$** | **$D^*_{max-2}$** |
| $r_3=3$ | **$\Delta(r_3)$** | $\Delta(r_2)$ | $\Delta(r_1)$ | **0** | **1** | . | **$D^*_{max-4}$** | **$D^*_{max-3}$** |
| $r_4=4$ | **$\Delta(r_4)$** | $\Delta(r_3)$ | $\Delta(r_2)$ | $\Delta(r_1)$ | **0** | . | **$D^*_{max-5}$** | **$D^*_{max-4}$** |
| ……………………………………………………………………………………………………… | | | | | | | | |
| $r_{max-1}$ | **$\Delta(r_{max-1})$** | $\Delta(r_{max-2})$ | $\Delta(r_{max-3})$ | $\Delta(r_{max-4})$ | $\Delta(r_{max-5})$ | . | | |
| $r_{max}$ | **$\Delta(r_{max})$** | $\Delta(r_{max-1})$ | $\Delta(r_{max-2})$ | $\Delta(r_{max-3})$ | $\Delta(r_{max-4})$ | . | | |

| $G[P^*_i;r_j] = 0$ | $G[P^*_i;r_j] = 1$ |
|---|---|

Designations adopted in Table 2:

$D^*_i$ = i-th quantity of non-zero demand (after putting it in order from $D^*=1$ to $D^*=D^*max$)
$f(D^*_i)$ = non-zero demand occurrence frequency $D^*_i$ (probability of demand $D = D^*_i$ occurrence)
$r_i$ = i-th difference between maximum level S and reorder level B.

The above elements allow presenting the rule for calculating expected value $E(\Delta_j)$ for the BS system in the following manner [Krzyżaniak 2015]:

$$E(\Delta_j) = \Delta(r_j) = \sum_{i=1}^{i_{max}} G[D^*_i; r_j] \cdot (D^*_i - r_j) \cdot f(D^*_i) + \sum_{i=1}^{i_{max}} \{1 - G[D^*_i; r_j]\} \cdot \Delta(r_j - D^*_i) \cdot f(D^*_i) \qquad (6)$$

where: $G[D^*_i; r_j] = \begin{cases} D^*_i \geq r_j \rightarrow G[D^*_i; r_j] = 1 \\ D^*_i < r_j \rightarrow G[D^*_i; r_j] = 0 \end{cases}$

On the basis of the above model, mean quantity of differences (transfers) Δ for the BQ system may be estimated using a simple recursive model.

$S_1 = Q$ *(momentary quantity of level S after the first delivery)*
$\Delta_1 = f_\Delta(S_1 - B)$ *(according to formula 6)*
$S_2 = B - \Delta_1 + Q$
.............................................
$\Delta_{N-1} = f_\Delta(S_{N-1} - B)$
$S_N = B - \Delta_{N-1} + Q$

Mean value: $\bar{\Delta} = \frac{\sum \Delta_i}{N}$ where N is the number of observed cycles (7)

Attention should be drawn to the fact that the scope of variability of delivery quantity Q has been chosen for the BQ system so that differences (Q - B) correspond to differences (S - B) for the BS system, which allows the comparison of results.

## INDICATING THE CAUSES OF OBSERVED INFLUENCE OF DELIVERY QUANTITY ON SERVICE LEVEL

To explain the observed influence of delivery quantity Q on service level $\alpha$SL, much greater than it might result from the changes to difference $\Delta$, a hypothesis was formulated that in certain situations difference $\Delta$ may significantly deviate from mean volume (Fig. 4, formula 7) and, therefore, affect service level.

Detailed analysis of courses of stock level changes in the function of time, obtained as a result of simulation, shows that stock deficit (stock-out) occurrence frequency in two successive replenishment cycles is depending on delivery quantity (Fig. 6). The courses presented refer to two delivery quantities $Q_1$=19 and $Q_2$=20. It might be assumed that such an insignificant change in delivery quantity cannot influence service level. Still, such a dependence has been observed. It may be explained in the following way: in the event of stock deficit occurrence in "i" cycles, following a delivery commencing the following cycle "i+1", stock level is equal to Q. Thus, the cycle may be compared to an stock replenishment cycle in the BS system, when S = Q. It means that expected transfer $\Delta$ will depend on difference (Q – B), as it was in the BS system in reference to dependence $\Delta$ = f(r) = f(S – B).

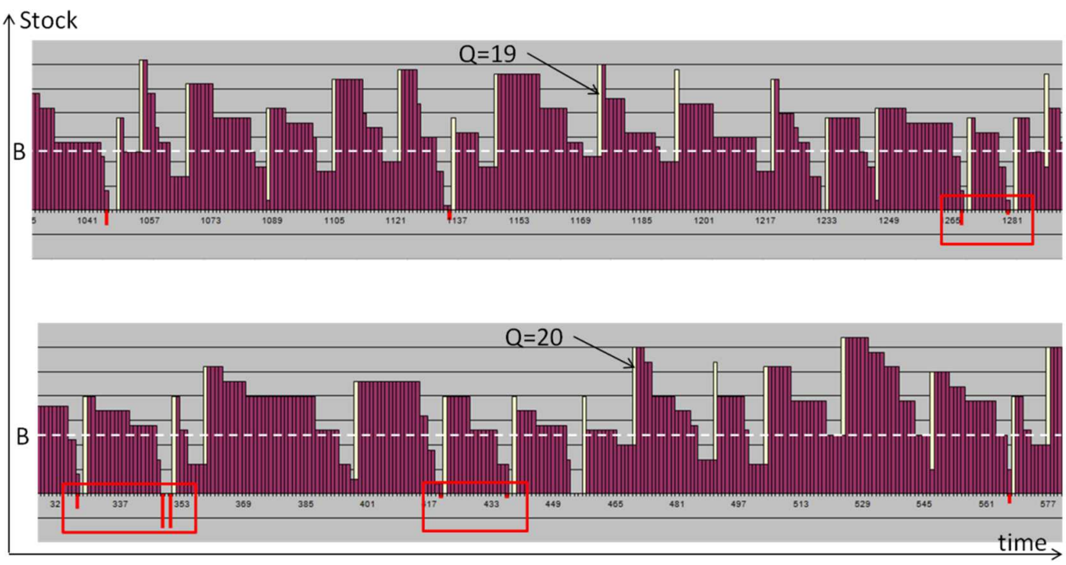


Fig. 6. Illustration of the influence of delivery quantity on stock deficit occurrence frequency in two successive replenishment cycles. The boxes present two successive replenishment cycles with stock deficits
Rys. 6. Ilustracja wpływu wielkości dostawy na częstotliwość (prawdopodobieństwo) występowania braku w zapasie w dwóch kolejnych cyklach uzupełnienia. W ramkach zaznaczono dwa kolejne cykle uzupełnienia, w których wystąpił brak w zapasie

To verify this conclusion, stock deficit occurrence frequencies in two successive cycles for Q=19 and Q=20 were studied by means of simulation. In accordance with the above consideration:

$$\Delta(Q = 19) = \Delta(19 - B) = \Delta(19 - 12) = \Delta(7) = 1{,}18,$$

$$\Delta(Q = 20) = \Delta(20 - B) = \Delta(20 - 12) = \Delta(8) = 3{,}96.$$

Event occurrence frequencies observed during simulation tests "in a cycle after a cycle with a stock deficit" (f') differed significantly depending on delivery quantity:

$$f''(Q = 19) = 0{,}094,$$

$$f''(Q = 20) = 0{,}242.$$

The above occurrence frequencies may be adopted as estimated probability of deficit occurrence in cycle "i+1" ($d_{i+1}$), if a deficit occurred in cycle "i" ($d_i$):

$$p'' = p(d_{i+1}/d_i) \qquad (8)$$

Obtained results have been presented in Figure 7.

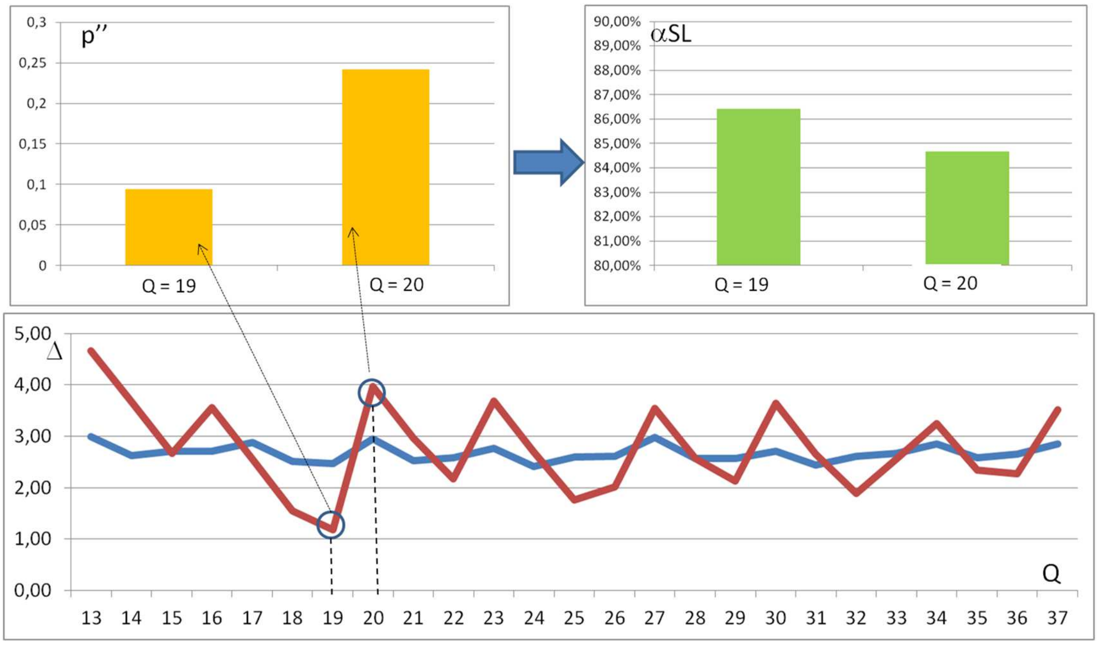


Fig. 7. Illustration of the influence of difference (transfer) Δ corresponding to difference S – B for the BS system BS (in accordance with Fig. 5) on stock deficit occurrence probability in two successive delivery cycles "p"', and, consequently, on service level αSL

Rys. 7. Ilustracja wpływu różnicy (przesunięcia) Δ odpowiadającej różnicy S – B dla systemu BS (zgodnie z rys. 5) na prawdopodobieństwo wystąpienia braku w zapasie w dwóch kolejnych cyklach dostawy p'', a w konsekwencji na poziom obsługi αSL

A simple service level model was developed on the basis of the above considerations:

$$\alpha SL = \alpha SL_0 \cdot \alpha SL_0 + (1 - \alpha SL_0) \cdot \alpha SL'' \qquad (9)$$

where:

$\alpha SL$ – expected mean service level,

$\alpha SL_0$– expected service level if a deficit did not occur in the previous cycle:

$$\alpha SL_0 = f[\bar{\Delta}(Q)],$$

$\alpha SL''$– expected service level if a deficit occurred in the previous cycle:

$$\alpha SL'' = f[\Delta(Q - B)].$$

Formula (9) shows that if the fact of stock deficit occurrence in cycle "i" did not affect deficit occurrence probability in the following cycle("i+1"), i.e. $p(d_{i+1}/d_i) = p(d_i)$ (formula 8) and $\alpha SL_0 = \alpha SL''$, the following would occur: $\alpha SL = \alpha SL_0$.

Practical application of formula (9), however, requires the knowledge of function $\alpha SL_0 = f[\bar{\Delta}(Q)]$ and $\alpha SL'' = f[\Delta(Q - B)]$, which should be the subject of further works. Thus, in its current shape, the model presented in formula (9) serves rather as an illustration of co-dependence between specific quantities, not an actual tool allowing the calculation of expected service level.

## FRAMEWORK RULES GOVERNING THE SELECTION OF DELIVERY QUANTITY

On the basis of the observations presented above, one might suggest the following course of action to choose delivery quantity which lower service level in the least significant way:

1. Define the distribution of "non-zero" demand, excluding periods without demand $f(D^*)$,
2. Using the algorithm presented in Table 2 (and discussed in detail in the author's earlier paper [Krzyżaniak 2015]), determine the course of dependence $\Delta = f(S - B)$, characteristic for the BS system,
3. On the basis of the course, identify quantities of difference $r = S - B$, for which $\Delta = f(S - B)$ demonstrates local minimums,
4. Taking advantage of the fact that for the BQ system considered, differences $S - B$ correspond to differences $Q - B$, select one or more quantity from a set of quantities $S - B$ $(Q - B)$ whose delivery quantity Q is allowed in technical, organisational and economic terms.

## CONCLUSIONS

Irregular demand distributions, characteristic for occasionally occurring demand may be related to the influence of delivery quantity Q on service level. A similar phenomenon was observed during the study of stock replenishment in the BS system, where service level depends, for certain courses of time series related to demand D = f(t), on the difference between levels of controlling parametres S and B. In the case of the system based on reorder level BQ, the dependence manifests itself in cycles occurring directly after the cycles in which stock deficits occurred (assuming that the so-called pent-up demand is not taken into consideration). As a consequence, observed service level (in this case measured with the probability to serve demand in a specific cycle) depends on order/delivery quantity. The main achievement of the presented study is the identification of factors causing observed dependencies of service level upon order quantity, elaboration of a general model of service level dependence on delivery quantity and a simple procedure supporting the choice of order/delivery quantity.

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# WPŁYW WIELKOŚCI DOSTAW NA POZIOM OBSŁUGI W SYSTEMIE ODNAWIANIA ZAPASU OPARTYM NA POZIOMIE INFORMACYJNYM DLA NIETYPOWYCH ROZKŁADÓW POPYTU

**STRESZCZENIE**. **Wstęp:** Jednym z dwóch klasycznych systemów odnawiania zapasu jest system oparty na poziomie informacyjnym, w którym zamówienie o stałej wielkości Q jest składane po osiągnięciu przez dostępny zapas poziomu B (poziomu informacyjnego, punktu ponownego zamówienia). Znana jest prosta zależność pomiędzy wartością poziomu informacyjnego B, a poziomem obsługi, najczęściej definiowanym jako prawdopodobieństwo obsłużenia popytu (POP) w danym cyklu uzupełniania zapasu. Zagadnienie komplikuje się jednak w przypadku, gdy rozkład częstości występowania popytu jest nietypowy i odbiega znacząco od normalnego, co jest charakterystyczne zwłaszcza w przypadku dóbr wolno rotujących. Ujawnia się tu istotny wpływ wielkości zamówienia na rzeczywisty poziom zapasu, przy którym rozpoczyna się cykl uzupełnienia (niższy niż założony poziom B), co znajduje swoje odzwierciedlenie w niższym niż się oczekuje poziomie obsługi. Celem naukowym badań przedstawionych w artykule było objaśnienie (w oparciu o przestawione wyniki badań symulacyjnych) obserwowanego zjawiska oraz przedstawienie rekomendacji dla wyznaczania wielkości dostaw, które w najmniejszym stopniu wpływają na poziom obsługi.

**Metody:** Przedmiotem prezentowanych badań było zidentyfikowanie skali zjawiska na wybranym przykładzie nietypowego rozkładu popytu. Dla wyznaczenia wpływu wielkości dostawy Q na wielkość różnicy Δ pomiędzy poziomem informacyjnym B (którego osiągnięcie lub zejście poniżej stanowi sygnał do złożenia zamówienia), a rzeczywistym poziomem zapasu w chwili rozpoczęcia cyklu uzupełnienia, przeprowadzono badania symulacyjne wykorzystując do tego autorskie narzędzie wykonane w arkuszu kalkulacyjnym EXCEL. Ponieważ stosunkowo niewielka skala zmienności różnicy Δ w funkcji wielkości zamówienia Q nie tłumaczy obserwowanej skali zmian poziomu obsługi, postawiono i potwierdzono (drogą badań symulacyjnych) hipotezę, zgodnie z którą

prawdopodobieństwo wystąpienia braku zapasu (lub przynajmniej sytuacji, w której zapas jest równy zero) w dwóch następujących po sobie cyklach uzupełnienia zależy od przebiegu zależności różnicy Δ od różnicy pomiędzy poziomami S i B stanowiącymi parametry sterujące odnawianiem zapasu w systemie BS, będącym jednym z systemów klasy MIN-MAX.
**Wyniki:** Przedstawiono wyniki badań wpływu wielkości dostawy Q na poziom obsługi POP Wykazano konieczność uwzględnienia przy wyznaczaniu wielkości dostawy Q różnicy Δ i jej zależności od różnicy pomiędzy poziomami S i B stanowiącymi parametry sterujące odnawianiem zapasu w systemie BS. Zaproponowano sposób postępowania pozwalający na unikanie zwiększonego ryzyka wystąpienia braku zapasu w dwóch następujących po sobie cyklach uzupełnienia, przekładającego się na obniżenie poziomu obsługi.
**Wnioski:** Przedstawione w artykule wyniki badań, a także proponowany sposób postępowania pozwolą na określenie właściwych wielkości dostaw przy odnawianiu zapasu w systemie opartym na poziomie informacyjnym (BQ), zwłaszcza w przypadku nietypowych rozkładów częstości występowania popytu. Umożliwi to unikanie takich wielkości dostaw, które rodzą ryzyko obniżenia poziomu obsługi. Dalszych prac wymaga opracowanie kompleksowego modelu określania oczekiwanego poziomu obsługi w funkcji wielkości dostawy.

**Słowa kluczowe:** zarządzanie zapasami, odnawianie zapasu w systemie opartym na poziomie informacyjnym BQ, punkt ponownego zamówienia, poziom obsługi, symulacja.

# EINFLUSS VON LIEFERGRÖßEN AUF DAS SERVICENIVEAU IM SYSTEM DER AN DAS INFORMATIONSNIVEAU FÜR UNTYPISCHE NACHFRAGEVERTEILUNGEN GESTÜTZTEN VORRATSERNEUE-RUNG

**ZUSAMMENFASSUNG.** Einleitung: Eines der zwei klassischen Systeme für die Vorratserneuerung ist das an das Informationsniveau gestützte System, in dem die Bestellung mit fester Größe Q nach dem Erreichen des Niveaus B (des Informationsniveaus, des Zeitpunktes einer erneuten Bestellung) durch den greifbaren Vorrat gestartet wird. Es ist offenbar bekannt der einfache Zusammenhang zwischen dem Wert des Informationsniveaus B und dem Serviceniveau, das am meisten als die Wahrscheinlichkeit der Nachfragebedienung (POP) im gegebenen Zyklus der Vorratsergänzung definiert wird. Das Problem kompliziert sich allerdings im Falle, wenn die Verteilung der Nachfrage-Frequenz untypisch ist und sich wesentlich von einer normalen Nachfrage unterscheidet, was besonders charakteristisch bei langsam rotierenden Gütern der Fall ist. Dann manifestiert sich der wesentliche Einfluss einer Bestellgröße auf das wirkliche Vorratsniveau, bei dem der Ergänzungszyklus (niedriger als das angenommene Niveau B) beginnt, was seine Widerspiegelung in einem niedrigeren als erwartet Serviceniveau findet. Das Ziel der im Artikel dargestellten Forschungen war es, die wahrgenommenen Erscheinungen (in Anlehnung an die projizierten Ergebnisse der Simulationsuntersuchungen) zu erklären und die Empfehlungen für die Bestimmung von Liefergrößen, die im kleinsten Ausmaße das Serviceniveau beeinflussen, zu unterbreiten.
**Methoden:** Der Gegenstand der projizierten Forschungen war die Ermittlung am ausgewählten Beispiel der Skala der Erscheinung einer untypischen Nachfrageverteilung. Zum Zwecke der Bestimmung des Einflusses einer Liefergröße Q auf die Größe der Differenz Δ zwischen dem Informationsniveau B (dessen Erreichen oder Unterschreiten ein Signal zum erneuten Bestellen gibt) und dem tatsächlichen Vorratsniveau zum Zeitpunkt des Anbeginns des Erneuerungszyklus wurde eine Simulationsuntersuchung mithilfe eines anhand der EXCEL-Kalkulationstabelle konzipierten Tools durchgeführt. Da die verhältnismäßig kleine Skala der Variabilität der Differenz Δ in der Funktion der Bestellgröße Q das beobachtete Ausmaß der Veränderungen des Serviceniveaus nicht erklärt, stellte und bestätigte man (mithilfe von Simulationsuntersuchungen) eine Hypothese, laut deren die Wahrscheinlichkeit des Auftretens eines Vorratsmangels (oder wenigstens einer Situation, in der der Vorrat einem Null gleicht) in zwei nacheinander folgenden Zyklen der Erneuerung vom Verlauf der Abhängigkeit der Differenz Δ von der Differenz zwischen den Niveaus S und B, die die Vorratserneuerung steuernden Parameter sind im System BS, das als eines der Systeme von der Klasse MIN-MAX besteht, abhängt.
**Ergebnisse:** Es wurden Forschungsergebnisse in Bezug des Einflusses der Liefergröße Q auf das Serviceniveau POP dargestellt. Dabei zeigte man die Notwendigkeit einer Berücksichtigung bei der Bestimmung der Liefergröße Q der Differenz Δ und ihrer Abhängigkeit von der Differenz zwischen den Niveaus S und B, die als die die Vorratserneuerung im System BS steuernden Parameter gelten, auf. Es wurde eine Vorgehensweise, die Vermeidung eines erhöhten Risikos des Auftretens eines Vorratsmangels in zwei nacheinander folgenden Erneuerungszyklen, die wiederum eine Herabsetzung des Serviceniveaus zu Folge haben, ermöglicht, vorgeschlagen.

**Fazit:** Die im Artikel projizierten Forschungsergebnisse und die vorgeschlagene Vorgehensweise erlauben die Bestimmung der richtigen Liefermengen bei der Vorratserneuerung innerhalb des an das Informationsniveau (BQ) gestützten Systems, insbesondere im Falle einer untypischen Frequenzverteilung bei der Nachfrage. Dies ermöglicht die Vermeidung solcher Liefermengen, die ein Risiko der Absenkung des Serviceniveaus verursachen können. Die Ausarbeitung eines komplexen Modells zur Bestimmung des erwarteten Serviceniveaus in der Funktion der Liefergröße bedarf noch weiterer Arbeiten.

**Codewörter:** Bestandsführung, Vorratserneuerung im an das Informationsniveau BQ gestützten System, Zeitpunkt der erneuten Bestellung, Serviceniveau, Simulation

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