LogForum

> Scientific Journal of Logistics <

2017, 13 (1), 61-76

http://dx.doi.org/10.17270/J.LOG.2017.1.6

http://www.logforum.net p-ISSN 1895-2038 e-ISSN 1734-459X

ORIGINAL PAPER

OPTIMIZATION IN FUZZY ECONOMIC ORDER QUANTITY (FEOQ) MODEL WITH PROMOTIONAL EFFORT COST AND UNITS LOST DUE TO DETERIORATION

Padmabati Gahan, Monalisha Pattnaik

Sambalpur University, Sambalpur, **India**

ABSTRACT. **Background:** This model presents a significant model for analyzing the effect of deteriorating items and promotional effort in fuzzy optimal instantaneous replenishment model for finite planning horizon. Accounting for holding cost per unit per unit time and ordering cost per order have traditionally been the case of modeling inventory systems in fuzzy environment. These imprecise parameters defined on a bounded interval on the axis of real numbers and the physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in the production system.

Methods: This model postulates the promotional effort cost to frame total inventory cost. Thus a modified fuzzy EOQ (FEOQ) model with promotional effort factor is introduced, it assumes that a percentage of the on-hand inventory is wasted due to deterioration and considered as an enhancement to EOQ model to determine the optimal promotional effort and the replenishment quantity so that the net profit is maximized. In theoretical analysis, the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions are proved and further the concavity of the fuzzy net profit function is established. Computational algorithm using the software LINGO 13.0 version is developed to find the optimal solution.

Results and conclusions: The results of the numerical analysis enable decision-makers to quantify the effect of promotion policy on optimizing the net profit for the retailer and wasting the percentage of on-hand inventory due to deterioration respectively. Finally, sensitivity analyses of the optimal solution with respect the major parameters are also carried out. Furthermore fuzzy decision making is shown to be superior then crisp decision making without promotional effort in terms of profit maximization.

Key words: logistics, optimization, fuzzy, FEOQ, promotion, units lost, deterioration.

INTRODUCTION

In the whole production system production function is the mid between the procurement function and physical distribution function. Other two functions are not processing in terms of production only they are facilitating for the smooth functioning and cost effecting of the production system in competitive advantage but production function processes to produce the finished products. So inventory plays a significant role in smooth functioning of the production function in a supply chain management. The physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in production system. The question is how reliable are the EOQ models when items stocked deteriorate one time.

Many models have been proposed to deal with a variety of inventory problems. Comprehensive reviews of inventory models can be found in Gupta and Gerchak [1995], Osteryoung et al. [1986] and Water [1994] and Tripathy et al. [2013] introduced a single item

Citation: Gahan P., Pattnaik M., 2017, Optimization in Fuzzy Economic Order Quantity (FEOQ) Model with Promotional Effort Cost and Units Lost Due to Deterioration. LogForum 13 (1), 61-76, http://dx.doi.org/10.17270/J.LOG.2017.1.6

Received: 18.04.16 Accepted: 22.09.2016, on-line: 30.12.2016.

EOQ model with two constraints. This model considers a continuous review, using fuzzy arithmetic approach to the system cost for instantaneous production process. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in Salameh et al. [1999]. From practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. Vujosevic et al. [1996] introduced the EOQ model where inventory system cost is fuzzy. Mahata and Goswami [2006] then presented production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating items where permissible delay in payments are allowed. Tripathy and Pattnaik [2011] presented an optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Later, Tripathy and Pattnaik [2009, 2011] also investigated fuzzy EOQ model with reliability consideration in instantaneous production plan. Again Tripathy and Pattnaik [2008, 2011] developed fuzzy entropic order quantity model for perishable items under two component demand and discounted selling price, where entropic means the amount of the disorder in the production system. Pattnaik [2013] discussed the fuzzy EOQ model with demand dependent unit price and variable setup cost, Pattnaik [2011, 2013, 2013] investigated the fuzzy method for supplier selection in manufacturing system for smooth function of supply chain management and manpower selection for micro, small and medium enterprises respectively. For this reason, this model considers the same by introducing the holding cost and ordering cost as with allowing promotion and wasting the percentage of the fuzzy numbers. Sahoo and Pattnaik [2013] developed linear programming problem and post optimality analyses in fuzzy space with case study applications. Pattnaik [2013] defined linear programming problems with crisp and fuzzy based optimization methods and sensitivity analyses have also evaluated for decision parameters. Pattnaik [2013] derived profit maximization fuzzy EOQ models for deteriorating items with two dimension sensitive demand. The model provides an approach for quantifying the benefits of nonrandom uncertainty which can be substantial, and should be reflected in fuzzy arithmetic system cost.

Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles is available in inventory literature considering deterioration. Interested readers may consult the survey model of Pattnaik [2011] investigated an entropic order quantity model for perishable items with pre and post deterioration discounts under two component demand in finite horizon. Pattnaik [2011] discussed an economic order quantity model for perishable items with constant demand where instant deterioration discount is allowed to obtain maximum profit. Goyal and Gunasekaran [1995] and Raafat [1991] surveyed for perishable items to optimize the EOQ model. The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Tripathy and Pattnaik [2008, 2011] studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where discounts are allowed for acquiring more profit. Pattnaik [2012] derived different types of typical deterministic EOQ models in crisp and fuzzy decision space.

Comprehensive reviews of inventory models under deterioration can be found in Bose et al. [1995]. In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver [1994] developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Gupta and Gerchak [1995] examined the simultaneous selection product durability and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Hariga [1995] considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga [1994] argued that the analysis of Bose et al. [1995] contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Padmanavan and Vrat [1995] presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. The most recent work found in the literature is that of Hariga [1996] who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Goyal et al. [2001] and Shah [2000] explored the inventory models for deteriorating items. Pattnaik [2010, 2011] studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where instant deterioration and post deterioration cash discounts respectively are allowed for acquiring more profit. Pattnaik [2011] developed an entropic order quantity model for deteriorating items where cash discounts are allowed but Pattnaik [2011] modified again to obtain the decision parameters for perishable items where instant deterioration discount is allowed in EOQ model. Pattnaik [2012] introduced a non linear profit maximization entropic order quantity model for deteriorating items with stock dependent demand rate. Pattnaik [2012] derived an EOQ model for perishable items with constant demand and instant deterioration.

Furthermore, retailer promotional activity has become more and more common in today's business world. For example, Wall Mart and Costco often try to stimulate demand for specific types of electric equipment by offering price discounts; clothiers Baleno and NET make shelf space for specific clothes items available for longer periods; McDonald's and Burger King often use coupons to attract consumers. Other promotional strategies include free goods, advertising, and displays and so on. The promotion policy is very important for the retailer. How much promotional effort the retailer makes has a big impact on annual profit. Residual costs may be incurred by too many promotions while too few may result in lower sales revenue. Tsao and Sheen [2008] discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Salameh et al. [1999] studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked. The effect of deteriorating items on the instantaneous profit maximization replenishment model under promotion is considered in this model. The market demand may increase with the promotion of the product over time when the units lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. Tripathy et al. [2012] investigated an optimal EOQ model for deteriorating items with promotional effort cost. Pattnaik [2012] explored the effect of promotion in fuzzy optimal replenishment model with units lost due to deterioration. Hence Pattnaik [2013] developed many instantaneous EOQ models and fuzzy EOQ models which are incorporated with promotional effort cost, fixed ordering cost, variable ordering cost and units lost due to deterioration. This model introduces a modified fuzzy EOQ model in which it assumes that a percentage of the on-hand inventory is wasted due to deterioration. There is hidden cost not account for when modeling inventory cost. This model studies the problem of promotion for a deteriorating item subject to loss of these deteriorated units. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with wasting the percentage of on-hand inventory due to deterioration and then compares the optimal results with none wasting the percentage of on-hand inventory due to deterioration traditional model. This model addresses the problem by proposing an inventory model under promotion by assuming that the units lost due to deterioration of the items. In this model, promotional effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. The objective of this model is to determine the optimal time length, optimal units lost due to deterioration, the promotional effort and the replenishment quantity with fixed ordering cost so that the net profit is maximized in an instantaneous replenishment fuzzy EOQ model and the numerical analysis show that an appropriate promotion policy can benefit the retailer and that promotion policy is important in fuzzy space, especially for deteriorating items. Finally, sensitivity analyses of the optimal solution with respect to the major parameters are also studied to draw the managerial insights. Furthermore crisp decision making is shown to be superior to crisp decision making without promotional effort cost in terms of profit maximization.

This model establishes and analyzes the fuzzy inventory model under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient EOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the EOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus.

In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality, promotion of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality and promotion of the product, these companies have mainly relied on qualitative judgment. The model tackles to investigate the effect of the wasting the percentage of on-hand inventory due to deterioration and the promotional effect cost for obtaining the optimum average payoff and the optimal values of the policy variables. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed and optimum solution is obtained through LINGO 13.0 version software. In order to make the comparisons equitable a particular evaluation function based on promotion is suggested. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer promotional effort in making optimum decision on promotion and replenishment with units lost due to deterioration. Numerical experiment is carried out to analyze the magnitude of the approximation error. However, adding of both promotional effort and wasting the percentage of on-hand inventory due to deterioration in fuzzy model might lead to super gain for the retailer. The major assumptions used in the above research articles are summarized in Table1.

The remainder of the model is organized as follows. In section 2 notations and assumptions are provided for the development of the model. The mathematical formulation is developed in section 3. Section 4 develops the fuzzy model. In section 5, the solution procedure is given. In section 6, an illustrative numerical analysis is given to illustrate the procedure of solving the proposed model. The sensitivity analysis is carried out in section 7 to observe the changes in the optimal solution. Finally section 8 deals with the summary and concluding remark of the FEOQ model.

Table 1. Summary of the related researches
Tabela 1. Zestawienie podobnych badań

Author(s) and published Year	**Structure of the model**	**Demand**	**Demand patterns**	**Deteriora-tion**	**Units Lost due to Deteriora-tion**	**Promotion al effort cost**	**Planning**	**Model**
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	No	No	Finite	Cost
Vujosevic et al. (1996)	Fuzzy (EOQ)	Constant (deterministic)	Constant	No	No	No	Infinite	Profit
Salameh et al. (1999)	Crisp (EOQ)	Constant (deterministic)	Constant	Yes	Yes	No	Finite	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant (deterministic)	Constant	Yes (Instant)	No	No	Finite	Profit
Pattnaik (2011)	Crisp (EOQ)	Constant (deterministic)	Constant	Yes (Instant)	No	No	Finite	Profit
Tsao et al. (2008)	Crisp (EOQ)	Time and Price	Linear and decreasing	Yes	No	Yes	Finite	Profit
Tripathy et al. (2009)	Fuzzy (FEOQ)	Constant (deterministic)	Constant	No	No	No	Finite	Cost
Present Model (2016)	Fuzzy (FEOQ)	Constant (deterministic)	Constant	Yes (Wasting)	Yes	Yes	Finite	Profit

ASSUMPTIONS AND NOTATIONS

- r Consumption rate,
- t_c Cycle length,
- h Holding cost of one unit for one unit of time,
- HC (q, ρ) Holding cost per cycle,
- K Setup cost per cycle,
- c Purchasing cost per unit,
- P_s Selling Price per unit,
- α Percentage of on-hand inventory that is lost due to deterioration,
- q Order quantity,
- q^{**} Modified economic ordering / production quantity (EOQ/EPQ),
- q^* Traditional economic ordering quantity (EOQ),
- ρ The promotional effort per cycle
- PE (ρ) The promotional effort cost, PE $(\rho) = K_1 (\rho-1)^2 r^{\alpha_1}$, where $K_1>0$ and α_1 is a constant
- $\varphi(t)$ On-hand inventory level at time t,
- $\pi_1(q, \rho)$ Net profit per unit of producing q units per cycle in crisp strategy,
- $\pi(q, \rho)$ Average profit per unit of producing q units per cycle in crisp strategy,
- $\tilde{\pi}_1(q, \rho)$ The net profit per unit per cycle in fuzzy decision space,
- $\tilde{\pi}(q, \rho)$ The average profit per unit per cycle in fuzzy decision space,
- $\tilde{h}$ Fuzzy holding cost per unit,
- $\tilde{K}$ Fuzzy setup cost per cycle.

MATHEMATICAL MODEL

Denote $\varphi(t)$ as the on-hand inventory level at time t. During a change in time from point t to t+dt, where t + dt > t, the on-hand inventory drops from $\varphi(t)$ to $\varphi(t+dt)$. Then $\varphi(t+dt)$ is given as:

$$\varphi(t+dt) = \varphi(t) - r\,\rho\,dt - \alpha\,\varphi(t)\,dt$$

$\varphi(t+dt)$ can be re-written as:

$$\frac{\varphi(t+dt) - \varphi(t)}{dt} = -r\rho - \alpha\varphi(t)$$

and $dt \to 0$, $\frac{\varphi(t+dt) - \varphi(t)}{dt}$ reduces to:

$$\frac{d\varphi(t)}{dt} + \alpha\varphi(t) + r\rho = 0$$

It is a differential equation, solution is

$$\varphi(t) = \frac{-r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right) \times e^{-\alpha t}$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. Behavior of the inventory level for the above model is illustrated in Fig. 1. The cycle length, t_c, is determined by first substituting t_c into $\varphi(t)$ and then setting it equal to zero to get: $t_c = \frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)$.

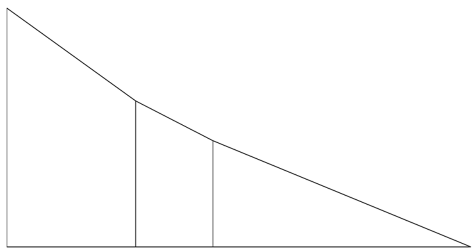

Fig. 1. Behavior of the inventory over a cycle for a deteriorating item
Rys. 1. Cechy zapasów ulegających zniszczeniu w okresie cyklu

Equation $\varphi(t)$ and t_c are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r\rho}$. Then the total number of units lost per cycle, L, is given as:

$$L = r\rho\left[\frac{q}{r\rho} - \frac{1}{\alpha}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$$

The total cost per cycle, TC(q), is the sum of the procurement cost per cycle, K+cq, the holding cost per cycle, HC(q), and the promotional effort cost per cycle, PE (ρ). HC (q) is obtained from equation $\varphi(t)$ as:

$$HC(q) = \int_0^{tc} h\varphi(t)dt = h\int_0^{\frac{1}{\alpha}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)}\left[-\frac{r\rho}{\alpha} + \left(q + \frac{r\rho}{\alpha}\right)\times e^{-\alpha t}\right]dt$$

$$= h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$$

PE (ρ) = $K_1(\rho - 1)^2 r^{\alpha_1}$

TC (q, ρ) = $K + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho - 1)^2 r^{\alpha_1}$

The total cost per unit of time, TCU (q,ρ), is given by dividing equation TC (q, ρ) by t_c to give:

$$TCU(q, \rho) = \left[K + cq + h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2}ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] + K_1(\rho - 1)^2 r^{\alpha_1}\right] \times \left[\frac{1}{\alpha}\ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]^{-1}$$

$$= \frac{K\alpha + (c\alpha + h)q}{ln\left(1 + \frac{\alpha q}{r\rho}\right)} - \frac{hr\rho}{\alpha} + \frac{K_1\alpha(\rho - 1)^2 r^{\alpha 1}}{ln\left(1 + \frac{\alpha q}{r\rho}\right)}$$

As α approaches zero and ρ = 1 equation TCU(q, ρ) reduces to TCU (q, ρ) = $\frac{Kr}{q} + cr + \frac{hq}{2}$

Whose solution is given by the traditional EOQ formula, $q^* = \sqrt{\frac{2Kr}{h}}$. The total profit per cycle is $\pi_1(q,\rho)$.

$\pi_1(q,\rho)$ = (q-L)×Ps – TC (q,ρ)

$= (q\text{-}L)\times Ps - K - cq - h \times \left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} \times ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right] - K_1(\rho - 1)^2 r^{\alpha 1}$

Where L, the number of units lost per cycle due to deterioration, and TC (q,ρ) the total cost per cycle, are calculated from equations L and TC(q,ρ), respectively. The average profit $\pi(q,\rho)$ per unit time is obtained by dividing t_c in $\pi_1(q,\rho)$. Hence the profit maximization problem is

Maximize π_1 (q,ρ)
∀ q > 0, $\rho > 0$
$\pi_1(q,\rho) = F_1(q,\rho) + F_2(q,\rho)h + F_3(q,\rho)K$

Where,
$F_1(q, \rho)$ = (q-L)×Ps- cq$-K_1(\rho - 1)^2 r^{\alpha 1}$
$F_2(q, \rho) = -\left[\frac{q}{\alpha} - \frac{r\rho}{\alpha^2} \times ln\left(\frac{\alpha q + r\rho}{r\rho}\right)\right]$
and F_3 (q, ρ) = -1

FUZZY MATHEMATICAL MODEL

The holding cost and ordering cost are replaced by fuzzy numbers $\tilde{h}$ and $\tilde{K}$ respectively. By expressing $\tilde{h}$ and $\tilde{K}$ as the normal triangular fuzzy numbers *(h_1, h_0, h_2)* and *(K_1, K_0, K_2)*, where, $h_1 = h - \Delta_1$, $h_0 = h$,

$h_2 = h + \Delta_2$, $K_1 = K - \Delta_3$, $K_0 = K$, $K_2 = K + \Delta_4$ such that $0 < \Delta_1 < h$, $0 < \Delta_2, 0 < \Delta_3 < K$, $0 < \Delta_4, \Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy ordering cost are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \dfrac{h - h_1}{h_0 - h_1}, & h_1 \le h \le h_0 \\ \dfrac{h_2 - h}{h_2 - h_0}, & h_0 \le h \le h_2 \\ 0, & otherwise \end{cases}$$

$$\mu_{\tilde{k}}(K) = \begin{cases} \dfrac{K - K_1}{K_0 - K_1}, & K_1 \le K \le K_0 \\ \dfrac{K_2 - K}{K_2 - K_0}, & K_0 \le K \le K_2 \\ 0, & \text{otherwise} \end{cases}$$

Then the centroid for $\tilde{h}$ and $\tilde{K}$ are given by

$$M_{\tilde{h}} = \frac{h_1 + h_o + h_2}{3} = h + \frac{\Delta_2 - \Delta_1}{3} \text{ and}$$

$$M_{\tilde{k}} = \frac{K_1 + K_o + K_2}{3} = K + \frac{\Delta_4 - \Delta_3}{3}$$

respectively.

For fixed values of q and ρ, let $\pi_1(h,K) = F_1(q,\rho) + F_2(q,\rho)h + F_3(q,\rho)K = y$

Let $h = \dfrac{y - F_1 - F_3K}{F_2}$, $\dfrac{\Delta_2 - \Delta_1}{3} = \psi_1$ and $\dfrac{\Delta_4 - \Delta_3}{3} = \psi_2$

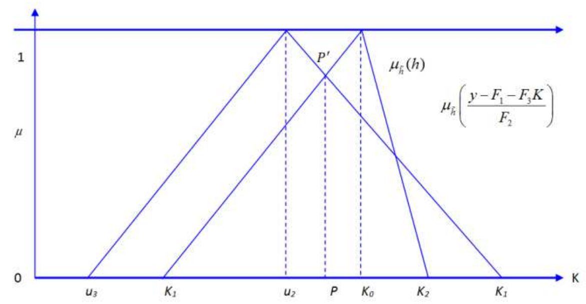

Fig. 2. Defuzzification by using Centroid Method
Rys. 2. Centralizowanie przy zastosowaniu metody Centroid

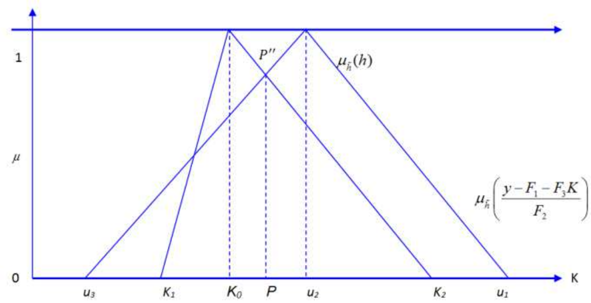

Fig. 3. Defuzzification by using Centroid Method
Rys. 3. Centralizowanie przy zastosowaniu metody Centroid

By extension principle the membership function of the fuzzy profit function is given by

$$\mu(\tilde{y})_{\pi_1(\tilde{h},\tilde{k})} = \sup_{(h,k)\in\pi_1^{-1}(y)} \{\mu_{\tilde{h}}(h) \wedge \mu_{\tilde{k}}(K)\}$$

$$= \sup_{k_1 \le k \le k_2} \left\{\mu_{\tilde{h}}\left(\frac{y - F_1 - F_3K}{F_2}\right) \wedge \mu_{\tilde{k}}(K)\right\}$$

Now,

$$\mu_{\tilde{h}}\left(\frac{y - F_1 - F_3K}{F_2}\right) = \begin{cases} \dfrac{y - F_1 - F_2h_1 - F_3K}{F_2(h_0 - h_1)}, & u_2 \le K \le u_1 \\ \dfrac{F_1 + F_2h_2 + F_3K - y}{F_2(h_2 - h_0)}, & u_3 \le K \le u_2 \\ 0, & \text{otherwise} \end{cases}$$

where, $u_1 = \dfrac{y - F_1 - F_2h_1}{F_3}$,

$u_2 = \dfrac{y - F_1 - F_2h_0}{F_3}$ and $u_3 = \dfrac{y - F_1 - F_2h_2}{F_3}$

Fig. 2 exhibits the graph of $\mu_{\tilde{h}}\left(\dfrac{y - F_1 - F_3K}{F_2}\right)$ and $\mu_{\tilde{h}}(h)$ when $u_2 \le K$ and $K \le u_1$ then $y \le F_1 + F_2h_0 + F_3K_0$ and $y \ge F_1 + F_2h_1 + F_3K_1$. It is clear that for every $y \in [F_1 + F_2h_1 + F_3K_1 \,,\, F_1 + F_2h_0 + F_3K_0]$, $\mu_y(y) = PP$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}\left(\dfrac{y - F_1 - F_3K}{F_2}\right)$ the value of PP may be found by solving the following equation:

$$\frac{K - K_1}{K_0 - K_1} = \frac{y - F_1 - F_2h_1 - F_3K}{F_2(h_0 - h_1)} \text{ or}$$

$$K = \frac{(y - F_1 - F_2h_1)(K_0 - K_1) + F_2K_1(h_0 - h_1)}{F_2(h_0 - h_1) + F_3(K_0 - K_1)}$$

Therefore,
$PP' = \frac{K-K_1}{K_0-K_1} = \frac{y-F_1-F_2h_1-F_3K}{F_2(h_0-h_1)+F_3(K_0-K_1)} = \mu_1(y)$, (say).

Fig. 3 exhibits the graph of $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$ and $\mu_{\tilde{h}}(h)$ when $u_3 \le K$ and $K \le u_2$ then $y \le F_1+F_2h_2+F_3K_2$ and $y \ge F_1+F_2h_0+F_3K_0$. It is evident that for every $y \in [F_1+F_2h_0+F_3k_0\ ,\ F_1+F_2h_2+F_3K_2], \mu_{\tilde{y}}(y) = PP'$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}}\left(\frac{y-F_1-F_3K}{F_2}\right)$, the value of PP' may be found by solving the following equation:

$$\frac{K_2-K}{K_2-K_0} = \frac{F_1+F_2h_2+F_3K-y}{F_2(h_2-h_0)} \text{ or,}$$

$$K = \frac{F_2K_2(h_2-h_0)-(F_1+F_2h_2-y)(K_2-K_0)}{F_2(h_2-h_0)+F_3(K_2-K_0)}$$

Therefore,
$PP'' = \frac{K_2-K}{K_2-K_0} = \frac{F_1+F_2h_2+F_3K_2-y}{F_2(h_2-h_0)+F_3(K_2-K_0)} = \mu_2(y)$, (say).

Thus the membership function for fuzzy total profit is given by

$$\mu_{\pi_1(\tilde{h},K)}(y) = \begin{cases} \mu_1(y); & F_1+F_2h_1+F_3K_1 \le y \le F_1+F_2h_0+F_3K_0 \\ \mu_2(y); & F_1+F_2h_0+F_3K_0 \le y \le F_1+F_2h_2+F_3K_2 \\ 0\ ; & \text{otherwise} \end{cases}$$

Now, let $P_1 = \int_{-\infty}^{\infty} \mu_{\pi_1(\tilde{h},\tilde{k})}(y)\,dy$ and

$$R_1 = \int_{-\infty}^{\infty} y\,\mu_{\pi_1(\tilde{h},\tilde{k})}(y)\,dy$$

Hence, the centroid for fuzzy total profit is given by $\tilde{\pi}_1(q,\rho) = M_{\tilde{TP}}(q,\rho) = \frac{R_1}{P_1}$

$$= F_1(q,\rho)+F_2(q,\rho)h+F_3(q,\rho)K + \psi_1F_2(q,\rho)+\psi_2F_2(q,\rho)$$

$$\tilde{\pi}_1(q,\rho) = M_{\tilde{TP}}(q,\rho) = F_1+(h+\psi_1)F_2+(K+\psi_2)F_3$$

where, $F_1(q,\rho)$, $F_2(q,\rho)$ and $F_3(q,\rho)$ are given by the equations.

Hence the profit maximization problem is Maximize $\tilde{\pi}_1(q,\rho) = M_{\tilde{TP}}(q,\rho)$ $\forall$ q ≥ 0, $\rho \ge 0$

OPTIMIZATION

The objective is to determine the optimal values of q and ρ to maximize the unit profit function. It is very difficult to derive the optimal values of q and ρ, hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variables.

The fuzzy optimal ordering quantity q and fuzzy promotional effort ρ per cycle can be determined by differentiating $\tilde{\pi}_1(q,\rho)$ with respect to q and ρ separately, then setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is jointly concave in terms of fuzzy ordering quantity q and fuzzy promotional effort ρ. The second partial derivates of $\tilde{\pi}_1(q,\rho)$ with respect to q and ρare strictly negative and the determinant of Hessian matrix is positive. Considering the following propositions;

Proposition 1 The net profit $\tilde{\pi}_1$ (q, ρ) per cycle is concave in q.

Conditions for optimal q

$$\frac{\partial\tilde{\pi}_1(q,\rho)}{\partial q} = \frac{r\rho}{\alpha q+r\rho}\left(P_s+\frac{h+\psi_1}{\alpha}\right)-\left(c+\frac{h+\psi_1}{\alpha}\right) = 0$$

The second order partial derivative of the net profit per cycle with respect to q can be expressed as

$$\frac{\partial^2\tilde{\pi}_1(q,\rho)}{\partial q^2} = -\frac{r\rho}{(\alpha q+r\rho)^2}(P_s\alpha+(h+\psi_1))$$

Since rρ> 0 and $P_s\alpha+(h+\psi_1)>0$, $\frac{\partial^2\tilde{\pi}_1(q,\rho)}{\partial q^2}$ is negative.

Proposition 2 The net profit $\tilde{\pi}_1$ (q, ρ) per cycle is concave in ρ.

Conditions for optimal ρ

$$\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial \rho} = \left(ln\left(\frac{\alpha q}{r\rho}+1\right) - \frac{\alpha q}{\alpha q + r\rho}\right)\left(P_s + \frac{(h+\psi_1)}{\alpha}\right)\frac{r}{\alpha} - 2K_1(\rho-1)r^{\alpha 1}=0$$

The second order partial derivative of the net profit per cycle with respect to ρ is

$$\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2} = -\left[\frac{q^2 r\left(P_s+\frac{(h+\psi_1)}{\alpha}\right)}{\rho(\alpha q+r\rho)^2} + 2K_1 r^{\alpha_1}\right].$$

Since $K_1 > 0$, $\rho > 0$, $q^2 r\left(P_s + \frac{(h+\psi_1)}{\alpha}\right) > 0$, so $\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2}$ is negative.

Propositions 1 and 2 show that the second partial derivatives of $\tilde{\pi}_1(q,\rho)$ with respect to q and ρ separately are strictly negative. The next step is to check that the determinant of the Hessian matrix is positive, i.e.

$$\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2} \times \frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2} - \left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q \partial \rho}\right)^2 > 0,$$

$\left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q^2}\right)$ and $\left(\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho^2}\right)$ shown in $\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial q}$ and $\frac{\partial \tilde{\pi}_1(q,\rho)}{\partial \rho}$, $\frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial q \partial \rho} = \frac{\partial^2 \tilde{\pi}_1(q,\rho)}{\partial \rho\, \partial q} = \frac{rq}{(\alpha q + r\rho)^2} \times [P_s\alpha + (h+\psi_1)]$

The average profit per unit time for the following maximization problem is

Maximize $\tilde{\pi}$ (q,ρ)

Subject to $\frac{r\rho}{\alpha q} < \left(\frac{1}{ln\left(\frac{\alpha q}{r\rho}+1\right)} - \frac{1}{2}\right)$

$\forall q,\rho \geq 0$

NUMERICAL EXAMPLE

Consider an inventory situation where K =Rs. 200 per order, h =Rs. 5 per unit per unit of time, r = 1200 units per unit of time, c =Rs. 100 per unit, P_s=Rs. 125 per unit, α= 5%, K_1 = 2.0, α_1 =1.0, Δ_1= 0.002, Δ_2= 0.02, Δ_3= 0.002 and Δ_4=0.2. The optimal solution that maximizes $\tilde{\pi}_1(q,\rho)^*$ and q^{**} and $\rho^*$are determined by using LINGO 13.0 version software and the results are tabulated in Table 2. The results indicate that the fuzzy total profit per unit is decreased as the percentage of on-hand inventory that is lost due to deterioration is increased as shown in Fig. 4. The three-dimensional mesh diagram of order quantity, promotional effort factor and the net profit per unit per cycle is shown in Fig. 6.

Table 2. Fuzzy Optimal Solutions of the Proposed Model
Tabela 2. Rozmyte optymalne rozwiązania dla proponowanego modelu

Model	Iteration	t_c^*	L^*	q^{**}	ρ^*	Promotional Effort Cost	$\tilde{\pi}_1(q,\rho)^*$	$\tilde{\pi}(q,\rho)^*$
Fuzzy	146	2.354328	1470.837	25489.47	8.50	135057.2	170864.7	72574.72
Fuzzy	52	0.258087	-	309.7040	1	0	7342.4685	28449.62

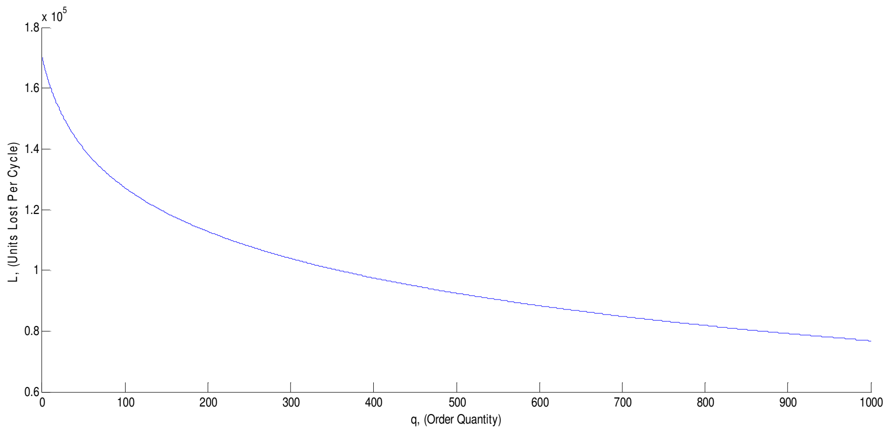

Fig. 4. Two Dimensional Plot of Order Quantity q and Units Lost per Cycle L
Rys. 4. Dwuwymiarowy wykres zależności wielkosci zamówienia q i strat w cyklu L

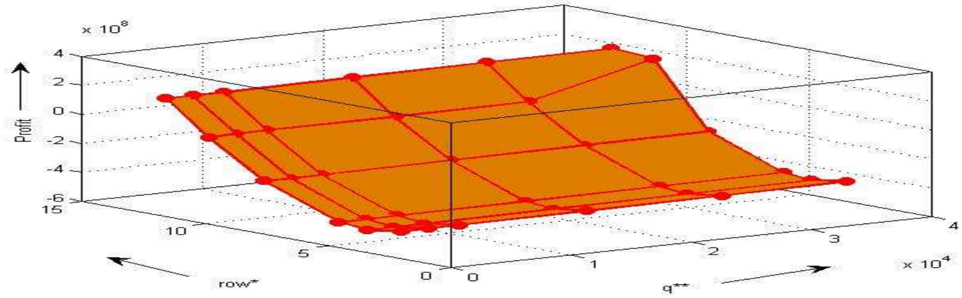

Fig. 5. The Three Dimensional Mesh Plot of Order Quantity q^*, Promotional Effort Factor ρ^* and Fuzzy Net Profit Per Unit Per Cycle $\widetilde{\pi_1}(q,\rho)^*$
Rys. 5. Trójwymarowy wykres zależności wielkosci zamówienia q^*, czynnika promocyjnego ρ^* oraz rozmytego zysku netto na jednostkę na cykl $\widetilde{\pi_1}(q,\rho)^*$

Table 3. Comparative Analysis with Percentage Change in Different Crisp Models with Fuzzy Model
Tabela 3. Analiza porównawcza ze zmianami procentowymi w różnych modelach Crisp dla modelu rozmytego

Model	Deterioration	Iteration	q^{**}	t_c^*	L^*	ρ^*	PE Cost	$\tilde{\pi}_1(q,\rho)^*$	$\tilde{\pi}(q,\rho)^*$
Fuzzy	Yes	146	25489.5	2.35432	1470.84	8.50	135057.2	170864.7	72574.72
Fuzzy	No	52	309.7040	0.258	-	1.0	0.0	7342.4685	28449.62
% Change	-	-	98.78	89.04	-	88.24	-	95.70	60.80
Crisp	Yes	773	13297.8	1.450	476.183	7.36939	97365.9	127738.98	88103.26
% Change	-	-	47.83	38.41	67.63	13.30	27.91	25.24	21.40
Crisp	Yes	-	220	0.183	1.00221	-	-	5074.5683	27806.128
% Change	-	-	99.14	92.23	99.93	-	-	97.03	61.69
Crisp	No	41	309.839	0.258	-	-	-	7345.9678	28450.81
% Change	-	-	98.78	89.04	-	-	-	95.70	60.80

COMPARATIVE ANALYSIS

From Table 3, it can be found that the retailer always includes the promotional effort cost and wastes the percentage of on-hand inventory due to deterioration in FEOQ model for obtaining the maximum net profit with less time consumption.

SENSITIVITY ANALYSIS

It is interesting to investigate the influence of α on retailer behavior. The computational results shown in Table 4 indicates the following managerial phenomena: when the percentage of on hand inventory that is lost due to deterioration α increases, the optimal replenishment cycle length, the replenishment quantity, the optimal total number of units lost per cycle, the optimal promotional effort and the optimal promotional effort cost will decrease but the optimal total profit per unit of time will be highly sensitive in nature respectively. So the percentage of loss due to deterioration reasons more sensitive to the other parameters. Fig. 6 is the sensitivity plotting of order quantity q, promotional effort factor ρ and fuzzy net profit per cycle $\tilde{\pi}_1(q,\rho)^*$.

Table 4. Sensitivity Analysis of the Parameter α
Tabela 4. Analiza wrażliwości dla parametru α

α%	Iteration	t^*	L^*	q^{**}	ρ^*	Promotional Effort Cost	$\tilde{\pi}(q,\rho)^*$
.02	130	3.446789	1735.444	50934.74	11.89496	284880.3	336976
.04	337	2.632347	1614.484	31213.92	9.370418	168153.4	208131.3
.10	89	1.540936	891.3179	11873.34	5.939046	58546.01	82053.37
.12	121	1.353883	745.2725	9429.727	5.345398	45317.95	65975.8
.15	86	1.145353	585.8022	7020.389	4.681664	32531.15	50003.07
.30	76	0.647086	248.524	2645.998	3.087527	10458.65	20278.71

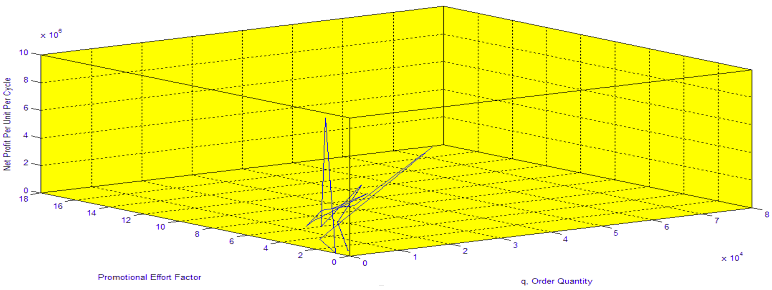

Fig. 6. Sensitivity Plotting of Order Quantity q, Promotional Effort Factor ρ and Fuzzy Net Profit per Cycle $\tilde{\pi_1}(q,\rho)$
Rys. 6. Wykres wrażliwości wielkości zamówienia q, czynnika promocyjnego ρ rozmytego zysku netto na cykl $\tilde{\pi_1}(q,\rho)$

Table 5. Sensitivity analyses of the parameters K,h,r,c,P_s,K_1 and α_1
Tabela 5. Analizy wrażliwości dla parametrów K,h,r,c,P_s,K_1 i α_1

Parameter	Value	Iteration	t^*	L^*	q^*	ρ^*	PE	$\pi_1(q,\rho)$	$\pi(q,\rho)$
K	150	86	2.354328	1470.837	25489.47	8.5016	135057.2	170914.7	72595.96
	160	77	2.354328	1470.837	25489.47	8.5016	135057.2	170904.7	72595.96
	210	190	2.354328	1470.837	25489.47	8.5016	135057.2	170854.7	72595.96
h	3	127	2.901615	2728.575	38546.30	10.287	206983.7	251359.9	86627.61
	8	263	1.835342	711.0935	15738.47	6.8232	81381.81	109132.9	59461.88
	10	93	1.600239	478.8019	12130.04	6.0674	61629.14	85752.72	53587.45
r	1100	124	2.354328	1348.267	23365.35	8.5016	123802.4	156609.3	66519.75
	1300	221	2.354328	1593.407	27613.59	8.5016	146311.9	185120.1	78629.70
	1400	89	2.354328	201.8419	3497.901	1.0000	0.00000	41808.82	17758.28
c	90	246	2.860377	3086.382	44214.04	11.982	289450.7	341964.2	119552.2
	110	248	1.379096	211.1158	6194.494	3.6155	16418.27	28772.69	20863.45
	120	247	0.449215	6.099399	545.1572	1.0000	0.00000	1152.623	2565.862
P_s	110	222	0.975232	64.64348	2673.132	2.2289	3624.745	9323.624	9560.415
	120	251	1.905113	656.3798	14003.72	5.8384	56183.94	79208.12	41576.59
	130	144	2.793675	2876.485	42167.4	11.720	275809.0	327065.4	117073.6
K_1	3	168	2.354328	1038.227	17992.38	6.0011	90038.10	125845.7	53452.90
	5	112	2.354328	692.1391	11994.71	4.0006	54022.86	89830.41	38155.44
	10	104	2.354328	432.5732	7496.455	2.5003	27011.43	62818.98	26682.34
α_1	1.5	104	2.354328	210.4724	3647.468	1.2166	3898.764	39706.32	16865.24
	1.8	55	2.354328	177.4728	3075.588	1.0258	464.6999	36272.25	15406.63
	2.2	90	2.354328	173.2693	3002.740	1.0015	27.25838	35834.81	15220.82

It is interesting to investigate the influence of the major parameters K, h, r, c, P_s, γ, $K_1 and\ \alpha_1$ on retailer's behavior. The computational results shown in Table 5 indicate the following managerial phenomena in fuzzy EOQ model:

– t_c the replenishment cycle length, L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$the optimal net profit per unit per cycle and π the optimal average profit per unit per cycle are insensitive to the parameter K.

- t_c the replenishment cycle length, L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter h.
- L units lost due to deterioration, q the optimal replenishment quantity, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter r but t_c the replenishment cycle length and ρ the promotional effort factor is insensitive to the parameter r.
- t_c the replenishment cycle length, L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter c.
- $t_c$the replenishment cycle length, L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter P_s.
- L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter K_1 but $t_c$the replenishment cycle length is insensitive to the parameter K_1.
- L units lost due to deterioration, q the optimal replenishment quantity, ρ the promotional effort factor, PE promotional effort cost, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter $\alpha_1$but $t_c$the replenishment cycle length is insensitive to the parameter α_1.

CONCLUSIONS

In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality, promotion of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality and promotion of the product, these companies have mainly relied on qualitative judgment. The model tackles to investigate the effect of the wasting the percentage of on-hand inventory due to deterioration and the promotional effect cost for obtaining the optimum average payoff and the optimal values of the policy variables. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision makers involved in the process. A policy iteration algorithm is designed and optimum solution is obtained through LINGO 13.0 version software. In order to make the comparisons equitable a particular evaluation function based on promotion is suggested. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer promotional effort in making optimum decision on promotion and replenishment with units lost due to deterioration. Numerical experiment is carried out to analyze the magnitude of the approximation error. However, adding of both promotional effort and wasting the percentage of on-hand inventory due to deterioration in fuzzy model might lead to super gain for the retailer. Traditional inventory models have not considered the promotional effort cost and units lost of sales due to deterioration simultaneously in the fuzzy environment. In this model, a modified FEOQ model is introduced which investigates the promotional effort parameter and it assumes that a percentage of the on-hand inventory is wasted due to deterioration as a characteristic feature and the inventory conditions govern the item stocked. This model provides some useful properties for finding the optimal profit, promotional effort and ordering quantity with deteriorated units of lost sales. A new mathematical model is developed and numerical example is provided to illustrate the solution procedure. An efficient computational algorithm is constructed to find the optimal solutions. A numerical example and sensitivity analyses are implemented to illustrate the model. The new modified FEOQ model is numerically compared to the traditional EOQ model. The economic order quantity and the net profit for the modified model, q^{**} were found to be more than that of the traditional, q^{*}, i.e. $q^{**} > q^{*}$ and the net profit respectively. Finally, the promotion effort parameter effect was demonstrated numerically to have an adverse effect on the average profit per unit. Hence the utilization of promotional effort and units lost due to deterioration make the scope of the application broader for achieving more profits than the traditional model. The model in this study is a general framework that considers wasting the percentage of on-hand inventory due to deterioration.

REFERENCES

Bose S., Goswami A., Chaudhuri K.S., 1995. An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. J. of Oper. Res. Soc., 46, 775-782.

Goyal S.K., Giri, B.C. 2001. Recent trends in modeling of deteriorating inventory. Euro. J.of Oper.Res., 134, 1-16.

Goyal S.K., Gunasekaran A., 1995. An integrated production-inventory-marketing model for deteriorating items. Comp.and Ind.Eng., 28, 755-762.

Gupta D., Gerchak Y., 1995. Joint product durability and lot sizing models. Euro. J. of Oper. Res., 84, 371-384.

Hariga M., 1995. An EOQ model for deteriorating items with shortages and time-varying demand. J. of Oper. Res. Soc., 46, 398-404.

Hariga M., 1996. An EOQ model for deteriorating items with time-varying demand. J. of Oper. Res. Soc., 47, 1228-1246.

Hariga M., 1994. Economic analysis of dynamic inventory models with non-stationary costs and demand. Inter. J. of Prod. Econ., 36, 255-266.

Jain K., Silver E., 1994. A lot sizing for a product subject to obsolescence or perishability. Euro. J. of Oper. Res., 75, 287-295.

Mahata G.C., Goswami A., 2006. Production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating item under permissible delay in payments. J. of Oper. Res. Soc .India, 43, 359-375.

Osteryoung J.S., Mc Carty D.E., Reinhart W.L., 1986. Use of EOQ models for inventory analysis. Prod. and Inv.Mgmt., 3rd Qtr: 39-45.

Padmanabhan G., Vrat P., 1995. EOQ models for perishable items under stock dependent selling rate. Euro.J. of Oper. Res., 86, 281-292.

Pattnaik M., 2013. A Framework of Dynamic Ordering Cost with Units Lost due to Deterioration in an Instantaneous Economic Order Quantity Model. Journal of Supply Chain and Operations Management, in press.

Pattnaik M., 2012. A Note on Non Linear Profit-Maximization Entropic Order Quantity (EnOQ) Model for Deteriorating Items with Stock Dependent Demand Rate. Oper. and Sup. Chain Mgmt., 5(2), 97-102.

Pattnaik M., 2011. A note on optimal inventory policy involving instant deterioration of perishable items with price discounts. The J. of Math. and Comp. Sc., 3(2), 145-155.

Pattnaik M., 2013. A note on profit-Maximization Fuzzy EOQ Models for Deteriorating Items with Two Dimension Sensitive Demand. Inter. J. of Mgmt. Sc. and Eng. Mgmt., in press.

Pattnaik M., 2010. An entropic order quantity (EnOQ) model under instant deterioration of perishable items with price discounts, Inter. Math. For., 5(52), 2581-2590.

Pattnaik M., 2011. An entropic order quantity (EnOQ) model with post deterioration cash discounts. Inter. J. of Cont. and Math. Sc., 6(19), 931-939.

Pattnaik M., 2012. An EOQ model for perishable items with constant demand and instant Deterioration. Dec., 39(1), 55-61.

Pattnaik M., 2011. Entropic order quantity (EnOQ) model under cash discounts. Thai. Stat. J., 9(2), 129-141.

Pattnaik M., 2013. Fuzzy Multi-objective Linear Programming Problems: A Sensitivity Analysis. The J. of Math. and Comp. Sc., 7(2), 131-137.

Pattnaik M., 2013. Fuzzy NLP for a Single Item EOQ Model with Demand – Dependent Unit Price and Variable Setup Cost. World J. of Model. and Simu., 9(1), 74-80.

Pattnaik M., 2013. Fuzzy Supplier Selection Strategies in Supply Chain Management. Inter. J. of Sup. Ch. Mgmt., 2(1), 30-39.

Pattnaik M., 2013. Linear Programming Problems in Fuzzy Environment: The Post Optimal Analyses. J. of Uncer. Sys., in press.

Pattnaik M., 2012. Models of inventory control. Lamb. Acad. Pub., Germany.

Pattnaik M., 2013. Optimal Decision-Making in Fuzzy Economic Order Quantity (EOQ) Model under Restricted Space: A Non-Linear Programming Approach. Inter. J. of Anal. and Appl., in press.

Pattnaik M., 2013. Optimization in an Instantaneous Economic Order Quantity (EOQ) Model Incorporated with Promotional Effort Cost, Variable Ordering Cost and Units Lost due to Deterioration. Uncer. Sup. Ch. Mgmt., 1(2), 57-66.

Pattnaik M., 2013. Skilled Manpower Selection for Micro, Small and Medium enterprises: A Fuzzy Decision Making Approach. Oper. and Sup. Ch. Mgmt., 6(2), 64-74.

Pattnaik M., 2011. Supplier Selection Strategies on Fuzzy Decision Space. Gen. Math. Notes., 4(1), 49-69.

Pattnaik M., 2012. The effect of promotion in fuzzy optimal replenishment model with units lost due to deterioration. Inter. J. of Mgmt. Sc. and Eng. Mgmt., 7(4), 303-311.

Pattnaik M., 2013. The Effect of Units Lost due to Deterioration in Fuzzy Economic Order Quantity (FEOQ) Model. Inter. J. of Anal. and Appl., 1(2), 128-146.

Pattnaik M., 2013. Wasting of Percentage On-hand Inventory of an Instantaneous Economic Order Quantity Model due to Deterioration. The J. of Math. and Comp. Sc., 7(3), 154-159.

Raafat F., 1991. Survey of literature on continuously deteriorating inventory models. J. of Oper. Res. Soc., 42, 89-94.

Sahoo P.K., Pattnaik M., 2013. Decision Making Approach to Fuzzy Linear Programming (FLP) Problems with Post Optimal Analysis. Inter. J. of Oper. Res. and Inf. Sys., in press.

Sahoo P.K., Pattnaik M., 2013. Linear Programming Problem and Post Optimality Analyses in Fuzzy Space: A Case Study of a Bakery Industry. J. of Bus. and Mgmt. Sc., 1(3), 36-43.

Salameh M.K., Jaber M.Y., Noueihed N., 1993. Effect of deteriorating items on the instantaneous replenishment model. Prod. Plan. and Cont., 10(2), 175-180.

Shah N., 2000. Literature survey on inventory models for deteriorating items. Econ. Annals., 44, 221-237.

Tripathy P.K., Pattnaik, M., 2011. A fuzzy arithmetic approach for perishable items in discounted entropic order quantity model. Inter. J. of Sc. Stat. Comp., 1(2), 7-19.

Tripathy P.K., Pattnaik M., 2011. A non-random optimization approach to a disposal mechanism under flexibility and reliability criteria. The Open Oper. Res. J., 5, 1-18.

Tripathy P.K., Pattnaik, M., 2008. An entropic order quantity model with fuzzy holding cost and fuzzy disposal cost for perishable items under two component demand and discounted selling price. Pak. J. of Stat. and Oper. Res., 4(2), 93-110.

Tripathy P.K., Pattnaik M., 2013. Fuzzy Supplier Selection Strategies in Supply Chain Management. Inter. J. of Sup.Ch. Mgmt., 2(1), 30-39.

Tripathy P.K., Pattnaik M., 2009. Optimal disposal mechanism with fuzzy system cost under flexibility & Reliability criteria in non-random optimization environment. Appl. Math. Sc., 3(37), 1823-1847.

Tripathy P.K., Pattnaik M., 2011. Optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Inter. J. of Mgmt. Sc. and Eng. Mgmt., 6(6), 412-420.

Tripathy P.K., Pattnaik M., Tripathy P., 2012. Optimal EOQ Model for Deteriorating Items with Promotional Effort Cost. Amer. J. of Oper. Res., 2(2), 260-265.

Tripathy P.K., Tripathy P., Pattnaik M., 2011. A Fuzzy EOQ Model with Reliability and Demand-dependent Unit Cost. Inter J. of Cont. Math. Sc., 6(30), 1467-1482.

Tsao Y.C., Sheen G.J., 2008. Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Comp. and Oper. Res., 35, 3562-3580.

Vujosevic M., Petrovic D., Petrovic, R., 1996. EOQ formula when inventory cost is fuzzy. Inter. J. of Prod. Econ., 45, 499-504.

Waters C.D.J., 1994. Inventory Control and Management. (Chichester: Wiley).

Wee H.M., 1993. Economic Production lot size model for deteriorating items with partial back-ordering. Comp. and Ind. Eng., 24, 449-458.

OPTYMALIZACJA MODELU ROZMYTEJ EKONOMICZNEJ WIELKOŚCI ZAMÓWIENIA Z EFEKTEM PROMOCYJNYM I STRATĄ SPOWODOWANĄ ZNISZCZENIEM

STRESZCZENIE. **Wstęp:** Tematem pracy jest model istotności służący do analizy efektów zniszczenia towarów oraz efektu promocyjnego w modelu rozmytym ciągłego uzupełniania w określonym horyzoncie czasu. Głównym parametrem tworzenia modelu zarządzania zapasem w rozmytym środowisku jest zazwyczaj koszt jednostkowy na jednostkę zamówienia w określonej jednostce czasu. Te nieprecyzyjne parametry, oparte o oś liczb rzeczywistych, tworzą otoczenie, w którym stosuje się system zarządzania zapasem.

Metody: Model określa wpływ promocyjny na całkowity koszt zapasu. W tym celu został stworzony rozmyty model ekonomicznej wielkości zakupu uwzględniający efekt promocyjny oraz procentowe zniszczenie zapasu, tak, aby osiągnąć maksymalny zysk netto. W trakcie analizy teoretycznej, niezbędne i wystarczające warunki istnienia i unikalności rozwiązań optymalizacyjnych wykazały słuszność dalszych badań nad funkcją rozmytej zyskowności netto. W celu znalezienie optymalnego rozwiązania stworzono algorytm przy użyciu oprogramowania LINGO 13.0.

Wyniki i wnioski: Wyniki analizy numerycznej umożliwiają oszacowanie efektu polityki promocyjnej na optymalizację zysku netto dla detalisty jak również poziomu zniszczeń towaru. Również zostały przeprowadzone analizy wrażliwości uwzględniające najważniejsze parametry, które wykazały celowość stosowania proponowanej metody dla maksymalizacji zysku przy uwzględnieniu efektu promocyjnego.

Słowa kluczowe: logistyka, optymalizacja, rozmyty, ekonomiczna wielkość zamówienia, promocja, straty, zniszczenia.

DIE OPTIMIERUNG DES MODELLS DER UNSCHARFEN WIRTSCHAFTLICHEN AUFTRAGSGRÖßE SAMT PROMOTIONSEFFEKT UND DEM DURCH DIE WARENVERNICHTUNG VERURSACHTEN VERLUST

ZUSAMMENFASSUNG. Einleitung: Zum Kernpunkt der vorliegenden Forschungsarbeit wurde ein Relevanzmodell, das zur Erfassung und Analyse des Warenvernichtungs-Effektes und des Promotionseffektes innerhalb eines unscharfen Modells für die Dauerergänzung von Beständen bei einem bestimmten Zeithorizont dient. Den Hauptparameter für die Schaffung des Modells für die Bestandsführung in einem Fuzzy-Umfeld machen gewöhnlich Einzelkosten für eine Bestelleinheit in einer bestimmten Zeiteinheit aus. Diese wenig präzisen Parameter, die auf die Achse der reellen Zahlen gestützt sind, bilden ein Umfeld, in dem das System der Bestandsführung in Funktion tritt.

Methoden: Das Modell bestimmt den Promotionseinfluss auf die Gesamtkosten des Bestandes. Zu diesem Zweck wurde das Fuzzy-Modell der wirtschaftlichen Einkaufsgröße geschaffen, das den Promotionseffekt und die prozentuelle Bestandsvernichtung berücksichtigt, und dies, um den maximalen Nettogewinn zu erzielen. Die unentbehrlichen und genügenden Bedingungen für das Vorhandensein und die Eigenartigkeit von Optimierungslösungen wiesen während der Analyse die Zweckmäßigkeit der weiteren Erforschung der Funktion einer unscharfen Netto-Rentabilität aus. Zwecks der Ermittlung einer optimalen Lösung wurde ein Algorithmus unter der Anwendung der Software LINGO 13.0 erstellt.

Ergebnisse und Fazit: Die aus der nummerischen Analyse resultierenden Ergebnisse ermöglichen die Einschätzung des Effektes der Promotionspolitik hinsichtlich der Optimierung des Nettogewinns für den Einzelhändler sowie des Niveaus von Warenvernichtungen. Es wurden auch die die wichtigsten Parameter berücksichtigenden Empfindlichkeitsanalysen, die auf die Brauchbarkeit der vorgeschlagenen Methode zwecks der Maximierung des Gewinns bei der Berücksichtigung des Promotionseffektes hingewiesen haben, durchgeführt.

Codewörter: Logistik, Optimierung, Fuzzy-Modell, wirtschaftliche Bestellgröße, Promotion, Verluste, Vernichtung

Padmabati Gahan
Dept. of Business Administration,
Sambalpur University, Jyotivihar, Burla
Sambalpur, **India**
e-mail: pgahan7@gmail.com

Monalisha Pattnaik
Dept. of Business Administration,
Sambalpur University, Jyotivihar, Burla
Sambalpur, **India**
e-mail: monalisha_1977@yahoo.com