



OPTIMAL INVENTORY CONTROL FOR PERISHABLE ITEMS UNDER ADDITIONAL COST FOR DETERIORATION REDUCTION

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ABSTRACT. Background: This paper analyses the problems of carrying out inventory control of perishable goods subject to deterioration during warehousing. It aims to propose a generalization of the classical models used in optimal inventory control theory for perishable goods adapted for the case of additional expenses incurred by supply companies to goods' deterioration reduction.

Methods: A generalized Wilson model for optimal lot sizing has been developed. It is assumed that the rate of deterioration depends non-linearly on the volume of investment intended for a reduction in goods deterioration. The case of inverse power dependence is analyzed in details. Firstly, the generalized Wilson model for a single item is examined, and then this model is considered for multi-item case.

Results and conclusions: For the joint optimization of lot sizes and volumes of investments, corresponding non-linear optimization problems are formulated. Numerical results are presented to illustrate the model. It is indicated that the proposed optimization model can be used as the methodical basis for supply companies in their investment activities aimed at improving storage technology of perishable products.

Key words: inventory management; perishable product; control of deterioration; the generalized Wilson model; lot sizing optimization.

INTRODUCTION

Recently, in logistics theory and practices the great attention has been paid to problems of transportation and storage of perishable products as an important part of population supply with food products [Kitinoja 2013, Lin et al. 2015, Postan and Filina-Dawidowicz 2013]. It is related, in particular, to the growing demand for food, including exotic fruit, increasing requirements for product quality as well as the need for supplied products variety. It is induced i. a. by the living standards growth and international trade development.

The main feature of perishable cargo is its limited shelf life, which causes the necessity to

assure the specific temperature and humidity conditions of these goods transportation and storage [Agreement 2014, Studziński 2005]. Failure to comply with the strict cargo requirements can result in products quality loss, which can lead to significant financial losses [Lin et al. 2015]. All this makes it necessary to improve the logistics systems of perishable goods delivery, including optimization of the inventory control process.

Inventory management, as an integral component of the perishable goods logistics, deals with the analysis of methods and techniques of planning and inventory control [Silver 1981, Venkata Subbaiah et al. 2004]. The main objective of this process is to provide the optimal balance of inventory levels, improvement of the service quality level and

minimizing the costs together with the smooth and continuous operation of enterprise [Bramel and Simchi-Levi 1997, Churchman et al. 1957, Hadley and Whitin 1963].

Inventory control problems are discussed in a number of sources [Almeder et al. 2009, Baran et al. 2010, Bed-Daya and Raouf 1994, Thomopoulos 2015, Thomopoulos 2016]. In these works particular attention is paid to the optimization of the ordered lot size, order execution time, the costs of goods storing and orders maintenance, etc. Moreover, these issues are provided in special studies on the problems of inventory control in warehouses in seaports, logistics centers, in storage facilities of distributors and retailers [Postan and Filina-Dawidowicz 2013, Song and Zhang 2011, Thomopoulos 2015].

The specificity of the perishable products imposes additional restrictions on the storage process and inventory control of these goods [Dash et al. 2014, Shah and Shah 2000, Li et al. 2010]. Perishable loads have to be stored in warehouses, providing the necessary cargo storage conditions [Ghare and Schrader 1963, Studziński 2005]. In the case of servicing of multi-item production, it is necessary to assure storage capacity (cold stores with microclimate required) for each type of cargo. Additionally, planning and orders implementation for each single product should be considered separately. Furthermore, the appropriate rotation of products in a warehouse, according to the FEFO (first expired, first out) rule has to be organized. These products should be ordered as close as possible to the moment of sale. The time of servicing of perishable commodity products in a warehouse is also important. This proves that these products are ordered with high frequency to reduce the possibility of products deterioration and their quality loss.

Deterioration deals with specificity of perishable goods and cannot be completely avoided in logistic chains. Ghare and Schrader [1963] proposed to divide inventory deterioration into following types: direct spoilage, physical depletion and deterioration. Direct spoilage determines the unstable state of inventory items caused by some breakage during transaction as well as sudden accidental events. For example, quality of food products

might be reduced due to non-functioning of a warehouse refrigerator or a transport vehicle cooling unit caused by absence of power supply or some refrigerated system damages [Filina and Filin 2008]. On the other hand, deterioration deals with slow but gradual loss of qualitative properties of products with the passage of time [Kundu et al. 2013]. Lin et al. [2015] discussed indicators that reflect and factors that influence the quality of perishable goods, such as food.

Rau et al. [2004] in his paper proposed deteriorating item inventory model with a shortage occurring at the supplier considering a supply chain between the producer and buyer. An alternative rate of production was analysed in inventory model for perishable items developed by Venkata Subbaiah et al. [2011]. Ghosh et al. [2011] considering model of a perishable product for the case of price dependent demand, partial backordering which depends on the length of the waiting time for the next replenishment, and lost sale. Moreover, Dalfard and Nosratian [2014] presented a pricing constrained single-product inventory-production model in perishable food for maximizing the total profit. A production inventory model for an item with three parameter Weibull deterioration taking into account price discount for partially deteriorated item has been developed by Pradhan and Tripathy [2012].

Ali et al. [2013] discussed problems of inventory management of perishable products based on logistic approach with regard to shortages and time decay functions. Bhunia et al. [2013] in his study showed two-warehouse partially backlogged deteriorating inventory models under inflation via particle swarm optimization. Kundu et al. [2013] implemented an economic order quantity (EOQ) model to analyse the inventory problem with alternating demand rate using a gradient based non-linear optimization technique (LINGO). Furthermore, the ability to quantify the effect of items lost due to deterioration by optimizing the fuzzy net profit for the retailer applying modified fuzzy EOQ (FEOQ) model was considered by Pattnaik [2014].

The literature review showed that there are also researches focusing on generalization of

well-known problems of inventory control of perishable products. The example of generalization of classical Wilson model used to optimize the lot size of goods deliveries exposed to deterioration during storage is given in [Dash et al. 2014, Lee and Nahmias 1993, Li et al. 2010, Shah and Shah 2000]. However, these studies do not take into account the possibility of reducing deterioration of stored products due to additional costs (investment), aimed at improving the technology of goods transportation and storage. An approach that takes account of these circumstances was proposed in an article [Postan and Filina-Dawidowicz 2013], where an example of generalization of the Wagner-Whitin classical model of inventory control in distribution centre is demonstrated.

The purpose of this article is to spread the approach mentioned in [Postan and Filina-Dawidowicz 2013] to the case of the Wilson model, taking into account the possibility of reducing deterioration of stored production in the warehouse due to additional costs (or investments). Firstly, the calculations results of the generalized Wilson model for perishable single item were presented, then the case of multi-item production was considered.

NOTATIONS AND ASSUMPTIONS

Let's consider a warehouse of supply company or retailer of the same item (homogeneous products), that faces the constant goods demand. The company periodically places orders for replenishment of perishable products with suppliers, which are assumed to deliver unlimited number of goods. Below, the principal notations and assumptions for model creation are listed:

- the order quantities are fixed at Q items per order;
- the planning horizon is T ;
- the demand for planning horizon is constant and equal D ; it means that products are distributed from warehouse uniformly with the rate $d = D/T$;
- the order execution time is zero;
- the order point is the time when the inventory level of products in warehouse is zero;
- the initial inventory level is zero;
- the set-up cost K is incurred every time the warehouse places an order;
- the per unit order cost is c ;
- the per unit time costs of storage per unit's product is equal to h ;
- the intensity of product deterioration (as the result of natural processes) at warehouse is proportional to inventory level with proportionality coefficient δ (growth rate of deterioration);
- the per unit's product sale price is s ;
- the size of company's investments at the beginning of the planning horizon aimed to reduce the losses of products deterioration, is v ;
- coefficient δ is a decreasing function of the parameter v .

As it is applied in different inventory control models, above assumptions let us simplify the real situation. There is an assumption that predetermined volume of demand is fixed for planning horizon, but it is not very realistic. It is assumed that fixed volume of delivered consignment is restrictive, the real execution order time is always positive.

However, during further improvement of relatively simple Wilson model for determining the optimal replenishment policy, all above assumptions may be relaxed. But for a better understanding of the main model's results, we will keep them.

THE GENERALIZED WILSON MODEL FOR SINGLE PERISHABLE ITEM

The main purpose of inventory management is to find an optimal replenishment and investment policy, that minimizes the overall costs per time unit for orders placement, goods purchase and warehousing, taking into consideration the possible market loss (or sale profits loss because of products deterioration) and the cost

of investment to reduce the products quality loss.

In order to find the optimal policy of ordering and investing in products deterioration reduction, we consider the inventory level as a function of time. Let $I(t)$ be the inventory level of acceptable (without deterioration and damage) quality products at time t . We refer to the time between two successive replenishments as a cycle time. Thus, total company's inventory cost in a cycle of length τ is

$$cQ + K + (h + \delta s) \int_0^{\tau} I(t) dt \quad (1)$$

According to above assumption

$$\frac{d}{dt} I(t) = -d - \delta I(t), t \in [0, \tau] \quad (2)$$

with initial condition

$$I(0) = Q. \quad (3)$$

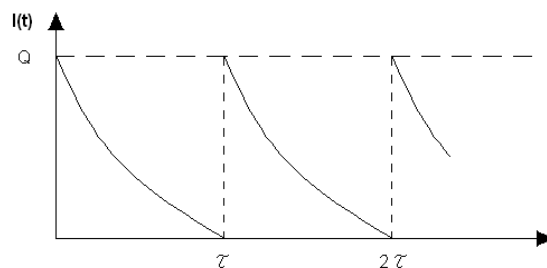
Note that the second term in the right-hand side of equation (2) determines the intensity of product's damage during period of its storage in a warehouse. The solution of initial-value problem for equation (2) is given by

$$I(t) = e^{-\delta t} \left(Q + \frac{d}{\delta} \right) - \frac{d}{\delta}. \quad (4)$$

According to Zero Inventory Ordering Policy (every order is received precisely when the inventory level drops to zero) $I(\tau) = 0$, hence from (4) we find the order cycle length

$$\tau = \frac{1}{\delta} \ln \left(1 + \frac{Q\delta}{d} \right). \quad (5)$$

From (4), it follows that changing the inventory level during the time is described by the curve presented on the Figure 1. This is type of so-called saw-toothed inventory pattern..



$I(t)$ - inventory level, t - time, Q - the size of the product ordered consignment, τ - order cycle length.

Fig. 1. Inventory level as function of time
 Rys. 1. Poziom zapasu jako funkcja czasu

Now we can re-write the expression (1) as follows

$$cQ + K + (h + \delta s) \tau \bar{I},$$

where $\bar{I} = \frac{1}{\tau} \int_0^{\tau} I(t) dt$ is average inventory

level for the cycle, besides

$$\bar{I} = \frac{Q}{\ln \left(1 + \frac{Q\delta}{d} \right)} - \frac{d}{\delta}.$$

After some calculations, taking into account (4) and (5), from (6), we obtain

$$cQ + K + \frac{(h + \delta s)}{\delta} (Q - \tau d).$$

Multiplying the last expression by the number of order cycles, that is equal T/τ , and adding the costs to reduce the product deterioration, we find the full average cost over the planning horizon (see(5))

$$\begin{aligned} \bar{C}(Q, v) &= \frac{T}{\tau} \left[cQ + K + \frac{(h + \delta s)}{\delta} (Q - \tau d) \right] + v = \\ &= T \left[\frac{Q(h + \delta s + \delta K)}{\ln \left(1 + \frac{Q\delta}{d} \right)} - \frac{(h + \delta s)d}{\delta} \right] + v, \end{aligned} \quad (7)$$

where $\delta \equiv \delta(v)$. The explicit form of the dependence $\delta(v)$ can be found using methods of mathematical statistics. The simplest forms of this dependence can be, for example, as follows

$$\begin{aligned} \delta(v) &= \frac{\delta_0}{(1 + \mu v)^\alpha}, \\ \delta(v) &= \delta_0 e^{-\mu v^\alpha}, \end{aligned} \quad (8)$$

where δ_0 is the value of the growth rate of deterioration with zero investment, i.e., suitable to standards of products natural loss using a given storage technology; μ is coefficient characterizing the decline rate of products deterioration for non-zero investments, aimed at more advanced products storage technology; α , $0 \leq \alpha \leq 1$, is given parameter.

For example, according to (8) the function (7) takes the following form:

$$\bar{C}(Q, v) = T \left\{ \frac{\delta_0 [Q(c+s) + K] + hQ(1+\mu v)^\alpha}{(1+\mu v)^\alpha \ln(1 + \frac{\delta_0 Q}{d(1+\mu v)^\alpha})} - [s + \frac{(1+\mu v)^\alpha}{\delta_0} hd] \right\} + v. \quad (9)$$

Thus, the optimal size of the products ordered consignment Q^* and the optimal amount of investment v^* are defined by a necessary condition for achieving a minimum of function (9), i.e.

$$\left. \frac{\partial \bar{C}(Q, v^*)}{\partial Q} \right|_{Q=Q^*} = 0, \quad \left. \frac{\partial \bar{C}(Q^*, v)}{\partial v} \right|_{v=v^*} = 0. \quad (10)$$

A sufficient requirement for existence of a minimum of this function is given by the condition

$$\left. \begin{array}{cc} \frac{\partial^2 \bar{C}(Q, v)}{\partial Q^2} & \frac{\partial^2 \bar{C}(Q, v)}{\partial Q \partial v} \\ \frac{\partial^2 \bar{C}(Q, v)}{\partial v \partial Q} & \frac{\partial^2 \bar{C}(Q, v)}{\partial v^2} \end{array} \right|_{Q=Q^*, v=v^*} > 0. \quad (11)$$

Using (9), we obtain a system of nonlinear equations to determine the optimal values of the parameters Q^*, v^* . However, its numerical calculation is very time consuming, so it's easier to solve the minimization problem using standard software, for example, Microsoft Excel. It should be taken into account that not all values of the initial parameters of optimization model satisfy the conditions (10) and (11).

NUMERICAL EXAMPLE

We consider a numerical example. Let $T = 365$ d., $c = 1,5$ th. \$/t, $s = 2,0$ th. \$/t, $h = 0,01$ \$/t.d., $\delta_0 = 0,2$ 1/d., $\mu = 0,003$ (th.\$)⁻¹, $\alpha = 1$, $K = 2$ th.\$.

The Table 1 shows the results of the calculation using the Microsoft Excel software for different values of the parameter d (daily demand for products). The data in the table shows that with the increase in demand the optimal size of ordered consignment, order cycle and investments in storage technology improvement have the largest value, while the demand is located in the range from 0,35 to 0,4 t/d.

Table 1. Calculation results for the optimization problem
 Tabela 1. Wyniki obliczeń problemu optymalizacyjnego

The values of d parameter, t/d.	The optimum value of ordered consignment size Q , t	The optimum value of investments v \$	The optimal value of delivery cycle τ , d.	The minimum value of the function (9), \$	The minimum value of the function(9) when $v=0$, \$
0,10	1,14	239,62	7,25	228536,2	292063,0
0,15	3,52	2674,50	18,91	169821,9	363494,4
0,20	3,76	2290,52	15,36	216353,9	427878,9
0,25	4,18	2290,52	13,94	255414,3	487791,0
0,30	4,27	2070,72	12,00	298635,8	544540,2
0,35	7,95	4749,62	19,87	296800,7	598899,6
0,40	8,14	4427,89	17,91	333983,0	651373,1
0,45	6,04	2587,23	11,71	395472,5	702305,8
0,48	6,47	2748,63	11,76	412678,5	732233,0
0,49	6,49	2748,63	11,65	419632,6	742114,7

The optimum value of the coefficient due to set initial data is changing in the range from 0,02 to 0,12. In the case of lack of investment the minimum costs exceed significantly the costs when the investment takes place (see the last column of the Table 1) and, in addition, increase faster while the demand increases. It is not surprising; since the lot size of the purchased consignment grows in the face of rising demand, respectively, the loss from product deterioration increases. These data illustrate the feasibility of such investments.

MULTI-ITEM INVENTORY CONTROL MODEL

The model examined in previous Section established optimal inventory policy for a single perishable item. It is obvious that without assumption of presence of joint purchase costs, a problem with several items each facing a constant demand can be solved by examination of each item's replenishment problem separately, if a warehouse capacity is large enough. But in reality, management of a single warehouse inventory system involves coordinating inventory orders to minimize costs without exceeding the warehouse capacity. The warehouse capacity limits the total volume held by the warehouse at any point in time. This constraint ties together the different items and necessitates careful scheduling of the orders [Bramel and Simchi-Levi 1997]. That is, not only it is important to know how often an item is ordered, but exactly the point in time at which each order takes place. This problem is called the Economic Warehouse Lot Scheduling Problem (EWLSP). The scheduling part, hereafter called the Staggering Problem, is exactly the problem of time-phasing the placement of the orders to satisfy the warehouse capacity constraint. This problem has no simple solution and it has attracted a considerable attention in the last decades even for nonperishable items.

The earliest known reference to the problem appears in the books of Churchman et al. [1957], Hadley and Whitin [1963]. These authors were concerned for determining lot sizes that made an overall schedule satisfactory for the capacity constraint and not with the

possibility of phasing the orders to avoid holding the maximum volume of each item at the same time. Thus, they only considered so called Independent Solutions, where every item is replenished without any regard for coordination with other items.

Below, we generalize these results for the case of several items of perishable products using the results of previous Sections.

Let $N = \{1, 2, \dots, n\}$ be a set of n items of perishable goods and the i th item facing a constant demand rate $d_i, i \in N$. The per unit cost C_i and ordering cost K_i are incurred each time an order for item i is placed. A holding cost h_i is accrued for each unit of item i held in inventory per unit of time. Demand for each item must be met over an infinite horizon without shortages or backlogging. The per unit sale price for item i is S_i .

Denote by $\gamma_i > 0$ the volume usage rate of item i . Then the volume of inventory of item i held at a given point in time is the product of its inventory level at that time I_i and γ_i . Note that the volume usage rate is defined as the volume displaced by one unit of item i .

As above, we denote by $I_i(t)$ the inventory level of item i at time t , and by τ_i the cycle time of item i . By definition, we assume that following differential equation is valid

$$\frac{d}{dt} I_i(t) = -d_i - \delta_i I_i(t), i \in N, \quad (12)$$

where δ_i is parameter which determines the intensity of the i th kind of product's damage during period of its storage in warehouse. Initial condition for equation (12) is

$$I_i(0) = Q_i, i \in N, \quad (13)$$

where Q_i is order quantity of item i . We denote the investments directed on decreasing of deterioration of the n th item by V_n . From (12), (13) it follows (see also (4) – (7)) that average total cost per unit of time for all items (including possible market loss) is

$$\bar{C}(Q_1, Q_2, \dots, Q_n) = \sum_{i=1}^n \left[\frac{Q_i(h_i + \delta_i c_i + \delta_i s_i) + \delta_i K_i - (h_i + \delta_i) d_i + v_i}{\ln \left(1 + \frac{Q_i \delta_i}{d_i} \right)} \right] \quad (14)$$

The control parameters Q_1, Q_2, \dots, Q_n are subjected to constraint

$$\sum_{i=1}^n \gamma_i \bar{I}_i \leq V$$

or in the evident form

$$\sum_{i=1}^n \gamma_i \left[\frac{Q_i}{\ln \left(1 + \frac{Q_i \delta_i}{d_i} \right)} - \frac{d_i}{\delta_i} \right] \leq V, \quad (15)$$

where V is the given value of warehouse cubic capacity.

The evident form of dependence $\delta_i(v_i)$ on the control parameter v_i may be taken, for example, in the form (see (8))

$$\delta_i(v_i) = \frac{\delta_{0i}}{(1 + \mu_i v_i)^{\alpha_i}}, \quad i \in N. \quad (16)$$

We arrive at the optimization problem: it is needed to find a positive vectors (Q_1, Q_2, \dots, Q_n) and (v_1, v_2, \dots, v_n) minimizing the objective function (14) under conditions (15), (16). This problem can be solved only numerically with the help of standard software (e.g. Microsoft Excel).

CONCLUSIONS AND DISCUSSIONS

Rational and efficient inventory control of perishable products ensures continuity of their supplies at the minimum cost of stocks maintaining.

The proposed generalized Wilson model can be applied to solve the problem of optimal inventory control of both single item and multi-item perishable production. Analysis of the inventory control process of perishable goods showed that the observed cargo deterioration can be reduced or even eliminated by the various types of investments. It could be the investments in storage facilities,

handling equipment, product packaging methods, etc.

The results shown above indicated that the proposed optimization model can form the methodical basis for supply companies in their investment activities and measures to improve storage technology of perishable products. For this purpose, it is necessary to create the relevant databases, collect essential information relating initial parameters of optimization models and implement the above optimization models in interested companies. The development and presentation of this methodology will be a subject of future scientific research and publications.

REFERENCES

- Agreement on the international carriage of perishable foodstuffs and on the special equipment to be used for such carriage (ATP), UNECE, 13 November 2014.
- Ali S.S., Madaan J., Chan F.T.S., Kannan S., 2013, Inventory management of perishable products: a time decay linked logistic approach. *International Journal of Production Research* 51(13), 3864-3879.
- Almeder C., Preusser M., Hartl R.F., 2009, Simulation and optimization of supply chains: alternative or complementary approaches? *OR Spectrum* 31, 95-119.
- Baran J., Jałowiecki P., Wysokiński M., 2010, Rozwiązania w zakresie sterowania zapasami w wybranych branżach agrobiznesu [Stock's management in agribusiness companies], XIV Konferencja Logistyki Stosowanej Total Logistic Management, Zakopane.
- Bed-Daya M., Raouf A., 1994, Inventory Models Involving Lead Time as a Decision Variable, *The Journal of Operational Research* 45 (5), 579-582.
- Bhunia A.K., Shaikh A.A., Gupta R.K., 2013, A study on two-warehouse partially backlogged deteriorating inventory models under inflation via particle swarm optimisation. *International Journal of*

- Systems Science.
DOI:10.1080/00207721.2013.807385: 1-15.
- Bramel J., Simchi-Levi D., 1997, *The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management*. Springer-Verlag, Berlin.
- Churchman C.W., Ackoff R.L., Arnoff E.L., 1957, *Introduction to Operations Research*, Jhon Wiley&Sons Ltd., New York.
- Dalfard V.M., Nosrati N.E., 2014, A new pricing constrained single-product inventory-production model in perishable food for maximizing the total profit. *Neural Computing and Applications* 24, 3, 735-743.
- Dash B.P., Singh T., Pattanayak H., 2014, An Inventory Model for Deteriorating Items with Exponential Declining Demand and Time-Varying Holding Cost. *American Journal of Operations Research*, 4, 1-7.
- Filina L., Filin S., 2008, An analysis of influence of lack of the electricity supply to reefer containers serviced at sea ports on storing conditions of cargoes contained in them. *Polish Maritime Research* 4, 96-102.
- Ghare P.M., Scharder G.F., 1963, A model for exponentially decaying inventory, *J. Indust. Engg* 14, 238-243.
- Ghosh S.K., Khanra S., Chaudhuri, K.S., 2011, Optimal price and lot size determination for a perishable product under conditions of finite production, partial backordering and lost sale. *Applied Mathematics and Computation* 217 (13), 6047 - 6053.
- Hadley G., Whitin T.M., 1963, *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, NJ.
- Kitinoja L., 2013, Use of cold chains for reducing food losses in developing countries. The Postharvest Education Foundation (PEF) White Paper 13-03, December 2013.
- Kundu A., Chakrabarti P., Chakrabarti T., 2013, An EOQ model for time-dependent deteriorating items with alternating demand rates allowing shortages by considering time value of money, *Yugoslav Journal of Operations Research* 23 (2), 263-278.
- Lee H.L., Nahmias S., 1993, Single Product, Single Location Models. In: Graves S.C., Rinnooy Kan A.H.G, Zipkin P.H. (Ed.), *Handbooks in Operations Research and Management Science, the volume on Logistics of Production and Inventory*, North-Holland, Amsterdam, 3-55.
- Li R., Lan H., Mawhinney J.R., 2010, A Review on Deteriorating Inventory. *European Journal of Operational Research* 3 (1), 117- 129.
- Lin X., Negenborn R.R., Lodewijks G., 2015, Survey on Operational Perishables Quality Control and Logistics. *Computational Logistics* 9335, 398-421.
- Pattnaik M., 2014, Optimization in Fuzzy Economic Order Quantity (FEOQ) model with deteriorating inventory and units lost. *LogForum* 10 (3), 247-262.
- Postan M.Ya., Filina-Dawidowicz L., 2013, Dynamiczny model optymalizacyjny procesu planowania zapasów i dostaw zróżnicowanych asortymentowo produktów szybko psujących się [The dynamic optimization model of inventory control and transportation planning of multi-item perishable products]. In: *Transport w Regionie Pomorza Zachodniego*, Szczecin, 13-20.
- Pradhan L.M., Tripathy C.K., 2012, A production inventory model for an item with three parameter Weibull deterioration and price discount. *LogForum* 8 (3), 257-266.
- Rau H., Wu M.Y., Wee H.M., 2004, Deteriorating item inventory model with shortage due to supplier in an integrated supply chain, *International Journal of System Science* 35 (5), 293-303.
- Shah N.H., Shah Y.K., 2000, Literature Survey on Inventory Models for Deteriorating Items. *Economics Annals* 44, 221-237.
- Silver E.A., 1981, Operations research in inventory management, *Opns. Res.*, 29, 628-645.
- Song D.-P., Zhang Q., 2011, Optimal Inventory control for empty containers in a port with random demands and repositioning delays, chapter 14, in

- International Handbook of Maritime Economics, from Edward Elgar.
- Studziński A., 2005, *Eksploatacja chłodniowców [Reefer ships exploitation]*, Trademar, Gdynia.
- Thomopoulos N. T., 2015, *Demand forecasting for inventory control*, Springer.
- Thomopoulos N. T., 2016, *Elements of Manufacturing, Distribution and Logistics. Quantitative Methods for Planning and Control*, Springer.
- Venkata Subbaiah K., Rao K.S., Satyanarayana B., 2004, *Inventory models for perishable items having demand rate dependent on stock level*, *Opsearch* 41 (4), 223-236.
- Venkata Subbaiah K., Rao S.V.U.M., Rao K.S., 2011, *An inventory model for perishable items with alternative rate of production*, *Advanced Operations Management* 3 (1), 66-87.

OPTIMALNE STEROWANIE ZAPASAMI PRODUKTÓW SZYBKOPSUJĄCYCH SIĘ Z UWZGLĘDNIENIEM DODATKOWYCH KOSZTÓW REDUKCJI PSUCIA SIĘ TOWARU

STRESZCZENIE. Wstęp: Artykuł analizuje problemy sterowania zapasami towarów szybko psujących się, których jakość może się pogorszyć podczas magazynowania. Ma on na celu zaproponowanie uogólnienia klasycznych modeli, stosowanych w teorii optymalnego sterowania zapasami produktów szybko psujących się, dostosowanych do sytuacji, w której dostawca ponosi dodatkowe koszty związane z redukcją psucia się towaru.

Metody: Opracowany został uogólniony model Wilsona dla optymalnego sterowania zapasami. Założono, że tempo pogorszenia jakości towaru zależy nieliniowo od wielkości inwestycji przeznaczonych na redukcję tego zjawiska. Najpierw, przedstawiono uogólniony model Wilsona dla jednej pozycji asortymentowej, następnie ukazano ten model dla przypadku wieloasortymentowej produkcji.

Wyniki i wnioski: W celu wspólnej optymalizacji wielkości partii zamówienia i wielkości inwestycji zostały sformułowane odpowiednio zadania nieliniowej optymalizacji. W celu zilustrowania poprawności funkcjonowania modelu przedstawiono wyniki obliczeń. Stwierdzono, że proponowany model optymalizacyjny może być wykorzystany przez dostawców jako metodyczna podstawa w ich działalności inwestycyjnej skierowanej na poprawę technologii przechowywania produktów szybko psujących się.

Słowa kluczowe: sterowanie zapasami; produkt szybko psujący się; kontrola psucia się; uogólniony model Wilsona; optymalizacja wielkości partii.

OPTIMALE BESTANDSKONTROLLE DER VERDERBLICHEN WAREN MIT BERÜCKSICHTIGUNG DER ZUSÄTZLICHEN KOSTEN DER VERDERBREDUKTION

ZUSAMMENFASSUNG. Einleitung: Die vorliegende Arbeit analysiert Probleme der Bestandskontrolle verderblicher Waren, die während der Lagerung verderben können. Der Artikel bezweckt eine Empfehlung von allgemeinen, klassischen Modellen, die in der Theorie der optimalen Bestandsführung von verderblichen Waren angewendet werden und auch an die Situationen angepasst sind, in denen der Lieferant Zusatzkosten für die Reduzierung des Warenverderbs tragen muss.

Methoden: Das verallgemeinerte Wilson-Modell für optimale Losgrößen wurde entwickelt. Es wird angenommen, dass die Rate des Verderbs nichtlinear von dem für die Reduktion des Warenverderbs bestimmten Investitionsvolumen abhängt. Der Fall der inversen Leistungsabhängigkeit wurde im Detail analysiert. Zunächst wurde das verallgemeinerte Wilson-Modell für ein einzelnes Element untersucht, und dann wurde dieses Modell für den Multi-Element-Fall in Anspruch genommen.

Ergebnisse und Fazit: Für die gemeinsame Optimierung der Losgrößen und Volumen der nötigen Investitionen wurden die entsprechenden nichtlinearen Optimierungsprobleme formuliert. Die numerischen Ergebnisse wurden dargestellt, um die Richtigkeit der Modell-Funktion zu veranschaulichen. Es wurde darauf hingewiesen, dass das vorgeschlagene Optimierungsmodell als eine methodische Grundlage für die Lieferanten in ihrer Investitionstätigkeit zur Verbesserung der Lagerungstechnologie bei verderblichen Produkten verwendet werden kann.

Codewörter: Bestandsmanagement, verderbliches Produkt, Kontrolle des Verderbs, das verallgemeinerte Wilson-Modell, Losgrößenoptimierung.

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