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ACCOUNTING THE SCALE AND SYNERGIES IN THE DEA-ANALYSIS

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ABSTRACT. **Background:** The proposal is to non-linear performance into account in terms of synergy when conducting DEA (Data Envelopment Analysis). The solution to the problem is produced for interacting schools, which can be regarded as business clusters. The inputs and outputs are selected by importance on the basis of the author's opinion. However, the technique does not change when taking into account other factors that are expressed numerically.
Methods: The proposal is to reduce the number of the inputs and outputs to one input and one output using weighting factors. Thus a solution can be found by linear programming. The DEA algorithm is easily coded in Mathcad.
Results: As a result, we obtain a vector of the effectiveness of each element in the business cluster, including stand-effective and super-efficient elements.
Conclusions: A model of DEA is proposed which takes into account the scale and synergies of the business cluster. This allows a performance rating against the collective interaction to be obtained.

Key words: DEA, business cluster, linear programming, inputs, outputs, synergies, efficiency, super-efficiency.

INTRODUCTION

A mathematical method of DEA (Data Envelopment Analysis) [Charnes, Cooper, Rhodes, 1978] allows us to make an adequate model assessment of business clusters, which are identical in the economic activity. The activity of the informationally interacting enterprises can be fully considered as the business activity of the business cluster, so the method of DEA with an appropriate choice of comparable inputs and outputs can be extended to these companies [Akbarian et al., 2015].

In the simplest case, the result of the business cluster's activity is defined by its single output (e.g. profit) depending on a single input (e.g. investment). To quantify the relative effectiveness on the plane (X, Y) it is constructed a scatter chart (input, output), as shown for the 5-members business cluster in Fig. 1.

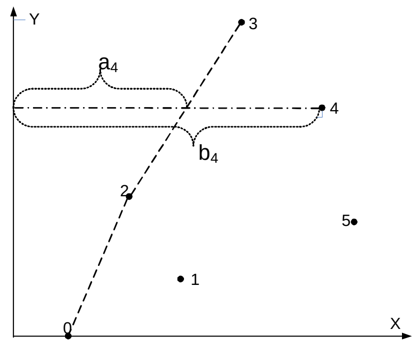

Fig. 1. Scatter plot of the input-output
Rys. 1. Schemat wejść-wyjść

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RESULTS AND DISCUSSION

Obviously, the participants of a business cluster which are the extreme left points will be the most effective, because their output values are maximum when the input values are minimum. In accordance with this principle, these points are the points 1, 2, 3. However, as we can see in Fig. 1, the point 1 is not the most efficient. Because despite having larger input than the point 2 has, its output is much less than of point 2. To avoid the possibility of including these points in the category of effective work [Novikov, Falko, Petrunya, 2009] it is proposed to introduce a fictitious point 0 on the axis X. The meaning of the 0 point is reduced to the minimum *allowable* input, for example according to the safety standard (floor space). In extreme cases, this point may be at the beginning of the coordinate system (X, Y). With the introduction of a fictitious point 0, as shown in Fig. 1, point 1 is automatically excluded from the category of the most effective. Leftmost points on the scatter chart are considered big-effective with the efficiency θ = 1 (points 0, 2, 3 in Fig. 1). These points form a convex polygon 0-2-3. In order to calculate the efficiency of the other points from the point *i*, we draw the line parallel to the axis X (see Fig. 1 point 4). Efficiency point *i* is given by:

$$\theta_i = \frac{a_i}{b_i} \; ,$$

where a_i, b_i correspond to the segments shown in Fig. 1 for point 4. Obviously, the efficiency ratio is in the range $0 \leq \theta_i \leq 1$. Than smaller θ_i we have, than lower the efficiency of the party business cluster we get. For example, if $\theta_i = 0{,}1$ the efficiency is 10 times lower than that of big-effective business cluster. That is, the coefficient of inefficiency can be defined as:

$$L_i = \frac{1}{\theta_i} \; .$$

The task of evaluating the effectiveness becomes much more complicated when there are multiple inputs X and multiple outputs Y. Taking all the information about the inputs and outputs into consideration it is necessary to evaluate relative measurement of the specific input and output in the form of weight ratio α_i β_j for inputs and outputs. Values α_i and β_j may be not-normalized from 0 to 1, and may take any positive value. Value $\alpha_i = 0$ means that the i-th input is not accounted in the evaluation of effectiveness. α_i = max value specifies the maximal evaluation degree to i-th input. After setting the weight ratio α_i, β_j inputs and outputs can be reduced by mixing weighting to one input and one output, as proposed in contrast to the known methods in [Novikov, Falko, Petrunya, 2009]. To provide adequacy of mixed inputs and outputs they are to be normalized to a maximum of 1, in other words should be deprived of dimensionality.

$$XM_i = \max_n X_{in}, \;\; \overline{X}_{in} = X_{in}/XM_i \; ,$$

where: i - i-th input, n - n-th value of the i-th input

$$YM_j = \max_k Y_{jk}, \;\; \overline{Y}_{jk} = Y_{jk}/YM_j \; ,$$

where: j - j-th input, k - k-th value of j-th input.

This method of calculation in DEA is attractive due to the fact that the higher accuracy allows to eliminate direct programming and to use Mathcad calculations only [Peiwu, Kai, Mei, 2015].

Main drawbacks of the above method are the following two problems.

Firstly, this method does not take into account the influence of the size of the members of the business cluster on efficiency. So almost always, the business cluster with bigger staff will be big-effective, since it is obvious that the sum of its output will be incommensurable with the business cluster with smaller staff. On the other hand the total input, in condition when the staff is not taken into account, leads towards the under-performance.

Thus, it is necessary to define two categories among the inputs and outputs. The first category includes the inputs and outputs that are independent of the number of team members. The second category includes the inputs i and outputs j, which values are necessary to be counted by one staff unit S_k for *k-th* member of the business cluster:

$$x_{ik} = x_{ik}/\,S_k, \quad y_{jk} = y_{jk}/S_k\,.$$

Secondly, method does not take into account the degree of contribution of an input and output to the synergy of the totality of the business cluster which contradicts to the law of Robert Metcalfe. It is obvious, for example, that the party of the business cluster with a large amount of investment is making a significant contribution to the development of new technologies for the whole system of the business cluster. The accounting of such an input in the method above is linear. The linear model in this case would significantly mark down the synergistically important inputs and outputs and generally underestimate the effectiveness of the participant of the business cluster. According to the law of Robert Metcalf it is necessary to take a non-linear relationship for these inputs i and outputs j:

$$x_{ik} = x_{ik}^2, \; y_{jk} = y_{jk}^2.$$

On the other hand in the system of business clusters there are inputs and outputs that negatively affect the synergy of the system and they need to be taken into account with the degree of ½:

$$x_{ik} = x_{ik}^{0.5}, \qquad y_{ik} = y_{ik}^{0.5}$$

In general, all inputs and outputs must be overridden by the formulas:

$$x_{ik} = x_{ik}^{\gamma_i}, \quad y_{jk} = y_{jk}^{\sigma_j},$$

where: $(\gamma_i, \delta_j) = [0.5;2]$.

The meaning of $(\gamma_i, \delta_j) = 2$ and $(\gamma_i, \delta_j) = 0.5$ is described above. The meaning of $(\gamma_i, \delta_j) = 1$ comes to the inputs and outputs, the effect on synergy system of which is independent of the value. For example, this input can be a bill of goods in the store, the number of cash registers in the store, etc.

In this problem, as in [Koster, Kosterin, 2006], we calculate the efficiency and super-efficiency for big-effective schools with Mathcad.

The calculations were performed for the 10 hypothetical schools with data, where the first school of [Novikov, Falko, Petrunya, 2009] is a dummy in accordance with the method and defines minimum permissible parameters on standards for the school:

ORIGIN := 1

The input: The number of graduates

x1:= $(5\ 52\ 23\ 82\ 23\ 18\ 12\ 45\ 32\ 53)^T$

The input: The number of visitors of the creative sections in the school

x2:= $(0\ 71\ 33\ 17\ 33\ 28\ 18\ 35\ 72\ 83)^T$

The input: The number of visitors of the sport sections

x3:= $(0\ 21\ 13\ 27\ 24\ 21\ 11\ 25\ 32\ 33)^T$

The input: The number of members of the youth organizations

x4:= $(0\ 41\ 23\ 42\ 54\ 51\ 35\ 31\ 20\ 45)^T$

The input: The number of excellent pupils among the graduates

x5:= $(0\ 12\ 3\ 6\ 9\ 11\ 5\ 13\ 18\ 16)^T$

The input: The number pupils with scores above 3.5

x6:= $(0\ 22\ 13\ 4\ 15\ 9\ 7\ 17\ 28\ 22)^T$

The output: The number of entrants to universities

y1:= $(0\ 15\ 1\ 46\ 5\ 4\ 5\ 22\ 1\ 16)^T$

The output: The number of entrants to colleges

$y2 := (0\ 25\ 11\ 16\ 15\ 11\ 5\ 24\ 21\ 20)^T$

In our opinion the most important parameters of the school are taken as the inputs and outputs, although there may be added other parameters that have numeric expressions. The only condition for the input selection is the condition of its positive efficiency with an increase in the numerical value of the condition, so, for example, the number of pupils with scores above 3.5 is taken as input, and not vice versa. Also the positive parameter of business cluster must be taken as output.

After making the initial data it is necessary to define the normalization ratios, which determine the importance of each input and output.

The normalization ratio

For inputs $a := (1\ 1.6\ 1.8\ 1.3\ 3\ 2.5)^T$

For outputs $b := (1\ 0.4)^T$

In this example, the significance of the fifth input (the number of excellent pupils among the graduates) is defined with a ratio 3 relatively to the first input (the number of graduates). And for outputs, the significance of the second output (the number of entrants to colleges) is defined with a ratio 0.4 relatively to the first output (number of entrants to universities).

The accounting of the collective impact leads the elements of the collective system to the same of conditions relative to the number of students in the school, which excludes the effect of the dependence of the efficiency on the size of the school in the analysis:

$i := 2..6$

The accounting of the collective impact $y_1 := \overrightarrow{\frac{y_1}{x_1}}$ $\quad y_2 := \overrightarrow{\frac{y_2}{x_1}}$ $\quad x_i := \overrightarrow{\frac{x_i}{x_1}}$

Where the vector x_1 determines the number of students in schools.

One of the novelties of the proposed method of calculation is the accounting of synergies associated with the nonlinear impact of some outputs on the collective self-organization of the whole system. In case of our analysis of schools it can be considered that the greatest synergy is provided by visiting creative sections by pupils (factor 2) and the number of excellent pupils at the school (factor 1.8). On the other hand, the number of graduates with a score below 3.5 obviously affects the whole synergy of the collective system, so for this input the synergy ratio is taken minimal:

The accounting of synergy $x_2 := (x_2)^2$ $\quad x_6 := (x_6)^{0.5}$ $\quad x_5 := (x_5)^{1.8}$

As suggested in [2], all the inputs and outputs need to be led to one input and one output using the normalization factors:

$j := 1..6 \quad m := 1..2$

calculation of the maximum values $x\_m_j := \max(x_j) \quad y\_m_m := \max(y_m)$

normalization $x_j := \overrightarrow{\frac{x_j}{x\_m_j}} \quad y_m := \overrightarrow{\frac{y_m}{y\_m_m}}$

reduction to one input $X := \sum_j \left[\overrightarrow{(x_j \cdot a_j)}\right] \quad X\_m := \max(X) \quad X := \frac{X}{X\_m}$

reduction to one output $y := y_1 \cdot b_1 + y_2 \cdot b_2 \quad y\_m := \max(y) \quad y := \frac{y}{y\_m}$

Note that when calculating the vectors x and y and the sum for X in Mathcad it is necessary to use the operation of vectorization.

The direct implementation of the method is implemented by solving the problem of linear programming, as shown for the 4th input (S = 4):

calculation of the efficiency ratio for S input

$S := 4 \quad m := \mathrm{rows}(y) \quad m = 10 \quad i := 1..m \quad \lambda_i := 0$

$$F(\lambda) := \frac{1}{X_S} \cdot \sum_{ii=1}^{m} (X_{ii} \cdot \lambda_{ii})$$

Given

$$\sum \lambda = 1 \quad y_S = \sum_{ii=1}^{m} (y_{ii} \cdot \lambda_{ii}) \quad \lambda \ge 0 \quad \lambda_S = 0$$

$\lambda := \mathrm{Minimize}(F, \lambda)$

$\theta := F(\lambda) \quad \theta = 1.683$

For clarity, Figure 2 shows the distribution of all the points of generalized normalized input and output in the plane, where the point at the origin corresponds to a fictitious school. For the calculation of big- effective schools S it is necessary to put λ_S S = 0 into the block Given.

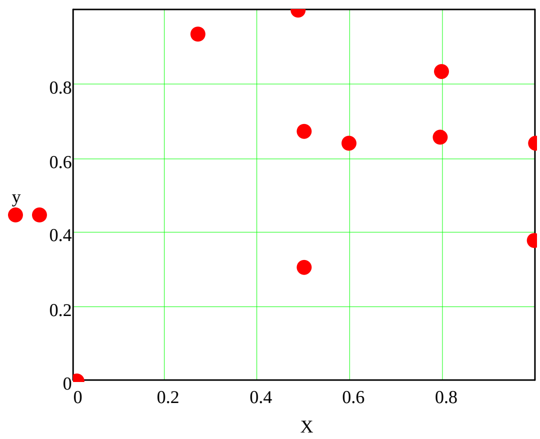

Fig. 2. Diagram of an input-output
Rys. 2. Wykres wejść-wyjść

This resulted in the following effectiveness parameters:

$$\Theta\Theta := (0\ \ 0.39\ \ 0.19\ \ 1\ \ 0.24\ \ 0.19\ \ 0.3\ \ 1\ \ 0.12\ \ 0.32)$$

$$\Theta\Theta\_super := (0\ \ 0.39\ \ 0.19\ \ 1.6\ \ 0.24\ \ 0.19\ \ 0.3\ \ 1\ \ 0.12\ \ 0.32)$$

According to these results, the least effective is the 8th school with efficiency ratio of 0.12, but the most effective is the third school with the super-efficiency ratio1.6.

If you place the assessment of super-efficiency in descending order, it is natural to get school's ranking score of rating from the leader to the outsider.

SUMMARY

The proposed methodology of DEA-analysis, in contrast to [Charnes, Cooper, Rhodes, 1978, Novikov, Falko, Petrunya, 2009] allows to take into account the synergy of the system in two directions. First, it allows eliminating the effect of the scale of school, which puts all the schools in the same conditions. Secondly, it allows to take into account the non-linear nature of the inputs and outputs that are either extremely needed for synergy, or make no effect on the synergy, or negatively affect the synergy.

The described method is interesting because it allows you to specify the rating not only of schools, as it is done in this work, but to make a full DEA in the form of rating for any group of objects that can be represented in the form of a business cluster. Note that the proposed method of DEA-analysis does not require a high-level programming tools and can be elegantly realized in MathCad.

REFERENCES

Akbarian M., Najafi E., Tavakkoli-Moghaddam R., Hosseinzadeh-Lotfi F., 2015, A Network-Based Data Envelope Analysis Model in a Dynamic Balanced Score Card.

Charnes A., Cooper W.W., Rhodes E. Measuring the efficiency of decision making unitSLL European Journal of operational Research, 2, 1978, 429-444.

Novikov V., Falko L., Petrunya G., Simplified method of DEA-analysis, Economic studies: analysis of state and development prospects, 19, 2009, 142-150.

Koster A., Kosterin I. Increase of store competitiveness based on DEA-analysis, Practical Marketing, 12, 2006, 30-44.

Peiwu D., Kai Q., Mei Y., 2015, Operational efficiency across the Chinese aerospace industry: a DEA and Malmquist analysis, Chinese Management Studies, 9, 4, 553-570.

OBLICZANIE SKALI I SYNERGII W ANALYZIE DEA

STRESZCZENIE. **Wstęp:** Poddano analizie metodę nieliniową dla synergii przy zastosowaniu metody DEA (Data Envelopment Analysis). Rozwiązanie zostało zrealizowane dla współpracujących szkół, które mogą być traktowane, jako klastery biznesowe. Dane wejściowe i wyjściowe zostały wyselekcjonowane według ważności (w opinii autora). Aczkolwiek należy zaznaczyć, że metoda postępowania nie zmienia się przy analizie innych czynników, mogących być przedstawione w formie liczbowej.

Metody: Zaproponowano redukcję liczby danych wejściowych i wyjściowych przy zastosowaniu współczynnika wagi. Następnie poszukano rozwiązania przy użyciu programowaniu liniowego. Algorytm DEA może być z łatwością zaimplementowany przy użyciu MATCAD.

Wyniki: Otrzymano wektor efektywności każdego elementu klastera biznesowego, łącznie z elementami o stałej i bardo dużej efektywności.

Wnioski: zaproponowano model DAE, uwzględniający skalę oraz synergię klastera biznesowego. Umożliwia to uszeregowanie zachowań wobec zbiorowych interakcji..

Słowa kluczowe: DEA, klastera biznesowy, programowanie liniowe, wejście, wyjście, synergia, efektywność, superefektywność.

DIE BERECHNUNG DER SKALA UND DER SYNERGIE ANGESICHTS DER DEA-ANALYSE

ZUSAMMENFASSUNG. Einführung: Die nichtlineare Methode für die Synergie wurde unter Anwendung der DEA-Methode (Data Envelopment Analysis) einer Analyse unterzogen. Die Lösung wurde für eng zusammen arbeitende Schulen, die dabei als Business-Cluster behandelt werden können, ausgearbeitet. Die In- und Output-Daten wurden nach deren Relevanz (nach der Ansicht des Autors) ausgesondert. Es muss dabei allerdings beachtet werden, dass die Vorgehensweise bei der Analyse von anderen Faktoren, die in zahlenmäßiger Form projiziert werden können, sich jedoch nicht verändert.

Methoden: Es wurde die Einschränkung der Anzahl von den betreffenden In- und Output-Daten unter Anwendung des Relevanz-Koeffizienten vorgeschlagen. Demzufolge suchte man nach einer Lösung mit der Inanspruchnahme von linearer Programmierung. Der DEA- Algorithmus kann durch die Anwendung vom MATCAD leicht implementiert werden.

Ergebnisse: Es wurde ein Effizienz-Vektor für jeden Bestandteil des Business-Clusters, einschließlich der Elemente von konstanter und sehr hoher Effizienz, erzielt.

Fazit: Es wurde ein DAE-Modell, das die Skala und die Synergie des Business-Clusters, mit berücksichtigen kann, vorgeschlagen. Dies ermöglicht eine angemessene Anordnung der einzelnen Verhaltensweisen den Sammel-Interaktionen gegenüber.

Codewörter: DEA, Business-Cluster, lineare Programmierung, Input, Output, Synergie, Effizienz, Supereffizienz

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