CYCLIC DELIVERY-SCHEDULING PROBLEM WITH SYNCHRONIZATION OF VEHICLES' ARRIVALS AT LOGISTIC CENTERS

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ABSTRACT. Background: In this paper a cyclic delivery-scheduling problem with vehicles serving fixed routes is presented. Each vehicle is assigned to one route to which some manufacturers' warehouses and logistics centers belong. A vehicle is to be loaded at a manufacturer's warehouse, then to deliver goods to a logistics center and may be also loaded there with other goods and to transport them to the next node along the route. One logistic center belongs to several routes, so the goods delivered by one vehicle may continue their journey by another truck. For every route the frequency of the vehicle is fixed and known. The objective here is to obtain such synchronization of vehicles arrivals in logistics centers, so that it is possible to organize their arrivals in repeatable blocks.

Methods: In the paper the cyclic delivery-scheduling problem with vehicles serving fixed routes is formulated as a MIP model. Due to the fixed routes and desirable synchronization of vehicles arrivals in shared points this problem seems to be similar to the public transit network timetabling problem. Because of that the model presented here was based on a model dedicated to the public transit network timetabling problem, where optimization criterion was to maximize synchronization of vehicles' arrivals at the shared nodes.

Results: Mixed integer programming model was employed for solving several cases of cyclic delivery-scheduling problem with vehicles serving fixed routes. Computational experiments are reported and obtained results are presented.

Conclusions: The mixed integer programming model for the cyclic delivery-scheduling problem with synchronization of vehicles arrivals at logistic centers presented in this paper can be utilized for generating schedules for a group of vehicles serving fixed long routes. It may result in reducing total operational cost related to this group of vehicles as well as in reducing the goods travel time from the place of origin to their destination.

Key words: cyclic delivery scheduling problem, mixed-integer programming, optimization, synchronization, logistics.

INTRODUCTION

This paper is devoted to the possible utilization of a MIP model for the cyclic delivery-scheduling problem for a network of manufacturers' plants and logistics centers where vehicles serve fixed routes. The objective of this formulation of cyclic delivery-scheduling problem is to obtain such synchronization of vehicles arrivals in logistics centers and load time, so that it is possible to organize their arrivals in repeatable modules.

In a logistics center all the processes of handling, loading and discharging goods as well as document circulation processes are controlled with an internal logistics management system. Efficiency of this system results in efficiency of the entire logistics center, therefore both separate modules as well as the complex system are designed and implemented with the aim to increase efficiency of warehousing and inventory management. These goals are obtained through shortening time of receipting and dispatching operations, speeding-up handling operations,
eliminating or reducing bottlenecks in handling and warehousing processes. In result the utilization rate of storage areas is supposed to increase, inventory costs are supposed to decrease, and level of customer service quality should raise [Chudzik 2006, Sitko and Gajdzik 2013].

In research on optimization of internal logistics management systems mathematical and IT tools are utilized. With these tools researchers are able to precisely identify warehousing and logistical capabilities of an enterprise, to monitor an individual consignment, to analyze size and due-dates of deliveries as well as to plan and schedule deliveries [Krystek 2011].

In modern approach to logistics systems loading, handling, dispatching, warehousing and inventory management processes are organized according to the 4R model: a consignment should be delivered in required quantity to right place in right time and all the deliveries should be served in right order [Gudehus and Kotzab 2009]. Right order and right time are crucial for scheduling routes for deliveries that need to be trans-shipped or handled in logistics centers or container terminals. In a system of cyclic-deliveries this problem is getting even more meaningful, since all the deliveries within the fixed network are being repeated with fixed frequencies. Therefore, when some consignments are to be handled between vehicles serving different fixed routes, then arrivals of these vehicles should be synchronized, so that the handling operations can be carried on quickly and goods can be temporarily stored in the storing area next to the loading ramp. Both the logistic center and a final customer benefit from such approach to cyclic-deliveries scheduling. First of all, the travel time of the consignment reduces, so the customers receive their delivery earlier. Secondly, the loading ramp devices are calibrated once and they can be utilized both for unloading one vehicle and reloading another, so the efficiency of the given loading ramp increases. Certainly, the above mentioned approach to the cyclic-deliveries scheduling process can be adopted when we can control vehicles’ departure times from the first point along their route. What is more the schedule has to include due-date and delivery orders as well as resources and specific character of a given logistics center [Ambroziak, Lewczuk 2008].

In general cyclic delivery-scheduling problem addresses numerous issues. Different mathematical models were developed for this problem, because different set of aspects were taken into account in each of them [Groenevelt et al.1992, Raa and Dullaert 2007]. Cyclic delivery-scheduling is important in food industry due to the problem of how to organize properly lots of perishable goods to be delivered [Akkerman et al. 2010, Chudzik 2006]. This problem is also vital in municipal services management [Kazan et al. 2012]. Synchronization of cyclic deliveries is crucial for supply chain management in a system with one producer and many customers, especially when demand changes on the seasonal basis [Chang and Chou 2012, Ekici et al. 2014].

The objective of research to which this paper is devoted was to synchronize arrival times at logistics centers, so that vehicles serving different routes can be organized in repeatable groups. Synchronization is understood here as a situation when time between arrivals of two vehicles of different routes at a logistics center does not exceed certain value. In this paper the cyclic delivery-scheduling problem with synchronization of vehicles arrivals at logistic centers was presented as a theoretical problem for which MIP model was developed. The model was employed for solving a set of exemplary situations - computational experiments are presented and a selection of obtained results is reported.
In the paper a cyclic delivery-scheduling problem with vehicles serving fixed routes is presented. Each vehicle is assigned to one route to which some manufacturers' warehouses and logistics centers belong. A vehicle is to be loaded at a manufacturer's warehouse, then to deliver goods to a logistics center and there may be loaded with other goods and transport them to the next node along the route. One logistic center belongs to several routes, so the goods delivered by one vehicle may continue their journey with another truck. In this problem routes are fixed and known in advance as well as delivery size. For every route the frequency of vehicles is fixed and known. The objective of this formulation of cyclic delivery-scheduling problem is to obtain such synchronization of vehicles arrivals in logistics centers and load time, so that it is possible to organize their arrivals in repeatable blocks. If vehicles, between which goods are handled, arrive in a logistics center simultaneously or one right after another, then goods-to-be-handled can be temporarily stored in the storing area next to the loading ramp. For every logistic center we can determine how long goods can be stored in the storing area next to the loading ramp, so we can compute a time window for two subsequent vehicles' arrivals to be considered as a synchronized pair. For cyclic deliveries the frequency of delivery of each route is also known, therefore we can try to establish departure times of every vehicle, so the total number of synchronizations in the entire network is maximal.

Every change of limits of a node's time window influences significantly on the total number of synchronizations. Obviously, when a time window for a single logistic center gets changed (either widened or narrowed) it may result in changing schedules of other routes as well as it may influence operational costs of the logistic center itself. Therefore, analysis of obtained solutions (delivery schedule for each route and number of synchronizations) may provide the management of a logistics center with information useful for both planning functioning of the center and planning departure times of the cyclic-delivery-vehicles from the first nodes of their routes, so that efficiency of the entire system may increase.

Due to the fixed routes and desirable synchronization of arrivals at nodes shared by different routes this approach to the cyclic delivery scheduling problem seems to be similar to city transit network timetabling problem [Ibarra-Rojas and Rios-Solis 2012, Ceder et al. 2001, Eranki 2004]. Therefore the model developed for the cyclic delivery scheduling problem was inspired by the model that was originally developed for bus synchronization timetabling problem. In both models the objective is to maximize number of synchronization of arrivals in shared nodes.

For the cyclic delivery scheduling problem with synchronization of vehicles' arrivals at logistic centers a MIP model based on the BTP model [Ibarra-Rojas i Rios-Solis 2012] was developed. In the model following sets, variables and parameters were utilized: \( I \) – set of routes, \( B \) – set of nodes (logistic centers), \( J^i \) – set of pairs \(<i, j>\), where \( i \)-th and \( j \)-th routes share a node, \( S^{ijb} \) – set of triples \(<i, j, b>\) where \( i \)-th and \( j \)-th routes share \( b \)-th node, \( T \) – parameter, planning horizon, that is the period during which all the deliveries must departure from the first nodes of their routes, \( f_{r_i} \) – parameter, number of the deliveries to be scheduled for the \( i \)-th route, \( H_i \) – parameter, fixed headway of the \( i \)-th route, \( t_{ib} \) – parameter, travel time between the first node of the \( i \)-th route and the \( b \)-th node, \( w_b \) – parameter, minimal interval between arrivals of consecutive deliveries in the \( b \)-th node, \( W_b \) – parameter, maximal interval between arrivals of consecutive deliveries in the \( b \)-th node, \( M \) – big number.

In this model two types of variables were utilized: \( X_{ip} \) – departure time of the \( p \)-th
delivery of the $i$-th route, $Y_{ibpq}$ – presence or absence of synchronization between every pair of deliveries that arrive in the $b$-th node.

As it was already mentioned, the objective is to maximize the number of synchronizations of the entire system. Synchronization is understood as simultaneous of consecutive arrivals of vehicles serving different routes in the shared node. Interval between these two arrivals should not exceed the range $(w_b; W_b)$.

Table 1. Mixed integer programming model for the cyclic-delivery scheduling problem with time windows

<table>
<thead>
<tr>
<th>Objective function:</th>
<th>$\text{max} \rightarrow \sum_{i \in I} \sum_{j \in J} \sum_{p \in P} \sum_{q \in Q} Y_{ibpq}$</th>
<th>$i, j \in I; b \in B; 1 \leq p \leq f_r; 1 \leq q \leq f_r$</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subject to:</td>
<td>$X_{i1} \leq H_i$</td>
<td>$i \in I$</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>$X_{ip}, \leq T$</td>
<td>$i \in I$</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>$T - H_i \leq X_{i,f_r}$</td>
<td>$i \in I$</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>$X_{ip1} - X_{ip} = H_i$</td>
<td>$i \in I; 1 \leq p \leq f_r - 1$</td>
<td>(5)</td>
</tr>
<tr>
<td></td>
<td>$(X_{ip} + f_r) - (X_{ip} + f_{pq}) \leq W_b + M \cdot (1 - Y_{ibpq})$</td>
<td>$i, j \in I; b \in B; 1 \leq p \leq p_r; 1 \leq q \leq f_r$</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>$(X_{ip} + f_r) - (X_{ip} + f_{pq}) \geq w_b - M \cdot (1 - Y_{ibpq})$</td>
<td>$i, j \in I; b \in B; 1 \leq p \leq f_r; 1 \leq q \leq f_r$</td>
<td>(7)</td>
</tr>
<tr>
<td></td>
<td>$Y_{ibpq} \leq 1 - Y_{ibpq}$</td>
<td>$i, j \in I; b \in B; 1 \leq p \leq f_r; 1 \leq q \leq f_r, i &lt; j, b \geq 0$</td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>$Y_{ibpq} \geq 0$</td>
<td>$i, j \in I; b \in B; 1 \leq p \leq f_r; 1 \leq q \leq f_r, i &lt; j, b \geq 0$</td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td>$X_{ip} + H = Z_{ip}$</td>
<td>$i \in I; i \in I; p \leq f_r$</td>
<td>(10)</td>
</tr>
<tr>
<td></td>
<td>$Y_{ipb} \in {0,1}$</td>
<td>$i \in I; b \in B; 1 \leq p \leq f_r; 1 \leq q \leq f_r$</td>
<td>(11)</td>
</tr>
<tr>
<td></td>
<td>$Z_{ipb} \in {0,1,2,...}$</td>
<td>$i \in I; b \in B; 1 \leq p \leq f_r$</td>
<td>(12)</td>
</tr>
</tbody>
</table>

Source: own work based on the BTP model [Ibarra-Rojas and Rios-Solis 2012]

**COMPUTATIONAL EXPERIMENTS - UTILISATION OF A MIP MODEL FOR THE CYCLIC DELIVERY SCHEDULING PROBLEM WITH SYNCHRONIZATION OF VEHICLES' ARRIVALS AT LOGISTIC CENTERS**

Cyclic deliveries are performed along three fixed routes. For each route a separate group of vehicle is assigned to serve deliveries along this route. Vehicles of a given group ($V_k$) transport goods between three producers ($M_i$) and four logistics centers ($LC_j$). The order of service (route) was defined for every group of vehicle and it cannot be changed. We assume that every group of vehicle is big enough, so for every delivery scheduled to any possible time there is a vehicle available to serve it.

Vehicles serve following routes:
- Route V1: $M1 \rightarrow LC2 \rightarrow LC4 \rightarrow M3 \rightarrow LC1 \rightarrow M1$
- Route V2: $M1 \rightarrow LC1 \rightarrow M2 \rightarrow LC4 \rightarrow M1$
- Route V3: $M2 \rightarrow LC3 \rightarrow LC4 \rightarrow M3 \rightarrow LC2 \rightarrow M2$.

During the planning horizon from the first node of the $i$-th route exactly $f_r$ deliveries must depart. For every route ($i$) their headways ($H_i$) – that is intervals between departures of consecutive deliveries from the first node – are known and they are fixed during the entire planning horizon.

As it was already mentioned, synchronization $Y_{ibpq}$ is understood as arrivals of the $p$-th delivery along the $i$-th route and the $q$-th delivery along the $j$-th route in the $b$-th node simultaneously of consecutively, if the interval between these two arrivals should not exceed the range $(w_b; W_b)$. For this problem a 60-hour planning horizon $T$ will be utilized in computations. What is more, it was assumed that time of delivery receiving and reloading may be neglected.

For solving the problem the MIP model presented in the previous chapter was utilized. After solving a problem we obtain departure time of each delivery from the first node of its route. On that basis we can compute the arrival time of each delivery at every node along its...
route. To simplify the problem we assumed that the travel time between each pair of subsequent nodes equals 1 hour. The scheme of this problem is presented in the Fig. 1.

![Source: own elaboration](Fig. 1. Scheme of a network for which cyclic delivery scheduling problem with synchronization of vehicles' arrivals at logistic centers is to be formulated)

In Table 2 and Table 3 data utilized in computational experiments are presented. In Table 3 number of deliveries to be scheduled for every route and headways adopted for each route are presented. In Table 3 information of upper (W_b) and lower (w_b) limits of time windows are provided. Upper limits of time windows may yield different values.

<table>
<thead>
<tr>
<th>Route</th>
<th>Headway H_i [h]</th>
<th>Number of deliveries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route V1</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Route V2</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>Route V3</td>
<td>3</td>
<td>10</td>
</tr>
</tbody>
</table>

*Source: own elaboration*

Computations were conducted with a computer equipped with a processor Intel® Core™i3 2.20 GHz and 4 GB RAM. Searching for a solution was limited in advance by time limit that was equal 1000 seconds. This amount of time was enough to obtain optimal solution for every case. For this problem 144 cases was solved – 144 is the total number of combinations of time windows’ limits, w_b and W_b (see Table 3).

Selected results are to be found in the Table 5. Results obtained for these selected cases show relations between the width of time windows and the number of synchronizations obtained for the entire network and for individual nodes. This information is important for managing the whole network. In Table 4 we present the solution of the case where maximal number of synchronizations was obtained (the highest result amongst 144 cases) and solutions of cases where maximal number of synchronizations was obtained for individual nodes.
As it was to be expected the solution with the highest value of the objective function (58 synchronizations) was obtained for the case with the widest time windows. In Table 4 this case is presented as a Case 1, and it is illustrated with Figure 2.
In Figure 2 moments of arrivals of every vehicle to every node belonging to its route are presented. Cyclic repetition of deliveries is to be observed for each route. This solution can be compared with the one obtained for Case 5 - the highest number of synchronizations in logistic center no. 4 (node no. 5) amongst all the 144 solved cases. LC4 is shared by all the three routes, so synchronization in this node can be crucial for the system. In Case 6 the number of synchronizations obtained in LC4 equaled 17 and the total number of synchronizations was 51, while in the Case 1 they were, respective, 14 and 54. What is to be emphasized is the fact that the maximal number of synchronizations in LC4 was achieved when some upper limits of time windows did not have their maximal width; in Case 6 upper limits of time windows in nodes M1 and LC1 were lower than in Case 1.

**CONCLUSIVE REMARKS**

In this paper results obtained for a small network were presented. Continuation of research in this field is recommended, since there are many other aspects of cyclic deliveries that were not taken into consideration here. Amongst them are to be listed: deliveries to warehouses or logistics centers where many deliveries can be received in the same time, costs of travel and waiting time or vehicle assignment problem.

The approach to the cyclic delivery scheduling problem with synchronization of vehicles' arrivals at logistic centers together with the MIP model may be utilized as a supportive tool for decision-making problems in delivery planning. With the model presented in this paper we can obtain solution in reasonable time, however, assumptions adopted here simplify scheduling in a logistic center, since we focus on a specific kind of deliveries. Therefore the research in this area should be continued and the model should be developed by adding further aspects of the problem.
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REFERENCES


Ching-Ter Ch., Hsiao-Ching Ch., 2013, A coordination system for seasonal demand problems in the supply chain, Applied Mathematical Modelling, 37, 3674-3686.


PROBLEM HARMONOGRAMOWANIA DOSTAW CYKLIczNYCH Z WARUNKIEM SYNCHRONIZACJI PRZYJAZDÓW DO CENTRÓW PRZEŁADUNKOWYCH

STRESZCZENIE. Wstęp: W pracy przedstawiono problem harmonogramowania dostaw cyklicznych wykonywanych przez pojazdy obsługujące ustalone i niezmienne trasy. Każdy pojazd obsługuje inną trasę, gdzie ma za zadanie dostarczyć towar do centrum logistycznego, a także załadować tam innym towar i przewieźć go do kolejnego punktu trasy lub wykonać pusty przejazd do kolejnego punktu załadunku. Wspólnymi punktami tras pojazdów są centra logistyczne, w których niejednokrotnie towar przywiezione przez jeden pojazd, wyrusza w dalszą drogę następnym pojazdem z rozpatrywanej grupy. Przejazdy po każdej trasie realizowane są ze stałą częstotliwością. Celem dla wspomnianego problemu harmonogramowania dostaw cyklicznych jest uzyskanie synchronizacji przyjazdów i pobytu pojazdów w centrach logistycznych tak, aby możliwe było grupowanie ich obsługi w bloki.

Metody: Ze względu na sztywno wyznaczone trasy oraz pożądaną synchronizację przyjazdów do punktów wspólnych trasy ten problem odnosi się do podobnych problemów układy rozkładów jazdy komunikacji miejskiej. Dlatego przy konstruowaniu modelu matematycznego dla tego problemu wykorzystano model przygotowany pierwotnie dla zadania układy rozkładów jazdy komunikacji miejskiej z kryterium optymalizacji związanym z synchronizacją przyjazdów na przystanki wspólne.

 Wyniki: Eksperyment obliczeniowy polegał na rozwiązywaniu i porównaniu uzyskanych wyników dla zbioru zadań programowania całkowitoliczbowego mieszанego dla problemu harmonogramowania cyklicznych dostaw z warunkiem synchronizacji przyjazdów do centrów przeładunkowych.

Wnioski: Przedstawiony model MIP dla zadania harmonogramowania cyklicznych dostaw z warunkiem synchronizacji przyjazdów do centrów przeładunkowych może być wykorzystywany do tworzenia harmonogramów do planowania kursów cyklicznych wykonywanych przez grupę pojazdów obsługujących ustalone długie trasy. Pozwoli to na racjonalne planowanie pracy centrum logistycznego i pośrednio wpłynie na obniżenie kosztów, a także skrócenie czasu podróży towaru z punktu wysyłki do odbiorcy.

Słowa kluczowe: harmonogramowanie dostaw cyklicznych, optymalizacja, synchronizacja, programowanie całkowitoliczbowe mieszane

DAS PROBLEM DER TERMINPLANUNG VON ZYKLISCHEN LIEFERUNGEN UNTER DER VOTRAUSSETZUNG DER SYNCHRONISATION VON ANKÜNFTEN IN GÜTERVERKEHRSZENTREN

ZUSAMMENFASSUNG. Einleitung: In der Arbeit wurde das Problem der Terminplanung von zyklischen Lieferungen dargestellt, die von Fahrzeugen mit festgelegten und unveränderten Routen bedient werden. Jedes Fahrzeug bedient anderen Weg, wo es die Aufgabe hat, die Ware zum Logistikzentrum zu liefern, dort eventuell andere Ware abzuholen und zum nächsten Punkt auf dem Weg weiter zu transportieren oder auch einen leeren Transport zum nächsten Lieferungspunkt auszuführen. Die gemeinsamen Punkte für diese Fahrzeuge sind Güterverkehrszentren, in denen die von einem Fahrzeug transportierte Ware mehrfach mit einem anderen Fahrzeug aus der berücksichtigten Gruppe weiter befördert wird. Die Fahrten sind auf jeder Strecke mit fester Frequenz realisiert. Das Ziel der genannten Problemstellung ist eine solche Synchronisation von Fahrten und Aufenthalten der Fahrzeuge in Logistikzentren, die ihre grupenweise Bedienung und Abfertigung ermöglicht.

Methoden: In Anbetracht der fest bestimmten Fahrstrecken und einer erwünschten Synchronisation der Fahrten zu gemeinsamen Punkten zeigt dieses Problem eine Ähnlichkeit zu den Fragestellungen, die bei der Zusammenstellung von städtischen Verkehrsfahrplänen auftauchen. Daher wurde bei der Ausgestaltung eines mathematischen Modells für die Lösung dieses Problems ein Modell verwendet, das ursprünglich für die Aufgabenlösung der Zusammenstellung von Verkehrsfahrplänen mit dem Kriterium der Optimierung für die Synchronisation von verbundenen Ankünften in gemeinsame Haltestellen vorbereitet wurde.

Ergebnisse: Das Berechnungsexperiment verließ sich auf die Lösung und Vergleichung der gewonnenen Ergebnissen für die Aufgabenstellung der ganzzahligen gemischten Optimierung des Problems der Terminplanung von zyklischen Lieferungen mit der Voraussetzung einer optimalen Durchfahrtsynchronisation zu Güterverkehrszentren.
**Fazit:** Das dargestellte Modell der ganzzahligen Optimierung für die Aufgabe der Terminplanung von zyklischen Lieferungen mit der Voraussetzung der Fahrsynchronisation zu Güterverkehrszentren kann zum Bilden von Zeitplänen für die Planung von zyklischen Kursen der Fahrzeuggruppen, die feste und lange Fahrstrecken bedienen, verwendet werden. Das erlaubt, die Arbeit des Logistikzentrums rationell zu planen sowie die Kosten und Transportzeiten zwischen Lieferanten und Empfängern zu reduzieren.

**Codewörter:** Terminplanung von zyklischen Lieferungen, die ganzzahlige Optimierung, Synchronisation.

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