



OPTIMIZATION IN FUZZY ECONOMIC ORDER QUANTITY (FEOQ) MODEL WITH DETERIORATING INVENTORY AND UNITS LOST

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ABSTRACT. Background: This model presents the effect of deteriorating items in fuzzy optimal instantaneous replenishment for finite planning horizon. Accounting for holding cost per unit per unit time and ordering cost per order have traditionally been the case of modeling inventory systems in fuzzy environment. These imprecise parameters defined on a bounded interval on the axis of real numbers and the physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in the production system.

Methods: The modified fuzzy EOQ (FEOQ) model is introduced, it assumes that a percentage of the on-hand inventory is wasted due to deterioration and considered as an enhancement to EOQ model to determine the optimal replenishment quantity so that the net profit is maximized. In theoretical analysis, the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions are proved and further the concavity of the fuzzy net profit function is established. Computational algorithm using the software LINGO 13.0 version is developed to find the optimal solution.

Results and conclusions: The results of the numerical analysis enable decision-makers to quantify the effect of units lost due to deterioration on optimizing the fuzzy net profit for the retailer. Finally, sensitivity analyses of the optimal solution with respect the major parameters are also carried out. Furthermore fuzzy decision making is shown to be superior than crisp decision making in terms of profit maximization.

Key words: Optimization, Fuzzy, FEOQ, Deterioration, Units Lost.

INTRODUCTION

In the whole production system production function is the mid between the procurement function and physical distribution function. Other two functions are not processing in terms of production only they are facilitating for the smooth functioning and cost effecting of the production system in competitive advantage but production function processes to produce the finished products. So inventory plays a significant role in smooth functioning of the production function in a supply chain management. The physical characteristics of stocked items dictate the nature of inventory policies implemented to manage and control in production system. The question is how

reliable are the EOQ models when items stocked deteriorate one time.

Many models have been proposed to deal with a variety of inventory problems. Comprehensive reviews of inventory models can be found in Gupta and Gerchak [1995], Osteryoung et al. [1986] and Water [1994] and Tripathy et al. [2013] introduced a single item EOQ model with two constraints. This model considers a continuous review, using fuzzy arithmetic approach to the system cost for instantaneous production process. In traditional inventory models it has been common to apply fuzzy on demand rate, production rate and deterioration rate, whereas applying fuzzy arithmetic in system cost usually ignored in Salameh et al. [1999]. From

practical experience, it has been found that uncertainty occurs not only due to lack of information but also as a result of ambiguity concerning the description of the semantic meaning of declaration of statements relating to an economic world. The fuzzy set theory was developed on the basis of non-random uncertainties. Vujosevic et al. [1996] introduced the EOQ model where inventory system cost is fuzzy. Mahata and Goswami [2006] then presented production lot size model with fuzzy production rate and fuzzy demand rate for deteriorating items where permissible delay in payments are allowed. Tripathy and Pattnaik [2011] presented an optimal inventory policy with reliability consideration and instantaneous receipt under imperfect production process. Later, Tripathy and Pattnaik [2009, 2011] also investigated fuzzy EOQ model with reliability consideration in instantaneous production plan. Again Tripathy and Pattnaik [2008, 2011] developed fuzzy entropic order quantity model for perishable items under two component demand and discounted selling price, where entropic means the amount of the disorder in the production system. Pattnaik [2013] discussed the fuzzy EOQ model with demand dependent unit price and variable setup cost, Pattnaik [2011, 2013, 2013] investigated the fuzzy method for supplier selection in manufacturing system for smooth function of supply chain management and manpower selection for micro, small and medium enterprises respectively. For this reason, this model considers the same by introducing the holding cost and ordering cost as with allowing promotion and wasting the percentage of the fuzzy numbers. Sahoo and Pattnaik [2013] developed linear programming problem and post optimality analyses in fuzzy space with case study applications. Pattnaik [2013] defined linear programming problems with crisp and fuzzy based optimization methods and sensitivity analyses have also evaluated for decision parameters. Pattnaik [2013] derived profit maximization fuzzy EOQ models for deteriorating items with two dimension sensitive demand. The model provides an approach for quantifying the benefits of nonrandom uncertainty which can be substantial, and should be reflected in fuzzy arithmetic system cost.

Product perishability is an important aspect of inventory control. Deterioration in general, may be considered as the result of various effects on stock, some of which are damage, decay, decreasing usefulness and many more. While kept in store fruits, vegetables, food stuffs etc. suffer from depletion by decent spoilage. Decaying products are of two types. Product which deteriorate from the very beginning and the products which start to deteriorate after a certain time. Lot of articles is available in inventory literature considering deterioration. Interested readers may consult the survey model of Pattnaik [2011] investigated an entropic order quantity model for perishable items with pre and post deterioration discounts under two component demand in finite horizon. Pattnaik [2011] discussed an economic order quantity model for perishable items with constant demand where instant deterioration discount is allowed to obtain maximum profit. Goyal and Gunasekaran [1995] and Raafat [1991] surveyed for perishable items to optimize the EOQ model. The EOQ inventory control model was introduced in the earliest decades of this century and is still widely accepted by many industries today. Tripathy and Pattnaik [2008, 2011] studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where discounts are allowed for acquiring more profit. Pattnaik [2012] derived different types of typical deterministic EOQ models in crisp and fuzzy decision space.

Comprehensive reviews of inventory models under deterioration can be found in Bose et al. [1995]. In previous deterministic inventory models, many are developed under the assumption that demand is either constant or stock dependent for deteriorated items. Jain and Silver (1994) developed a stochastic dynamic programming model presented for determining the optimal ordering policy for a perishable or potentially obsolete product so as to satisfy known time-varying demand over a specified planning horizon. They assumed a random lifetime perishability, where, at the end of each discrete period, the total remaining inventory either becomes worthless or remains usable for at least the next period. Gupta and Gerchak [1995] examined the simultaneous selection product durability

and order quantity for items that deteriorate over time. Their choice of product durability is modeled as the values of a single design parameter that effects the distribution of the time-to-onset of deterioration (TOD) and analyzed two scenarios; the first considers TOD as a constant and the store manager may choose an appropriate value, while the second assumes that TOD is a random variable. Hariga [1995] considered the effects of inflation and the time-value of money with the assumption of two inflation rates rather than one, i.e. the internal (company) inflation rate and the external (general economy) inflation rate. Hariga [1994] argued that the analysis of Bose et al. [1995] contained mathematical errors for which he proposed the correct theory for the problem supplied with numerical examples. Padmanavan and Vrat [1995] presented an EOQ inventory model for perishable items with a stock dependent selling rate. They assumed that the selling rate is a function of the current inventory level and the rate of deterioration is taken to be constant. The most recent work found in the literature is that of Hariga [1996] who extended his earlier work by assuming a time-varying demand over a finite planning horizon. Goyal et al. [2001] and Shah [2000] explored the inventory models for deteriorating items. Pattnaik [2010, 2011] studied profit maximization entropic order quantity model for deteriorated items with stock dependent demand where instant deterioration and post deterioration cash discounts respectively are allowed for acquiring more profit. Pattnaik [2011] developed an entropic order quantity model for deteriorating items where cash discounts are allowed but Pattnaik [2011] modified again to obtain the decision parameters for perishable items where instant deterioration discount is allowed in EOQ model. Pattnaik [2012] introduced a non linear profit maximization entropic order quantity model for deteriorating items with stock dependent demand rate. Pattnaik [2012] derived an EOQ model for perishable items with constant demand and instant deterioration.

Furthermore, retailer promotional activity has become more and more common in today's business world. For example, Wall Mart and Costco often try to stimulate demand for specific types of electric equipment by offering

price discounts; clothiers Baleno and NET make shelf space for specific clothes items available for longer periods; McDonald's and Burger King often use coupons to attract consumers. Other promotional strategies include free goods, advertising, and displays and so on. The promotion policy is very important for the retailer. How much promotional effort the retailer makes has a big impact on annual profit. Residual costs may be incurred by too many promotions while too few may result in lower sales revenue. Tsao and Sheen (2008) discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Salameh et al. [1999] studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked. The effect of deteriorating items on the instantaneous profit maximization replenishment model under promotion is considered in this model. The market demand may increase with the promotion of the product over time when the units lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. Tripathy et al. [2012] investigated an optimal EOQ model for deteriorating items with promotional effort cost. Pattnaik [2012] explored the effect of promotion in fuzzy optimal replenishment model with units lost due to deterioration. Hence Pattnaik [2013] developed many instantaneous EOQ models and fuzzy EOQ models which are incorporated with promotional effort cost, fixed ordering cost, variable ordering cost and units lost due to deterioration. This model introduces a modified fuzzy EOQ model in which it assumes that a percentage of the on-hand inventory is wasted due to deterioration. There is hidden cost not account for when modeling inventory cost. This model studies the problem of promotion for a deteriorating item subject to loss of these deteriorated units. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with wasting the percentage of on-hand inventory due to deterioration and then compares the optimal results with none wasting

the percentage of on-hand inventory due to deterioration traditional model. This model addresses the problem by proposing an inventory model under promotion by assuming that the units lost due to deterioration of the items. In this model, promotional effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. The objective of this model is to determine the optimal time length, optimal units lost due to deterioration, the promotional effort and the replenishment quantity with fixed ordering cost so that the net profit is maximized in an instantaneous replenishment fuzzy EOQ model and the numerical analysis show that an appropriate promotion policy can benefit the retailer and that promotion policy is important in fuzzy space, especially for deteriorating items. Finally, sensitivity analyses of the optimal solution with respect to

the major parameters are also studied to draw the managerial insights. Furthermore crisp decision making is shown to be superior to crisp decision making without promotional effort cost in terms of profit maximization.

This model establishes and analyzes the fuzzy inventory model under profit maximization which extends the classical economic order quantity (EOQ) model. An efficient FEOQ does more than just reduce cost. It also creates revenue for the retailer and the manufacturer. The evolution of the FEOQ model concept tends toward revenue and demand focused strategic formation and decision making in business operations. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus.

Table 1. Summary of the Related Researches
 Tabela 1. Podsumowanie prac o podobnej tematyce

Author(s) and published Year	Structure of the model	Demand	Demand patterns	Deterioration	Planning	Units Lost due Deterioration	Model
Hariga (1994)	Crisp (EOQ)	Time	Non-stationary	Yes	Finite	No	Cost
Tsao et al. (2008)	Crisp (EOQ)	Time and Price	Linear and Decreasing	Yes	Finite	No	Profit
Pattnaik (2009)	Crisp (EnOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Finite	No	Profit
Pattnaik (2011)	Crisp (EOQ)	Constant (Deterministic)	Constant	Yes (Instant)	Finite	No	Profit
Present model (2013)	Fuzzy (FEOQ)	Constant (Deterministic)	Constant	Yes (Wasting)	Finite	Yes	Profit

In recent years, companies have started to recognize that a tradeoff exists between product varieties in terms of quality of the product for running in the market smoothly. In the absence of a proper quantitative model to measure the effect of product quality of the product, these companies have mainly relied on qualitative judgment. The model tackles to investigate the effect of the wasting the percentage of on-hand inventory due to deterioration for obtaining the optimum average payoff and the optimal values of the policy variables. The problem consists of the optimization of fuzzy EOQ model, taking into account the conflicting payoffs of the different decision

makers involved in the process. A policy iteration algorithm is designed and optimum solution is obtained through LINGO 13.0 version software. In order to make the comparisons equitable a particular evaluation function based on units lost is suggested. This model postulates that measuring the behavior of production systems may be achievable by incorporating the idea of retailer promotional effort in making optimum decision on replenishment units lost due to deterioration. Numerical experiment is carried out to analyze the magnitude of the approximation error. However, adding wasting the percentage of on-hand inventory due to deterioration in fuzzy model might lead

to super gain for the retailer. The major assumptions used in the above research articles are summarized in Table 1.

The remainder of the model is organized as follows. In Section 2 assumptions and notations are provided for the development of the model. The mathematical formulation is developed in Section 3 and the fuzzy mathematical model is developed in 4. The solution procedure is given in Section 5. In Section 6, numerical example is presented to illustrate the development of the model. The sensitivity analyses are studied in Section 7 to observe the changes in the optimal solution for change in major parameter. Finally in Section 8 the summary and the concluding remarks are explained.

ASSUMPTIONS AND NOTATIONS

r	Consumption rate,
t_c	Cycle length,
h	Holding cost of one unit for one unit of time,
HC (q)	Holding cost per cycle,
K	Setup cost per cycle,
c	Purchasing cost per unit,
P_s	Selling Price per unit,
α	Percentage of on-hand inventory that is lost due to deterioration,
q	Order quantity,
q^{**}	Modified economic ordering / production quantity (EOQ/EPQ),
q^*	Traditional economic ordering quantity (EOQ),
$\varphi(t)$	On-hand inventory level at time t,
$\pi_1(q)$	Net profit per unit of producing q units per cycle in crisp strategy,
$\pi(q)$	Average profit per unit of producing q units per cycle in crisp strategy,
$\tilde{\pi}_1(q, \rho)$	The net profit per unit per cycle in fuzzy decision space,
$\tilde{\pi}(q, \rho)$	The average profit per unit per cycle in fuzzy decision space,
\tilde{h}	Fuzzy holding cost per unit,
$\tilde{K} \times (q^{\gamma-1})$	Fuzzy setup cost per cycle.

MATHEMATICAL MODEL

Denote $\varphi(t)$ as the on-hand inventory level at time t. During a change in time from point t to $t+dt$, where $t + dt > t$, the on-hand inventory drops from $\varphi(t)$ to $\varphi(t+dt)$.

Then $\varphi(t+dt)$ is given as:

$$\varphi(t+dt) = \varphi(t) - r dt - \alpha \varphi(t) dt$$

can be re-written as:

$$\frac{\varphi(t+dt) - \varphi(t)}{dt} = -r - \alpha \varphi(t)$$

and $dt \rightarrow 0$, equation $\frac{\varphi(t+dt) - \varphi(t)}{dt}$ reduces to: $\frac{d\varphi(t)}{dt} + \alpha \varphi(t) + r = 0$

It is a differential equation, solution is

$$\varphi(t) = \frac{-r}{\alpha} + \left(q + \frac{r}{\alpha} \right) \times e^{-\alpha t}$$

Where q is the order quantity which is instantaneously replenished at the beginning of each cycle of length t_c units of time. The stock is replenished by q units each time these units are totally depleted as a result of outside demand and deterioration. Behavior of the inventory level for the above model is illustrated in Fig. 1. The cycle length, t_c , is determined by first substituting t_c into equation $\varphi(t)$ and then setting it equal to zero to get:

$$t_c = \frac{1}{\alpha} \ln \left(\frac{\alpha q + r}{r} \right)$$

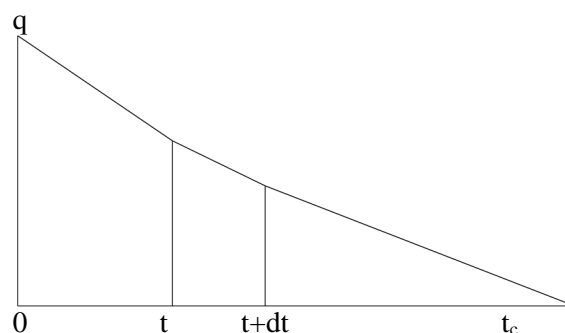


Fig. 1. Behavior of the Inventory over a Cycle for a Deteriorating Item

Rys. 1. Poziom zapasu badanego artykułu w czasie cyklu

Equation $\varphi(t)$ and t_c are used to develop the mathematical model. It is worthy to mention that as α approaches to zero, t_c approaches to $\frac{q}{r}$. Then the total number of units lost per cycle, L , is given as:

$$L = r \left[\frac{q}{r} - \frac{1}{\alpha} \ln \left(\frac{\alpha q + r}{r} \right) \right]$$

The total cost per cycle, $TC(q)$, is the sum of the procurement cost per cycle, $K+cq$ and the holding cost per cycle, $HC(q)$. $HC(q)$ is obtained from equation $\varphi(t)$ as:

$$\begin{aligned} HC(q) &= \int_0^{t_c} h\varphi(t)dt \\ &= h \int_0^{\frac{1}{\alpha} \ln \left(\frac{\alpha q + r}{r} \right)} \left[-\frac{r}{\alpha} + \left(q + \frac{r}{\alpha} \right) \times e^{-\alpha t} \right] dt \\ &= h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln \left(\frac{\alpha q + r}{r} \right) \right] \end{aligned}$$

$$TC(q) = K + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln \left(\frac{\alpha q + r}{r} \right) \right]$$

The total cost per unit of time, $TCU(q)$, is given by dividing equation $TC(q)$ by t_c to give:

$$\begin{aligned} TCU(q) &= \left[K + cq + h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln \left(\frac{\alpha q + r}{r} \right) \right] \right] \times \left[\frac{1}{\alpha} \ln \left(\frac{\alpha q + r}{r} \right) \right]^{-1} \\ &= \frac{K\alpha + (c\alpha + h)q}{\ln \left(1 + \frac{\alpha q}{r} \right)} - \frac{hr}{\alpha} \end{aligned}$$

As α approaches zero in equation $TCU(q)$ reduces to $TCU(q) = \frac{Kr}{q} + cr + \frac{hq}{2}$, whose solution is given by the traditional EOQ formula, $q^* = \sqrt{\frac{2Kr}{h}}$. The total profit per cycle is $\pi_1(q)$.

$$\begin{aligned} \pi_1(q) &= (q-L) \times P_s - TC(q) \\ &= (q-L) \times P_s - K - cq - h \times \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln \left(\frac{\alpha q + r}{r} \right) \right] \end{aligned}$$

Where L , the number of units lost per cycle due to deterioration, and $TC(q)$ the total cost per cycle, are calculated from equations L and $TC(q)$, respectively. The average profit $\pi(q)$

per unit time is obtained by dividing t_c in $\pi_1(q)$. Hence the profit maximization problem is:

$$\begin{aligned} &\text{Maximize } \pi_1(q) \\ &\forall q \geq 0. \end{aligned}$$

$$\pi_1(q) = F_1(q) + F_2(q)h + F_3(q)K$$

where,

$$F_1(q) = ((q-L) \times P_s) - cq, \quad F_2(q) = - \left[\frac{q}{\alpha} - \frac{r}{\alpha^2} \ln \left(\frac{\alpha q + r}{r} \right) \right], \quad \text{and } F_3(q, \rho) = -1$$

FUZZY MATHEMATICAL MODEL

The holding cost and ordering cost are replaced by fuzzy numbers \tilde{h} and \tilde{K} respectively. By expressing \tilde{h} and \tilde{K} as the normal triangular fuzzy numbers (h_1, h_0, h_2) and (K_1, K_0, K_2) , where, $h_1 = h - \Delta_1$, $h_0 = h$, $h_2 = h +$

Δ_2 , $K_1 = K - \Delta_3$, $K_0 = K$, $K_2 = K + \Delta_4$ such that $0 < \Delta_1 < h$, $0 < \Delta_2$, $0 < \Delta_3 < K$, $0 < \Delta_4$, $\Delta_1, \Delta_2, \Delta_3$ and Δ_4 are determined by the decision maker based on the uncertainty of the problem.

The membership function of fuzzy holding cost and fuzzy ordering cost are considered as:

$$\mu_{\tilde{h}}(h) = \begin{cases} \frac{h-h_1}{h_0-h_1}, & h_1 \leq h \leq h_0 \\ \frac{h_2-h}{h_2-h_0}, & h_0 \leq h \leq h_2 \\ 0, & \text{otherwise} \end{cases}$$

$$\mu_{\tilde{K}}(K) = \begin{cases} \frac{K-K_1}{K_0-K_1}, & K_1 \leq K \leq K_0 \\ \frac{K_2-K}{K_2-K_0}, & K_0 \leq K \leq K_2 \\ 0, & \text{otherwise} \end{cases}$$

Then the centroid for \tilde{h} and \tilde{K} are given by

$$M_{\tilde{h}} = \frac{h_1 + h_o + h_2}{3} = h + \frac{\Delta_2 - \Delta_1}{3} \quad \text{and}$$

For fixed values of q and ρ , let $\pi_1(h, K) = F_1(q) + F_2(q)h + F_3(q)K = y$

$$M_{\tilde{k}} = \frac{K_1 + K_o + K_2}{3} = K + \frac{\Delta_4 - \Delta_3}{3}$$

Let $h = \frac{y - F_1 - F_3K}{F_2}$, $\frac{\Delta_2 - \Delta_1}{3} = \psi_1$ and

$$\frac{\Delta_4 - \Delta_3}{3} = \psi_2$$

respectively.

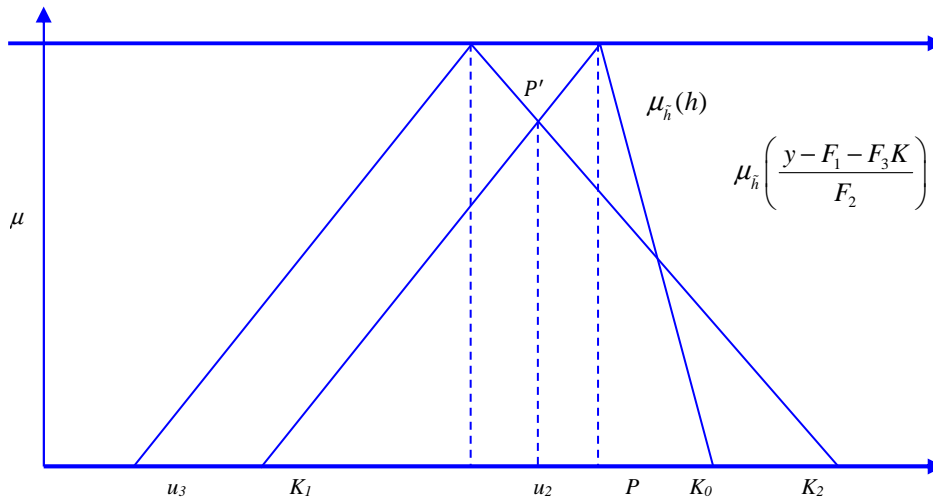


Fig. 2. Defuzzification by using Centroid Method
 Rys. 2. Dezufikacja przy zastosowaniu metody centroidowej

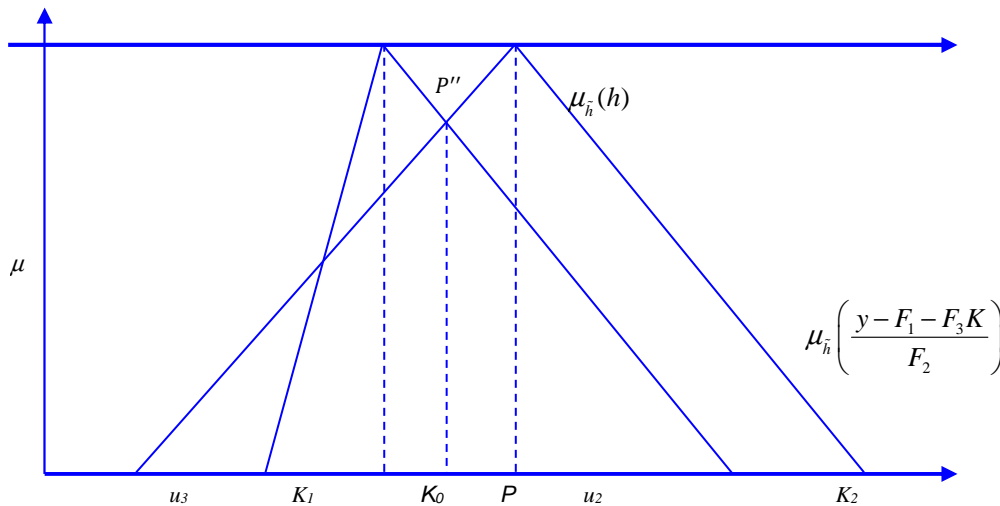


Fig. 3. Defuzzification by using Centroid Method
 Rys. 3. Dezufikacja przy zastosowaniu metody centroidowej

By extension principle the membership function of the fuzzy profit function is given by

$$\begin{aligned} \mu_{\pi(\tilde{h}, \tilde{k})}^{(y)} &= \text{Sup}_{(h,k) \in \pi_1^{-1}(y)} \left\{ \mu_{\tilde{h}}(h) \wedge \mu_{\tilde{k}}(K) \right\} \\ &= \text{Sup}_{k_1 \leq k \leq k_2} \left\{ \mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right) \wedge \mu_{\tilde{k}}(K) \right\} \end{aligned}$$

Now,

$$\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right) = \begin{cases} \frac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1)} & , u_2 \leq K \leq u_1 \\ \frac{F_1 + F_2 h_2 + F_3 K - y}{F_2 (h_2 - h_0)} & , u_3 \leq K \leq u_2 \\ 0 & , \text{otherwise} \end{cases}$$

where, $u_1 = \frac{y - F_1 - F_2 h_1}{F_3}$, $u_2 = \frac{y - F_1 - F_2 h_0}{F_3}$ and $u_3 = \frac{y - F_1 - F_2 h_2}{F_3}$

Fig. 2 exhibits the graph of $\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right)$ and $\mu_{\tilde{h}}(h)$ when $u_2 \leq K$ and $K \leq u_1$ then

$y \leq F_1 + F_2 h_0 + F_3 K_0$ and $y \geq F_1 + F_2 h_1 + F_3 K_1$. It is clear that for every

$y \in [F_1 + F_2 h_1 + F_3 K_1, F_1 + F_2 h_0 + F_3 K_0]$, $\mu_y(y) = PP'$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right)$

the value of PP' may be found by solving the following equation:

$$\frac{K - K_1}{K_0 - K_1} = \frac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1)} \text{ or } K = \frac{(y - F_1 - F_2 h_1)(K_0 - K_1) + F_2 K_1 (h_0 - h_1)}{F_2 (h_0 - h_1) + F_3 (K_0 - K_1)}$$

Therefore, $PP' = \frac{K - K_1}{K_0 - K_1} = \frac{y - F_1 - F_2 h_1 - F_3 K}{F_2 (h_0 - h_1) + F_3 (K_0 - K_1)} = \mu_1(y)$, (say).

Fig. 3 exhibits the graph of $\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right)$ and $\mu_{\tilde{h}}(h)$ when $u_3 \leq K$ and $K \leq u_2$ then

$y \leq F_1 + F_2 h_2 + F_3 K_2$ and $y \geq F_1 + F_2 h_0 + F_3 K_0$.

It is evident that for every $y \in [F_1 + F_2 h_0 + F_3 K_0, F_1 + F_2 h_2 + F_3 K_2]$, $\mu_y(y) = PP'$. From the $\mu_{\tilde{h}}(h)$ and $\mu_{\tilde{h}} \left(\frac{y - F_1 - F_3 K}{F_2} \right)$, the value of PP' may be found by solving the following equation:

$$\frac{K_2 - K}{K_2 - K_0} = \frac{F_1 + F_2 h_2 + F_3 K - y}{F_2 (h_2 - h_0)} \text{ or, } K = \frac{F_2 K_2 (h_2 - h_0) - (F_1 + F_2 h_2 - y)(K_2 - K_0)}{F_2 (h_2 - h_0) + F_3 (K_2 - K_0)}$$

Therefore, $PP' = \frac{K_2 - K}{K_2 - K_0} = \frac{F_1 + F_2 h_2 + F_3 K_2 - y}{F_2 (h_2 - h_0) + F_3 (K_2 - K_0)} = \mu_2(y)$, (say).

Thus the membership function for fuzzy total profit is given by

$$\mu_{\pi_1(\tilde{h}, \tilde{K})}(y) = \begin{cases} \mu_1(y); & F_1 + F_2 h_1 + F_3 K_1 \leq y \leq F_1 + F_2 h_0 + F_3 K_0 \\ \mu_2(y); & F_1 + F_2 h_0 + F_3 K_0 \leq y \leq F_1 + F_2 h_2 + F_3 K_2 \\ 0; & \text{otherwise} \end{cases}$$

$$\text{Now, let } P_1 = \int_{-\infty}^{\infty} \mu_{\pi_1(\tilde{h}, \tilde{K})}(y) dy \text{ and } R_1 = \int_{-\infty}^{\infty} y \mu_{\pi_1(\tilde{h}, \tilde{K})}(y) dy$$

$$\text{Hence, the centroid for fuzzy total profit is given by } \tilde{\pi}_1(q) = M_{TP}(q) = \frac{R_1}{P_1}$$

$$= F_1(q) + F_2(q)h + F_3(q)K + \psi_1 F_2(q) + \psi_2 F_2(q)$$

$$\tilde{\pi}_1(q) = M_{TP}(q) = F_1 + (h + \psi_1)F_2 + (K + \psi_2)F_3$$

where, $F_1(q)$, $F_2(q)$ and $F_3(q)$ are given by the equations.

Hence the profit maximization problem is

$$\text{Maximize } \tilde{\pi}_1(q) = M_{TP}(q) \quad \forall q \geq 0.$$

OPTIMIZATION

The optimal ordering quantity q per cycle can be determined by differentiating equation $\tilde{\pi}_1(q)$ with respect to q , then setting these to zero.

In order to show the uniqueness of the solution in, it is sufficient to show that the net profit function throughout the cycle is concave in terms of ordering quantity q . The second order derivatives of equation $\tilde{\pi}_1(q)$ with respect to q are strictly negative. Consider the following proposition.

Proposition 1 The net profit $\tilde{\pi}_1(q)$ per cycle is concave in q .

Conditions for optimal q

$$\frac{d\tilde{\pi}_1(q)}{dq} = \frac{r}{\alpha q + r} \left(P_s + \frac{(h + \psi_1)}{\alpha} \right) - \left(c + \frac{(h + \psi_1)}{\alpha} \right) = 0$$

The second order derivative of the net profit per cycle with respect to q can be expressed as:

$$\frac{d^2\tilde{\pi}_1(q)}{dq^2} = - \frac{r}{(\alpha q + r)^2} (P_s \alpha + (h + \psi_1))$$

Since, $r > 0$ and $P_s \alpha + (h + \psi_1) > 0$ equation $\frac{d^2\tilde{\pi}_1(q)}{dq^2}$ is negative.

Proposition 1 shows that the second order derivative of equation $\tilde{\pi}_1(q)$ with respect to q are strictly negative.

The objective is to determine the optimal values of q to maximize the unit profit function of equation $\tilde{\pi}_1(q)$. It is very difficult to derive the optimal values of q , hence unit profit function. There are several methods to cope with constraints optimization problem numerically. But here LINGO 13.0 software is used to derive the optimal values of the decision variables.

NUMERICAL EXAMPLE

Consider an inventory situation where K is Rs. 200 per order, h is Rs. 5 per unit per unit of time, r is 1200 units per unit of time, c is Rs. 100 per unit, the selling price per unit P_s is Rs. 125 $\Delta_1 = 0.002$, $\Delta_2 = 0.02$, $\Delta_3 = 0.002$ and $\Delta_4 = 0.2$ and α is 5%. Fig. 2 shows the relationship between the order quantity q and units lost per cycle due to deterioration L and Fig. 3 represents the three dimensional

mesh plot of units lost per cycle due to deterioration L, order quantity q and net profit per cycle $\tilde{\pi}_1(q)$. Fig. 4 is the sensitivity plotting of units lost per cycle L, order quantity q and net profit per cycle $\tilde{\pi}_1(q)$.

The optimal solution that maximizes $\tilde{\pi}_1(q)$ and q^{**} and q^* are determined by using LINGO 13.0 version software and the results are tabulated in Table 2.

Table 2. Optimal values of the proposed model
 Tabela 2. Wartości optymalne proponowanego modelu

Model	Iteration	t^*	L^*	Q	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
Fuzzy	90	2.354328	173.0073	$q^{**} = 2998.201$	35807.55	15209.25
Crisp	87	2.355661	173.2071	$q^* = 3000$	35828.39	15209.49
	-	0.05662	0.1155	0.06	0.0582	0.00158
Crisp	41	0.2581989	-	$q^* = 309.8387$	28450.81	7345.96784 6
% Change	-	89.021	-	89.666	20.5452	51.701

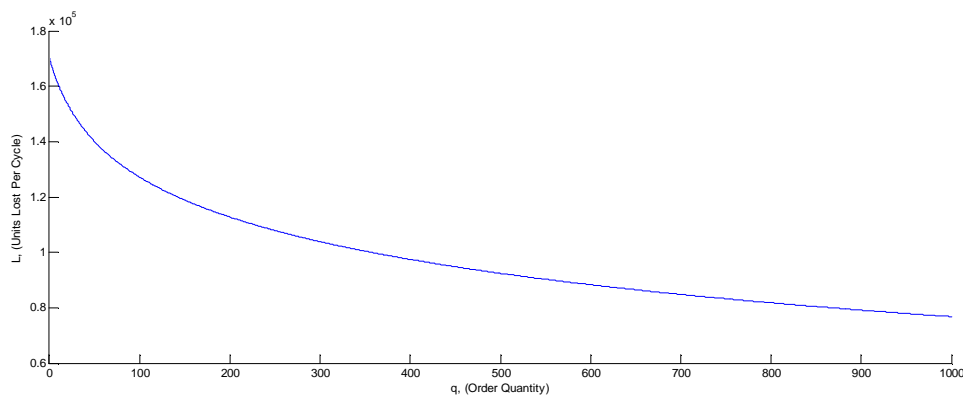


Fig. 4. Two Dimensional Plot of Order Quantity q and Units Lost per Cycle L
 Rys. 4. Dwuwymiarowy wykres zależności wielkości zamówienia q i strat w cyklu L

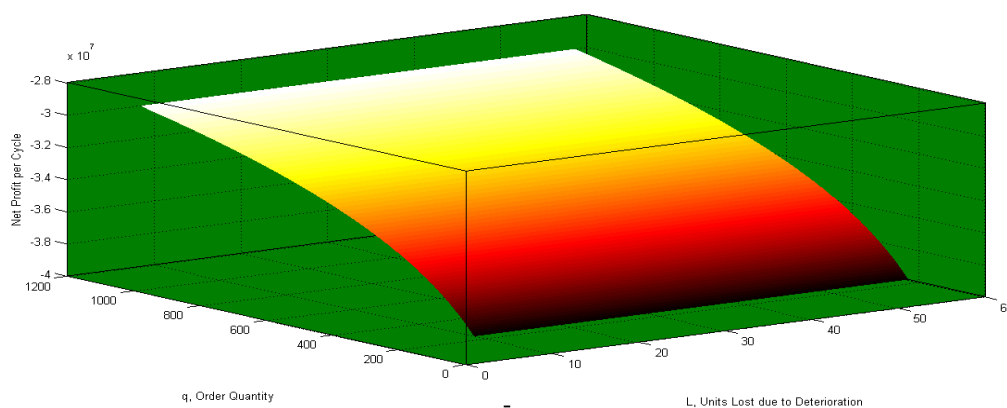


Fig. 5. Three Dimensional Mesh Plot of Units Lost per Cycle L, Order Quantity q and Fuzzy Net Profit per Cycle $\tilde{\pi}_1(q)$
 Rys. 5. Trójwymiarowy wykres zależności wielkości strat w cyklu L, wielkości zamówienia q oraz zysku netto w cyklu $\tilde{\pi}_1(q)$

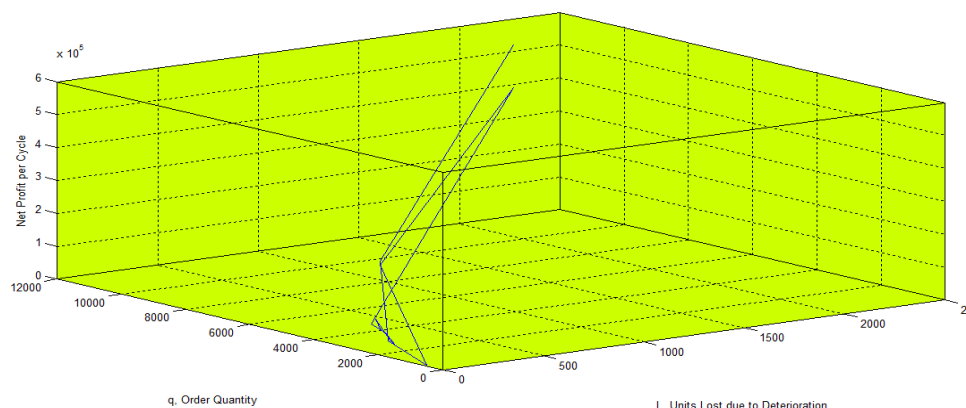


Fig. 6. Sensitivity Plotting of Units Lost due to Deterioration L, Order Quantity q and Fuzzy Net Profit per Cycle $\tilde{\pi}_1(q)$

Rys. 6. Wykresu wrażliwości strat spowodowanych psuciem się L, wielkości zamówienia q oraz zysku netto w cyklu $\tilde{\pi}_1(q)$

SENSITIVITY ANALYSIS

It is interesting to investigate the influence of α on retailer behaviour. The computational results shown in Table 3 indicates the following managerial phenomena: when the percentage of on-hand inventory that is lost

due to deterioration α increases, the replenishment cycle length, the optimal replenishment quantity and optimal net profit per unit per cycle decrease respectively. The optimal total number of units lost per cycle and optimal average profit per unit per cycle are fluctuated with increase in the percentage value of the major parameter α .

Table 3. Sensitivity analysis of α
 Tabela 3. Analiza wrażliwości α

$\alpha\%$	Iteration	t^*	L^*	q^{**}	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
.01	75	4.078203	101.161	4995.005	61388.77	15052.81
.10	66	1.540936	150.0776	1999.2	23507.35	15255.25
.20	73	0.9114078	106.0227	1199.712	13886.15	15235.94
.30	73	0.6470859	80.49288	856.9959	9820.065	15175.83
.40	64	0.5016161	64.63842	666.5778	7575.626	15102.44

It is interesting to investigate the influence of the major parameters \tilde{K} , \tilde{h} , r, c and P_s on retailer's behavior. The computational results shown in Table 4 indicate the following managerial phenomena:

- t_c the replenishment cycle length, q the optimal replenishment quantity, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are insensitive to the parameter \tilde{K} .
- t_c the replenishment cycle length, q the optimal replenishment quantity and $\tilde{\pi}_1$ the optimal net profit per unit per cycle are sensitive to the parameter h but $\tilde{\pi}$

the optimal average profit per unit per cycle is insensitive to the parameter \tilde{h} .

- q the optimal replenishment quantity, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to the parameter r but t_c the replenishment cycle length is insensitive to the parameter r.
- t_c the replenishment cycle length, q the optimal replenishment quantity, $\tilde{\pi}_1$ the optimal net profit per unit per cycle and $\tilde{\pi}$ the optimal average profit per unit per cycle are sensitive to both the parameters c and P_s .

Table 4. Sensitivity analyses of the parameters \tilde{K} , \tilde{h} , r, c and P_s
Tabela 4. Analiza wrażliwości parametrów \tilde{K} , \tilde{h} , r, c oraz P_s

Parameter	Value	Iteration	t^*	L^*	q^*	$\tilde{\pi}_1(q)$	$\tilde{\pi}(q)$
\tilde{K}	150	80	2.354328	173.0073	2998.201	35857.55	15230.48
	155	82	2.354328	173.0073	2998.201	35852.55	15228.36
	210	87	2.354328	173.0073	2998.201	35797.55	15205.00
\tilde{h}	3	89	2.901615	265.2521	3747.19	44376.20	15293.62
	4	88	2.599	211.7875	3331.113	39635.89	15247.87
	6	75	2.151501	143.9851	2725.786	32650.95	15175.90
r	1100	74	2.354328	158.5900	2748.351	32806.92	13934.73
	1300	73	2.354328	187.4246	3248.051	38808.19	16483.76
	1400	73	2.354328	201.8419	3497.901	41808.82	17758.28
c	50	65	8.103972	2265.641	11990.41	389039.4	48006.02
	80	72	4.460206	643.7555	5996.003	124697.8	27957.86
	120	74	0.4492148	6.099399	545.1572	1152.623	2565.862
P_s	120	74	1.905113	112.4249	2398.561	23024.18	12085.47
	150	75	4.460472	643.8354	5996.402	138583.9	31069.34
	200	86	8.105304	2266.439	11992.80	518876.6	64016.92

CONCLUSION

In this model, a modified FEOQ model is introduced which investigates the optimal order quantity assumes that a percentage of the on-hand inventory is wasted due to deterioration as a characteristic feature and the inventory conditions govern the item stocked in fuzzy space. This model provides a useful property for finding the optimal profit and ordering quantity with units lost due to deterioration. A new mathematical model is developed and compared to the traditional EOQ model numerically. The economic order quantity, q^{**} and the net profit for the modified model, were found to be less than that of the traditional, q^* , i.e. $q^{**} < q^*$ and the net profit respectively. But the modified average profit per unit per cycle is less than that of the traditional profit per unit per cycle. Finally, wasting the percentage of on-hand inventory due to deterioration effect was demonstrated numerically to have an adverse effect on the average profit per unit per cycle. Hence the utilization of units lost due to deterioration makes the scope of the application broader.

Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analysis with respect to the major parameters. The model in this study is a general framework that considers wasting/ none wasting the percentage of on-hand inventory due to deterioration simultaneously.

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OPTYMIZACJA MODELU ZMIENNEJ WIELKOŚCI EKONOMICZNEJ ZAMÓWIENIA (FEOQ) DLA PRODUKTÓW PODLEGAJĄCYCH PSUCIU SIĘ ORAZ UWZGLĘDNIĄCY STRATY TOWARU

STRESZCZENIE. Wstęp: Model ten prezentuje wpływ psucia się produktów w systemie ciągłego uzupełniania dla skończonego horyzontu planowania. Tradycyjnie zostały wyliczone w tym modelowym systemie koszty magazynowania na jednostkę artykułu, na jednostkę czasu oraz koszt zamówienia na zamówienie. Te nieprecyzyjne parametry zdefiniowane w określonych przedziałach osi dla rzeczywistych wartości i fizycznych charakterystyk magazynowanych produktów określają zasady zarządzania zapasami stosowanymi w danym systemie produkcyjnym.

Metody: Zastosowano zmodyfikowany model zmiennej ekonomicznej wielkości zamówienia (FEOQ), zakładający, że pewien odsetek zapasów jest tracony w wyniku psucia się wyrobów. Model ten został tak zmieniony aby uzyskać optymalną wielkość zamówienia przy maksymalizacji zysku netto. W analizie teoretycznej, koniecznym i wystarczającym warunkiem istnienia i unikalności optymalnego rozwiązania jest znalezienie przegięcia funkcji zysku netto. Opracowano algorytm obliczeniowy w celu znalezienia optymalnego rozwiązania przy zastosowaniu oprogramowania LINGO 13.0.

Wyniki i wnioski: Wyniki analizy matematycznej umożliwiają osobom podejmującym decyzję określenie wielkości wpływu psucia się zapasów na optymalizację zysku netto detalisty. Przeprowadzono również analizę wrażliwości dla optymalnego rozwiązania uwzględniając istotne parametry. Przedstawiono dowody, że podejmowanie decyzji na zasadzie prawdopodobieństwa jest istotniejsze w procesie maksymalizacji zysku od decyzji typu Crisp.

Słowa kluczowe: optymalizacja, FEOQ, psucie się, straty jednostkowe

OPTIMALISIERUNG DES MODELLS DER VARIABLEN WIRTSCHAFTLICHEN GRÖ?E EINER BESTELLUNG (FEOQ) FÜR VERDERBANFÄLLIGE PRODUKTE UNTER BERÜCKSICHTIGUNG VON WARENVERUSTEN

ZUSAMMENFASSUNG. Einleitung: Das Modell präsentiert die Einflussnahme des Produktenverderbs innerhalb des Systems einer ständigen Vervollständigung der Vorräte für den finiten Zeitplan-Horizont. In diesem Modellsystem wurden die Lagerkosten traditionsgemäß auf die Artikeleinheit, ferner auf die Zeiteinheit und die Auftragskosten auf die Bestellungseinheit ausgerechnet. Diese unpräzisen Parameter, die innerhalb der bestimmten Intervalle der Achse für die Echtwerte sowie für physische Charakteristika von gelagerten Produkten definiert wurden, bestimmen die im jeweiligen Produktionssystem angewendeten Prinzipien der Bestandsführung.

Methoden: Es wurde ein modifiziertes Modell der variablen wirtschaftlichen Größe einer Bestellung (FEOQ), in dem angenommen wird, dass eine gewisse Losgröße von Vorräten infolge des Verderbs von Produkten verloren geht, angewendet. Das Modell wurde so verändert, dass man die optimale Losgröße der Bestellung bei Maximalisierung des Netto-Gewinns erzielen konnte. In der theoretischen Analyse besteht die unentbehrliche und genügende Voraussetzung für das Bestehen und die Einmaligkeit der meist optimalen Lösung in der Ermittlung der Verbeugung von Funktion des Netto-Gewinns. Für die Ermittlung einer optimalen Lösung wurde ein Berechnungsalgorithmus unter Anwendung der Software LINGO 13.0 ausgearbeitet.

Ergebnisse und Fazit: Die Ergebnisse der mathematischen Analyse befähigen die Entscheidungsträger zur Bestimmung der Einflussnahme des Verderbs von Vorräten auf die Optimierung des Netto-Gewinns beim Einzelhändler. Darüber hinaus wurde auch eine Analyse der Empfindlichkeit für die optimale Lösung unter Berücksichtigung von relevanten Parametern durchgeführt. Man stellte dabei die Beweise dafür dar, dass das auf das Wahrscheinlichkeitsprinzip gestützte Entscheidungstreffen im Prozess der Maximalisierung des Gewinns mehr brauchbar ist als die Crisp-Entscheidungen selbst.

Codewörter: Optimierung, FEOQ, Verderb, einheitliche Verluste

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