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AN EOQ MODEL FOR WEIBULL DETERIORATING ITEM WITH RAMP TYPE DEMAND AND SALVAGE VALUE UNDER TRADE CREDIT SYSTEM

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ABSTRACT. Background: In the present competitive business scenario researchers have developed various inventory models for deteriorating items considering various practical situations for better inventory control. Permissible delay in payments with various demands and deteriorations is considerably a new concept introduced in developing various inventory models. These models are very useful for both the consumers and the manufacturer.

Methods: In the present work an inventory model has been developed for a three parameter Weibull deteriorating item with ramp type demand and salvage value under trade credit system. Here we have considered a single item for developing the model.

Results and conclusion: Optimal order quantity, optimal cycle time and total variable cost during a cycle have been derived for the proposed inventory model. The results obtained in this paper have been illustrated with the help of numerical examples and sensitivity analysis.

Key words: EOQ, Weibull deterioration, ramp type demand, salvage value, trade credit.

INTRODUCTION

In the traditional EOQ inventory model, the purchaser pays for his items as soon as the items are received. However in real competitive business world, the supplier may allow a credit period to encourage the customers to buy in bulk. It is also named as permissible delay in payments. Delay in payments to the supplier is an alternative way of price discount. Hence paying later in directly, reduces the purchase cost which attracts the customers to enhance their ordering quantity. Generally retailers are encouraged towards bulk purchasing due to the trade credit given by suppliers. No interest is charged if the account is settled within the credit period. However if the payment is not settled during the period, then interest is charged. Salvage value is the estimated resale value of an asset at the end of its useful life. Researchers are being engaged in developing various inventory models taking various practical situations.

Abad et.al [2003] developed an inventory model of joint approach for setting unit price and the length of the credit period for a seller when end demand is price sensitive. Similarly Ordering policies of deteriorating items under permissible delay in payments studied by Aggarwal [1995]. Chang C.T, [2004] invented an EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. Chung, K. J. [2009] developed an ordering policy with allowable shortage and permissible delay in payments. Huang, Y.F, [2003] studied an inventory model for optimal retailer's ordering policies in the EOQ model under trade credit financing. Hwang, H. and

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Shinn, S. W., [1997] proposed a model for retailer's pricing and lot-sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Jamal, A. M. M., Sarker, B. R., and Wang, S., [1997] studied an ordering policy for deteriorating items with allowable shortages and permissible delay in payment. Meher M.K., Panda G.Ch., Sahu S.K., [2012], have studied an inventory model with Weibull deterioration rate under the delay in payment in demand declining market. Shah, Nita H. and Raykundaliya, Nidhi, [2010], proposed a model of retailers pricing and ordering strategy for Weibull distribution deterioration under trade credit in declining market. Tripathy C.K. and Pradhan L.M., [2010], [2011] and [2012] have developed some EOQ and EPQ models considering various aspects like trade credit, permissible delay in payments, salvage value and price discount under different situations. Tripathy C. K., and Mishra U. [2011] studied an EOQ model with time dependent Weibull deterioration and ramp type demand for constant deterioration.

In the present paper an economic ordered quantity model has been developed considering three parameter Weibull deterioration where salvage value is considered but shortages are not allowed. Here the demand is assumed to be ramp type demand and holding cost is constant. In section 2 assumptions and notations required for the development of the model have been given. The optimum cycle time, optimal ordered quantity and total average cost of the model have been derived in the Section 3. Illustrative numerical examples, sensitivity analysis and conclusion have been given in section 4, 5 and 6 respectively..

BASIC ASSUMPTIONS AND NOTATIONS

The following are the assumptions required for developing the model:

1. The model deals with a single item.
2. Replenishment rate is infinite.
3. Lead time is zero and shortages are not allowed.
4. Ramp-type demand rate

   $f(t) = D_0\left[t-(t-\mu)H(t-\mu)\right]$, $D_0 > 0$,

   Here $H(t-\mu)$ is a Heaviside's unit function which is defined as follows:

   $$H(t-\mu) = \begin{cases} 1 & t > \mu \\ 0 & t \le \mu \end{cases}$$
5. Unit cost of generated sales revenue is deposited in an interest bearing account at the time of fixed period μ .To meet the day-to-day expenses of the system, the difference between sales price and unit cost is retained. At the end of the credit period the account is settled and interest charges are payable on the account in stock.
6. Deterioration rate is a three parameter Weibull function.
7. The salvage value aC (where $0 \le a < 1$) is associated to deteriorated units during the cycle time.

The notations that are employed here:

A: ordering cost per order.

a: constant or a real number, where $0 \le a < 1$.

C: Purchase cost per unit.

P: Unit selling price P>C

h: Inventory holding cost per unit per unit time excluding interest charges.

θ: Weibull three parameter deterioration rate (unit/unit time), $\theta = \alpha\beta(t-\gamma)^{\beta-1}$, where $0 < \alpha < 1$, $\beta > 0$, and $0 < \gamma$, where α is called scale parameter, β is called shape parameter and γ is called the location parameter.

T: The length of cycle time.

D(T) It is the Number of units that deteriorate during one cycle.

I_c Interest charged per unit in stock per annum by the supplier to the retailer.

I_e Interest earned per unit per annum. ($I_c > I_e$).

μ: Permissible delay period for settling accounts in time units.

$\phi_1(T)$: Total average cost per time unit when $\mu < T$.

$\phi_2(T)$: Total average cost per time unit when $\mu > T$.

MATHEMATICAL MODEL

For developing mathematical model we consider two cases as follows.

Case 1 When μ < T

Let $I(t)$ denote the on hand inventory of the system at any time t $(0 \le t \le T)$. Let the initial inventory be Q. Depletion due to demand and deterioration occur simultaneously. In this case the permissible delay period for settling accounts is less than the total cycle time of the system. The differential equation that describes the instantaneous state of $I(t)$ in the interval $0 \le t \le T$ is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D_0 t \quad , \quad 0 \le t \le \mu \qquad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -D_0 \mu \quad , \quad \mu \le t \le T \qquad (2)$$

Where $\theta = \alpha\beta(t-\gamma)^{\beta-1}$, $0 < \alpha < 1$, $\beta > 0$ and $0 < \gamma$ called the scale, shape and location parameter respectively.
Here the boundary conditions are

$$I(0) = Q \text{ and } I(T) = 0 \qquad (3)$$

Where Q is the inventory order quantity.

Equation (1) is a linear differential equation. Its integrating factor is given by

$$= e^{\int \alpha\beta(t-\gamma)^{\beta-1} dt} = e^{\alpha(t-\gamma)^{\beta}} .$$

Hence the solution of equation (1) can be written as

$$I(t)\, e^{\alpha(t-\gamma)^{\beta}} = \int -D_0\, t\, e^{\alpha(t-\gamma)^{\beta}} dt + c \ ,$$

where c is the constant of integration.

Since α is very small, neglecting the terms involving second and higher powers α of from the series expansion of the exponential function and then integrating we get

$$I(t)\, e^{\alpha(t-\gamma)^{\beta}} = \int -D_0 t\left[1 + \alpha(t-\gamma)^{\beta}\right]dt + c$$

$$\Rightarrow I(t) e^{\alpha(t-\gamma)^{\beta}} = -D_0\left\{\frac{t^2}{2} + \frac{\alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right\} + c$$

Using the given boundary condition $I(0) = Q$ in the above we get the required solution of equation (1) as

$$I(t) e^{\alpha(t-\gamma)^{\beta}} = -D_0\left\{\frac{t^2}{2} + \frac{\alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right\} + Q + Q\alpha(-\gamma)^{\beta} - \frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}$$

$$\Rightarrow I(t) = e^{-\alpha(t-\gamma)^{\beta}}\left[-D_0\left\{\frac{t^2}{2} + \frac{\alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right\} + Q + Q\alpha(-\gamma)^{\beta} - \frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

Since $0 < \alpha <$,1so taking the first two terms from the series expansion of the exponential function and multiplying we get,

$$\Rightarrow I(t) = \left[1 - \alpha(t-\gamma)^{\beta}\right]\left[-D_0\left\{\frac{t^2}{2} + \frac{\alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} - \frac{\alpha(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right\} + Q + Q\alpha(-\gamma)^{\beta} - \frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

Again neglecting the terms involving second and higher powers of α the above equation can be written as

$$I(t) = \frac{-D_0 t^2}{2} - \frac{D_0\alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0\alpha(t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + Q + Q\alpha(-\gamma)^{\beta}$$

$$- \frac{D_0\alpha\,(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 t^2\alpha(t-\gamma)^{\beta}}{2} - Q\alpha(t-\gamma)^{\beta}, \quad 0 \le t \le \mu \qquad (4)$$

Similarly using the condition $I(T)=0$ and solving the equation (2) we can find

$$I(t)=D_0\mu(T-t)+\frac{D_0\mu\alpha\,(T-\gamma)^{\beta+1}}{(\beta+1)}-\frac{D_0\mu\alpha\,(t-\gamma)^{\beta+1}}{(\beta+1)}-D_0\mu\alpha(t-\gamma)^{\beta}(T-t),\quad \mu\le t\le T \qquad (5)$$

Similarly using the condition $I(T)=0$ and solving the equation (2) we can find

$$I(t)=D_0\mu(T-t)+\frac{D_0\mu\alpha\,(T-\gamma)^{\beta+1}}{(\beta+1)}-\frac{D_0\mu\alpha\,(t-\gamma)^{\beta+1}}{(\beta+1)}-D_0\mu\alpha(t-\gamma)^{\beta}(T-t),\quad \mu\le t\le T \qquad (5)$$

Substituting $t=\mu$ in equation (4) and (5) and then equating both the equation we get

$$\Rightarrow\frac{-D_0\mu^2}{2}-\frac{D_0\alpha\,\mu(\mu-\gamma)^{\beta+1}}{(\beta+1)}+\frac{D_0\alpha(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+Q+Q\alpha(-\gamma)^{\beta}$$
$$-\frac{D_0\alpha\,(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{D_0\mu^2\alpha(\mu-\gamma)^{\beta}}{2}-Q\alpha(\mu-\gamma)^{\beta}=D_0\mu(T-\mu)+\frac{D_0\mu\alpha\,(T-\gamma)^{\beta+1}}{(\beta+1)}$$
$$-\frac{D_0\mu\alpha\,(\mu-\gamma)^{\beta+1}}{(\beta+1)}-D_0\mu\alpha(\mu-\gamma)^{\beta}(T-\mu)$$

$$\Rightarrow Q\left[1+\alpha(-\gamma)^{\beta}-\alpha(\mu-\gamma)^{\beta}\right]=D_0\mu T-\frac{D_0\mu^2}{2}+\frac{D_0\mu^2\alpha\,(\mu-\gamma)^{\beta}}{2}+\frac{D_0\alpha\,\mu(T-\gamma)^{\beta+1}}{(\beta+1)}$$
$$-D_0\mu T\alpha(\mu-\gamma)^{\beta}-\frac{D_0\alpha(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}$$
$$\Rightarrow Q=\left[1+\left\{\alpha(-\gamma)^{\beta}-\alpha(\mu-\gamma)^{\beta}\right\}\right]^{-1}\left[D_0\mu T-\frac{D_0\mu^2}{2}+\frac{D_0\mu^2\alpha\,(\mu-\gamma)^{\beta}}{2}+\frac{D_0\alpha\,\mu(T-\gamma)^{\beta+1}}{(\beta+1)}\right.$$
$$\left.-D_0\mu T\alpha(\mu-\gamma)^{\beta}-\frac{D_0\alpha(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

Now using the series expansion formula $(1+x)^{-1}=1-x+x^2-x^3+.....$ and neglecting the terms involving second and higher powers of α (i.e. taking the first two terms of the series) the above equation can be written as

$$\Rightarrow Q=\left[1-\alpha(-\gamma)^{\beta}+\alpha(\mu-\gamma)^{\beta}\right]\left[D_0\mu T-\frac{D_0\mu^2}{2}+\frac{D_0\mu^2\alpha\,(\mu-\gamma)^{\beta}}{2}+\frac{D_0\alpha\,\mu(T-\gamma)^{\beta+1}}{(\beta+1)}\right.$$
$$\left.-D_0\mu T\alpha(\mu-\gamma)^{\beta}-\frac{D_0\alpha(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{D_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

Now multiplying the above two brackets and neglecting the terms involving second and higher powers of α (since α is very small) the above equation can be written as

$$Q=D_0\mu T-\frac{D_0\mu^2}{2}+\frac{D_0\mu\alpha\,(T-\gamma)^{\beta+1}}{(\beta+1)}-\frac{D_0\alpha\,(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}$$
$$+\frac{D_0\alpha\,(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)}-D_0\mu\alpha\,T(-\gamma)^{\beta}+\frac{D_0\mu^2\alpha(-\gamma)^{\beta}}{2} \qquad (6)$$

Now the number of units that deteriorate during one cycle is given by

$$D(T) = Q - \left[\int_0^{\mu} D_0 t \; dt + \int_{\mu}^{T} D_0 \mu \; dt \right]$$

$$= Q - \left[\frac{D_0 \mu^2}{2} + D_0 \mu T - D_0 \mu^2 \right] = Q - D_0 \mu T + \frac{D_0 \mu^2}{2}$$

Using the value of Q from equation (6) the above equation becomes

$$D(T) = \frac{D_0 \mu \alpha \; (T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0 \alpha \; (\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 \alpha \; (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - D_0 \mu \alpha \; T(-\gamma)^{\beta} + \frac{D_0 \mu^2 \alpha (-\gamma)^{\beta}}{2}$$

Hence cost due to deterioration during one cycle is given by

$$CD(T) = \frac{CD_0 \mu \alpha \; (T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{CD_0 \alpha \; (\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{CD_0 \alpha \; (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - CD_0 \mu \alpha \; T(-\gamma)^{\beta} + \frac{CD_0 \mu^2 \alpha (-\gamma)^{\beta}}{2}, \qquad (7)$$

Now the salvage value of deteriorated units is given by

$$SV = a\,CD(T) = \frac{a\,CD_0 \mu \alpha \; (T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{a\,CD_0 \alpha \; (\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{aCD_0 \alpha \; (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - aCD_0 \mu \alpha \; T(-\gamma)^{\beta} + \frac{aCD_0 \mu^2 \alpha (-\gamma)^{\beta}}{2} \qquad (8)$$

Putting the value of Q from equation (6) in the equation (4) we get

$$I(t) = \frac{-D_0 t^2}{2} - \frac{D_0 \alpha t \; (t-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0 \alpha (t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 \alpha t^2 \; (t-\gamma)^{\beta}}{2} + D_0 \mu T - \frac{D_0 \mu^2}{2} + \frac{D_0 \alpha \mu (T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0 \alpha (\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - D_0 \mu T \alpha (t-\gamma)^{\beta} + \frac{D_0 \mu^2 \alpha (t-\gamma)^{\beta}}{2}, \quad 0 \le t \le \mu \qquad (9)$$

Total inventory holding cost per cycle is

$$IHC = h \int_0^T I(t) dt = h \left[\int_0^{\mu} I(t) dt + \int_{\mu}^{T} I(t) dt \right]$$

Using the value of $I(t)$ from equation (9) and (5) in the above equation and then integrating we get,

$$IHC = h \left[\frac{-D_0 \mu^3}{6} - \frac{D_0 \alpha \mu (\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3D_0 \alpha (\mu-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} - \frac{3D_0 \alpha (-\gamma)^{\beta+3}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{D_0 \alpha \mu T (-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0 \mu^2 \alpha (-\gamma)^{\beta+1}}{2(\beta+1)} + \frac{D_0 T^2}{2} + \frac{D_0 \mu \alpha T (T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{2D_0 \alpha \mu (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} \right] \qquad (10)$$

Ordering cost per cycle is, $OC = A$ (11)

Total interest earned during one cycle time is

$$IE_1 = PI_e \int_0^{\mu} D_0 t \; dt = \frac{PI_e D_0 \mu^2}{2}, \tag{12}$$

Total interest paid during one cycle time is

$$\begin{aligned} IC_1 &= CI_c \int_{\mu}^{T} I(t) dt \\ &= CI_c \left[\frac{D_0 \mu T^2}{2} - \frac{2D_0 \alpha\mu (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 \mu\alpha T (T-\gamma)^{\beta+1}}{(\beta+1)} - D_0\mu^2 T + \frac{D_0\mu^3}{2} \right. \\ &\left. + \frac{2D_0\alpha\mu(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0\mu\alpha T(\mu-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0\mu^2\alpha\left\{(T-\gamma)^{\beta+1} + (\mu-\gamma)^{\beta+1}\right\}}{(\beta+1)} \right] \end{aligned} \tag{13}$$

Therefore the total average cost per unit time is

$$\begin{aligned} \phi_1(T) &= \frac{1}{T}\left[OC + IHC + CD + IC_1 - IE_1 - SV\right] \\ &= \frac{A}{T} + \frac{h}{T}\left[\frac{-D_0\mu^3}{6} - \frac{D_0\alpha\mu(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3D_0\alpha\left\{(\mu-\gamma)^{\beta+3} - (-\gamma)^{\beta+3}\right\}}{(\beta+1)(\beta+2)(\beta+3)} \right] \\ &\quad + \frac{D_0\alpha\mu h(-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0\mu^2\alpha h(-\gamma)^{\beta+1}}{2(\beta+1)T} + \frac{D_0\mu T h}{2} + \frac{D_0\mu\alpha h(T-\gamma)^{\beta+1}}{(\beta+1)} \\ &\quad - \frac{2D_0\alpha\mu h(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{CD_0\mu\alpha(1-a)(T-\gamma)^{\beta+1}}{(\beta+1)T} - CD_0\mu\alpha(1-a)(-\gamma)^{\beta} \\ &\quad - \frac{CD_0\alpha(1-a)\left\{(\mu-\gamma)^{\beta+2} - (-\gamma)^{\beta+2}\right\}}{(\beta+1)(\beta+2)T} + \frac{CD_0\mu^2\alpha(1-a)(-\gamma)^{\beta}}{2T} \\ &+ CI_c\left[\frac{D_0\mu T}{2} - \frac{2D_0\alpha\mu(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} + \frac{D_0\mu\alpha(T-\gamma)^{\beta+1}}{(\beta+1)} - D_0\mu^2 + \frac{D_0\mu^3}{2T} + \frac{2D_0\alpha\mu(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T} \right. \\ &\left. - \frac{D_0\mu^2\alpha\left\{(T-\gamma)^{\beta+1} - (\mu-\gamma)^{\beta+1}\right\}}{(\beta+1)T} + \frac{D_0\mu\alpha(\mu-\gamma)^{\beta+1}}{(\beta+1)} \right] - \frac{PI_e D_0\mu^2}{2T} \end{aligned} \tag{14}$$

The necessary condition for the total average cost to be minimized is $\frac{\partial \phi_1}{\partial T} = 0$

$$\begin{aligned} &\Rightarrow \frac{-A}{T^2} - \frac{h}{T^2}\left[\frac{-D_0\mu^3}{6} - \frac{D_0\alpha\mu(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3D_0\alpha\left\{(\mu-\gamma)^{\beta+3} - (-\gamma)^{\beta+3}\right\}}{(\beta+1)(\beta+2)(\beta+3)} \right] \\ &+ \frac{D_0\mu^2\alpha h(-\gamma)^{\beta+1}}{2(\beta+1)T^2} + \frac{D_0\mu h}{2} + D_0\mu\alpha h(T-\gamma)^{\beta} - \\ &\frac{2D_0\alpha\mu h}{(\beta+1)(\beta+2)T^2}\left\{T(\beta+2)(T-\gamma)^{\beta+1} - (T-\gamma)^{\beta+2}\right\} \\ &+ \frac{CD_0\mu\alpha(1-a)}{(\beta+1)T^2}\left\{T(\beta+1)(T-\gamma)^{\beta} - (T-\gamma)^{\beta+1}\right\} + \frac{CD_0\alpha(1-a)}{(\beta+1)(\beta+2)T^2}\left\{(\mu-\gamma)^{\beta+2} - (-\gamma)^{\beta+2}\right\} \\ &- \frac{CD_0\mu^2\alpha(1-a)(-\gamma)^{\beta}}{2T^2} + CI_c\left[\frac{D_0\mu}{2} - \frac{2D_0\alpha\mu h}{(\beta+1)(\beta+2)T^2}\left\{T(\beta+2)(T-\gamma)^{\beta+1} - (T-\gamma)^{\beta+2}\right\} \right. \\ &+ D_0\mu\alpha(T-\gamma)^{\beta} - \frac{D_0\mu^3}{2T^2} - \frac{2D_0\alpha\mu(\mu-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T^2} - \frac{D_0\mu^2\alpha}{(\beta+1)T^2}\left\{T(\beta+1)(T-\gamma)^{\beta} - (T-\gamma)^{\beta+1}\right\} \\ &\left. + \frac{D_0\mu^2\alpha(\mu-\gamma)^{\beta+1}}{(\beta+1)T^2} \right] + \frac{PI_e D_0\mu^2}{2T^2} = 0 \end{aligned} \tag{15}$$

The value of T found from equation (15) will minimize the average total variable cost if the second order derivative is positive.

Case 2 When μ > T

Let $I(t)$ denote the on hand inventory of the system at any time t $(0 \le t \le T)$. Let the initial inventory be Q. Depletion due to demand and deterioration occur simultaneously. In this case the permissible delay period for settling accounts is greater than the total cycle time of the system. Hence there is no interest charged during the cycle. The differential equation that describes the instantaneous state of $I(t)$ in the interval $0 \le t \le T$ is given by

$$\frac{dI(t)}{dt} + \theta I(t) = -D_0 t \quad , \qquad 0 \le t \le T \tag{16}$$

Using the given boundary condition $I(0) = Q$ the solution of equation (1) is obtained as

$$\begin{aligned} I(t) = & \frac{-D_0 t^2}{2} - \frac{D_0 \alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0 \alpha (t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + Q + Q\alpha(-\gamma)^{\beta} \\ & - \frac{D_0 \alpha\,(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 t^2 \alpha (t-\gamma)^{\beta}}{2} - Q\alpha (t-\gamma)^{\beta}, \quad 0 \le t \le T \end{aligned} \tag{17}$$

Now using the condition $I(T) = 0$, in equation (17) the value of Q can be found as

$$Q = \frac{D_0 T^2}{2} + \frac{D_0 \alpha T\,(T-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0 \alpha (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{D_0 \alpha (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha(-\gamma)^{\beta} D_0 T^2}{2} \tag{18}$$

Using this value of Q in equation (17) we get

$$\begin{aligned} I(t) = & \frac{-D_0 t^2}{2} - \frac{D_0 \alpha t\,(t-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0 \alpha (t-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{D_0 t^2 \alpha\,(t-\gamma)^{\beta}}{2} \\ & + \frac{D_0 T^2}{2} + \frac{D_0 \alpha T\,(T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{D_0 \alpha (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{D_0 T^2 \alpha (t-\gamma)^{\beta}}{2}, \; 0 \le t \le T \end{aligned} \tag{19}$$

Now the number of units that deteriorate during one cycle is given by

$$\begin{aligned} D(T) &= Q - \int_0^T D_0 t\, dt \;= Q - \frac{D_0 T^2}{2} \\ &= \frac{D_0 \alpha T\,(T-\gamma)^{\beta+1}}{(\beta+1)} + \frac{D_0 \alpha (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{D_0 \alpha (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{D_0 T^2 \alpha (-\gamma)^{\beta}}{2} \end{aligned} \tag{20}$$

Hence cost due to deterioration during one cycle is given by

$$CD(T) = \frac{C D_0 \alpha T\,(T-\gamma)^{\beta+1}}{(\beta+1)} + \frac{C D_0 \alpha (-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{C D_0 \alpha (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{C D_0 T^2 \alpha (-\gamma)^{\beta}}{2} \tag{21}$$

Now the salvage value of deteriorated units is given by

$$SV = a\,CD(T)$$

$$= \frac{aCD_0\alpha T\ (T-\gamma)^{\beta+1}}{(\beta+1)} + \frac{aCD_0\alpha(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{aCD_0\alpha(T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{aCD_0T^2\alpha(-\gamma)^{\beta}}{2} \quad (22)$$

Total inventory holding cost per cycle is

$$IHC = h\int_0^T I(t)dt$$

Using the value of $I(t)$ from equation (19) and then integrating we get,

$$IHC = h\left[\frac{D_0T^3}{3} - \frac{3D_0\alpha T\ (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3D_0\alpha\{(T-\gamma)^{\beta+3}-(-\gamma)^{\beta+3}\}}{(\beta+1)(\beta+2)(\beta+3)} + \frac{D_0\alpha T^2}{(\beta+1)}\left\{(T-\gamma)^{\beta+1} + \frac{(-\gamma)^{\beta+1}}{2}\right\}\right] \quad (23)$$

Ordering cost per cycle is, $OC = A$ (24)

Total interest earned during one cycle time is

$$IE_2 = PI_e\int_0^T D_0t\ dt + PI_eD_0T(\mu - T) = PI_eD_0T\left(\mu - \frac{T}{2}\right), \quad (25)$$

Total interest paid during one cycle time is $IC_2 = 0$ (26)

Therefore the total average cost per unit time is

$$\phi_2(T) = \frac{1}{T}[OC + IHC + CD - SV + IC_2 - IE_2]$$

$$\Rightarrow \phi_2(T) = \frac{1}{T}[OC + IHC + (1-a)CD - IE_2]$$

$$= \frac{A}{T} + \frac{hD_0T^2}{3} - \frac{3hD_0\alpha\ (T-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{3hD_0\alpha\{(T-\gamma)^{\beta+3}-(-\gamma)^{\beta+3}\}}{(\beta+1)(\beta+2)(\beta+3)T}$$

$$+ \frac{hD_0\alpha T}{2(\beta+1)}\{2(T-\gamma)^{\beta+1} + (-\gamma)^{\beta+1}\}$$

$$+ (1-a)CD_0\alpha\left\{\frac{(T-\gamma)^{\beta+1}}{(\beta+1)} - \frac{T(-\gamma)^{\beta}}{2} - \frac{(T-\gamma)^{\beta+2}-(-\gamma)^{\beta+2}}{(\beta+1)(\beta+2)T}\right\} - PI_eD_0\left(\mu - \frac{T}{2}\right) \quad (27)$$

The necessary condition for the total average cost to be minimized is $\frac{\partial\phi_2}{\partial T} = 0$ (28)

The value of T found from equation (28) will minimize the average total variable cost $\phi_2(T)$ if the second order derivative is positive.

To illustrate these we have given a numerical example and a sensitivity analysis in the following section.

NUMERICAL EXAMPLE

Case 1 When μ < T

Let us consider an inventory system with the following parametric values in their proper units.

$[A, C, h, P, I_c, I_e, a, D_0, \mu, \alpha, \beta, \gamma]$

$= [500, 10, 5,\ 100,\ 0.16, 0.12,\ 0.4, 20, 0.2, 0.4, 4, 0.6]$

Using these values in equation (15) we get, T=2.25362. For this value of T the second order derivative found to be 275.674 which is greater than zero. So this value of T will minimize the total variable cost. Putting the optimum values of T in equation (6) and (14) we get, Q=121359 and φ_1=276.624 and respectively.

Case 2 When μ > T

Let us consider an inventory system with the following parametric values in their proper units.

$$[A, C, h, P, I_e, a, \mu, D_0, \alpha, \beta, \gamma] = [2000, 10, 4, 50, 0.24, 0.4, 5, 16, 0.6, 4, 0.4].$$

Using these values in equation (28) we get T=1.82892. For this value of T the second order derivative found to be 1990,33>0. So this value of T will minimize the total variable cost. Putting the optimum values of T in equation (18) and (27) we get, Q=55.5447 and φ_2=493.388 respectively

SENSITIVITY ANALYSIS

For study of sensitivity analysis change in one parameter is considered at a time keeping the other parameters unchanged. The original values of all the parameters for sensitivity analysis have been taken from the example given in section 3 above. Sensitivity analysis is performed by changing the values of all the parameters from -50% to +50%, one by one in the model which are given in the following tables. Table 1 is for Case 1 and Table 2 is for Case 2.

Table 1. When μ < T
Tabela 1. Sytuacja gdy μ < T

Parameter	% change	T	ϕ_1	Q
D_o	-50	2.51187	242.767	8.66642
	-25	2.35809	261.635	10.4626
	0	2.25362	276.624	12.1359
	25	2.17487	289.355	13.7258
	50	2.11181	300.585	15.2529
μ	-50	2.50997	243.814	8.73996
	-25	2.35739	262.386	10.5255
	0	2.25362	276.624	12.1359
	25	2.17511	288.136	13.6092
	50	2.11187	297.666	14.9648
α	-50	2.47713	257.628	12.9967
	-25	2.34356	268.562	12.501
	0	2.25362	276.624	12.1359
	25	2.18654	283.029	11.8403
	50	2.13344	288.347	11.5881
β	-50	2.91949	242.541	16.3787
	-25	2.4794	267.726	15.3033
	0	2.25362	276.624	12.1359
	25	2.11761	287.111	11.5743
	50	2.02787	292.023	10.3358
γ	-50	2.05037	306.614	13.0341
	-25	2.15079	291.113	12.6249
	0	2.25362	276.624	12.1359
	25	2.35877	262.516	11.3801
	50	2.46628	248.029	10.1079

Table 2. When μ > T
Tabela 2. Sytuacja gdy μ > T

Parameter	% change	T	ϕ_2	Q
D_o	-50	2.0411	760.96	36.6079
	-25	1.91591	636.841	40.8599

	0	1.82892	493.388	44.5447
	25	1.76213	338.419	47.8517
	50	1.7078	175.639	50.8782
α	-50	1.98051	420.623	47.3964
	-25	1.89091	461.847	45.6981
	0	1.82892	493.388	44.5447
	25	1.78185	519.144	43.6759
	50	1.7441	541.013	42.9815
β	-50	2.1036	417.688	58.5671
	-25	1.93201	470.805	52.4949
	0	1.82892	493.388	44.5447
	25	1.75678	515.921	40.4415
	50	1.70531	530.929	36.8627
γ	-50	1.71723	576.435	46.1735
	-25	1.77252	533.552	45.3117
	0	1.82892	493.388	44.5447
	25	1.88653	455.204	43.697
	50	1.94556	418.04	42.519

From the table 1 given below we can conclude the following:

The optimal time T increases as γ increases but it decreases as D_o, μ, α, β increases. Again the optimal ordered quantity Q per cycle increases as D_o, μ increases but it decreases as α, β, γ increases. The total average cost ϕ_1 of the system increases as D_o, μ, α, β increases but it decreases as γ increases.

From the table 2 given below we can conclude the following:

The optimal time T increases as γ increases but it decreases as D_o, α, β increases. Again the optimal ordered quantity Q per cycle increases as D_o increases but it decreases as α, β, γ increases. The total average cost ϕ_2 of the system increases as α, β increases but it decreases as γ and D_o increases.

CONCLUSIONS

In the present paper an economic ordered quantity model has been developed for an item considering three parameter Weibull deterioration, ramp type demand, salvage value and permissible delay in payments. The optimal cycle time, optimal ordered quantity per cycle and total optimal cost has been derived for the model. Sensitivity analysis shows how the different parameters affect the optimal cycle time, ordered quantity per cycle and total optimal cost. In both the cases of the present model it can be concluded that to minimise the total cost, it is required to minimize the value of the location parameter whereas we need to maximise the value of scale parameter, shape parameter and the constant coefficient D_o present in the ramp type demand function.

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MODEL EOQ DLA ASORTYMENTÓW ULEGAJĄCYCH NISZCZENIU WEDŁUG SCHEMATU WEIBULLA ZE SPADAJĄCYM POPYTEM I WARTOŚCIĄ REZYDUALNĄ DLA SYSTEMU KREDYTU KUPIECKIEGO

STRESZCZENIE. **Wstęp:** Środowisko rynkowe, na którym działają obecnie firmy charakteryzuje się duża konkurencyjnością i właśnie dla takiego rynku zostały opracowane różne modelu zarządzania zapasem dla asortymentów ulegających zniszczeniu. Opcja opóźnionej płatności w takich modelach jest nowo pojawiającym się elementem. Modele tego typu są bardzo praktyczne w zastosowaniu zarówno dla konsumentów jak i dla wytwórcy.
Metody: W prezentowanej pracy został przedstawiony opracowany model uwzględniający trójparametrowy system dla asortymentów podlegających niszczeniu Weibulla, charakteryzujący się spadającym popytem oraz uwzględniający oferowany kredyt kupiecki. Metoda została zaprezentowana dla jednego artykułu.
Wyniki i wnioski: Optymalna wielkość zamówienia, optymalny czas cyklu zamówienia oraz całkowite koszty zmienne w trakcie trwania cyklu zostały wyliczone dla proponowanego modelu zarządzania zapasem. Otrzymane wyniki zostały dodatkowo zaprezentowane w formie przykładu liczbowego oraz analizy wrażliwości..

Słowa kluczowe: EOQ, obniżenie wartości według schematu Weibulla, popyt spadający, wartość rezydualna, kredyt kupiecki.

DAS EOQ-MODELL FÜR DIE EINEM VERDERB UNTERLIEGENDEN SORTIMENTE NACH DEM WEIBULL-MODELL MIT SINKENDER NACHFRAGE UND RESIDUELLEM WERT FÜR DAS KAUFMANNSKREDIT-SYSTEM

ZUSAMMENFASSUNG. Einleitung: Das Umfeld des Marktes, auf dem gegenwärtig Firmen und Unternehmen aktiv sind, charakterisiert ein großer Wettbewerb und daher gerade für einen solchen Markt wurden unterschiedliche Modelle für Vorratshaltung der einem Verderb unterliegenden Sortimente ausgearbeitet. Das Verfahren einer verzogenen Zahlung stellt bei solchen Modellen ein neues Element dar. Die betreffenden Modelle sind in der Anwendung sowohl für die Verbraucher als auch für die Produzenten sehr brauchbar.

Methoden: Im Rahmen der vorliegenden Arbeit wurde ein konzipiertes Modell, welches das Dreiparameter-System für die dem Verderb nach dem Weibull-Modell unterliegenden Sortimente berücksichtigt, dargestellt. Das Weibull-Modell charakterisiert sich durch eine sinkende Nachfrage und nimmt den angebotenen Kaufmannskredit in Anspruch. Diese Methode wurde anhand nur eines Sortiment-Artikels präsentiert.

Ergebnisse und Fazit: Für das vorgeschlagene Modell der Vorratshaltung wurden die optimale Größe der Bestellung, die optimale Zeit des Bestellungszyklus und die variablen Gesamtkosten innerhalb eines Zyklus berechnet. Die ermittelten Ergebnisse wurden anhand eines zahlmäßigen Beispiels und in Form einer Empfindlichkeitsanalyse dargestellt..

Codewörter: EOQ, Wertsenkung nach dem Weibull-Modells, sinkende Nachfrage, residueller Wert, Kaufmannskredit

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